Name Key Date ________ Period ________

12.1 Small Investment, Big Reward
Exponential Functions

Vocabulary

Define each term in your own words.

1. exponential function

A geometric sequence written in function notation. It gets its name because the variable is in the exponent.

2. half-life

The amount of time it takes a substance to decay to half of its original amount.

Problem Set

Write the explicit formula for each geometric sequence. Then, use the equation to determine the 10th term. Round answers to the nearest thousandth, if necessary.

3.

1	2	3	4	5	6	10
5	15	45	135	405	1,215	98,415

$a_n = 5 \cdot 3^{n-1}$
$a_{10} = 5 \cdot 3^{10-1}$
$= 5 \cdot 3^9$
$= 5 \cdot 19{,}683$
$= 98{,}415$

4.

1	2	3	4	5	6	10
200	100	50	25	12.6	6.25	0.391

$a_n = 200 \cdot (0.5)^{n-1}$
$a_{10} = 200(0.5)^{10-1}$
$= 200(0.5)^9$
$\approx 200(0.001953)$
≈ 0.391

5.

1	2	3	4	5	6	10
1	1.25	1.563	1.953	2.441	3.052	7.451

$a_n = 1 \cdot 1.25^{n-1}$
$a_{10} = 1 \cdot 1.25^{10-1}$
$= 1 \cdot 1.25^9$
≈ 7.451

6.

1	2	3	4	5	6	10
1	0.8	0.64	0.512	0.410	0.328	0.134

$a_n = 1 \cdot 0.8^{n-1}$
$a_{10} = 1 \cdot 0.8^{10-1}$
$= 1 \cdot 0.8^9$
≈ 0.134

7.

1	2	3	4	5	6	10
0.4	0.8	1.6	3.2	6.4	12.8	204.8

$a_n = 0.4 \cdot 2^{n-1}$
$a_{10} = 0.4 \cdot 2^{10-1}$
$= 0.4 \cdot 2^9$
$= 0.4 \cdot 512$
$= 204.8$

8.

1	2	3	4	5	6	10
27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{729}$

$a_n = 27 \cdot \left(\frac{1}{3}\right)^{n-1}$
$a_{10} = 27\left(\frac{1}{3}\right)^{10-1}$
$= 27\left(\frac{1}{3}\right)^9$
$\approx 27(0.00005)$
≈ 0.0014
$= \frac{1}{729}$

Write an exponential function to represent each geometric sequence.
Evaluate the function for the given value of n. Round to the nearest thousandth, if necessary.

9. $a_n = 4 \cdot 2.5^{n-1}$

$n = 10$

$f(n) = 4 \cdot 2.5^{n-1}$

$= 4 \cdot 2.5^n \cdot \left(\frac{5}{2}\right)^{-1}$

$= 4 \cdot 2.5^n \cdot \frac{2}{5}$

$f(n) = 1.6 \cdot 2.5^n$

$f(10) = 1.6 \cdot 2.5^{10}$

$\approx 1.6 \cdot 9536.743$

$\approx 15{,}258.789$

12. $a_n = 0.05 \cdot 1.25^{n-1}$

$n = 24$

$f(n) = .05(1.25)^{n-1}$

$= .05(1.25)^n \cdot (1.25)^{-1}$

$= .05(1.25)^n(.8)$

$f(n) = .04(1.25)^n$

$f(24) = .04(1.25)^{24}$

≈ 8.470

10. $a_n = 0.3 \cdot 8^{n-1}$

$n = 3$

$f(n) = 0.3 \cdot 8^{n-1}$

$= 0.3 \cdot 8^n \cdot 8^{-1}$

$= 0.3 \cdot 8^n \cdot \frac{1}{8}$

$f(n) = 0.0375 \cdot 8^n$

$f(3) = 0.0375 \cdot 8^3$

$= 0.0375 \cdot 512$

$= 19.2$

13. $a_n = 10 \cdot 4^{n-1}$

$n = 7$

$f(n) = 10 \cdot 4^{n-1}$

$= 10 \cdot 4^n \cdot 4^{-1}$

$= 10 \cdot 4^n \cdot \frac{1}{4}$

$f(n) = 2.5 \cdot 4^n$

$f(7) = 2.5 \cdot 4^7$

$= 2.5 \cdot 16{,}384$

$= 40{,}960$

11. $a_n = 150 \cdot 0.8^{n-1}$

$n = 2$

$f(n) = 150 \cdot 0.8^{n-1}$

$= 150 \cdot 0.8^n \cdot 0.8^{-1}$

$= 150 \cdot 0.8^n \cdot \frac{10}{8}$

$f(n) = 187.5 \cdot 0.8^n$

$f(2) = 187.5 \cdot 0.8^2$

$= 187.5 \cdot 0.64$

$= 120$

14. $a_n = 1{,}000 \cdot 0.5^{n-1}$

$n = 5$

$f(n) = 1000(.5)^{n-1}$

$= 1000(.5)^n \cdot (.5)^{-1}$

$= 1000(.5)^n \cdot 2$

$f(n) = 2000(.5)^n$

$f(5) = 2000(.5)^5 = 62.5$

Write an exponential function $A(t)$, where t represents elapsed time, to represent each half-life situation.
Then, use the function to complete each table. Round as necessary.

15.

Elapsed Time (hours)	0	2	4	6	8	20
Drug in Bloodstream (mg)	120	60	30	15	7.5	0.1172
Number of Half-Life Cycles	0	1	2	3	4	10

$A(t) = 120\left(\frac{1}{2}\right)^{\frac{t}{2}}$

$A(20) = 120\left(\frac{1}{2}\right)^{\frac{20}{2}}$

$= 120\left(\frac{1}{2}\right)^{10}$

$\approx 120(0.00098)$

≈ 0.1172

16.

Elapsed Time (minutes)	0	5	10	15	20	100
Bacteria Subject to Growth Inhibitor	800	400	200	100	50	0.000763
Number of Half-Life Cycles	0	1	2	3	4	20

$A(t) = 800\left(\frac{1}{2}\right)^{\frac{t}{5}}$

$A(100) = 800\left(\frac{1}{2}\right)^{\frac{100}{5}}$

$= 800\left(\frac{1}{2}\right)^{20} \approx 800(0.00000095) \approx 0.000763$

17.

Elapsed Time (years)	0	14	21	28	42	56
Strontium in Rock Sample (grams)	16	8	5.657	4	2	1
Number of Half-Life Cycles	0	1	1.5	2	3	4

$A(t) = 16\left(\frac{1}{2}\right)^{\frac{t}{14}}$

$A(21) = 16\left(\frac{1}{2}\right)^{\frac{21}{14}}$

$= 16\left(\frac{1}{2}\right)^{\frac{3}{2}}$

$\approx 16(0.354)$

≈ 5.657

18.

Elapsed Time (years)	0	5,700	11,400	15,675	17,100	22,800
C-14 in Rock Sample (grams)	1	0.5	0.25	0.1487	0.125	0.0625
Number of Half-Life Cycles	0	1	2	2.75	3	4

$A(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{5,700}}$

$A(15,675) = 1\left(\frac{1}{2}\right)^{\frac{15,675}{5,700}}$

$= 1\left(\frac{1}{2}\right)^{\frac{11}{4}}$ $\approx 1(0.1487)$ ≈ 0.1487

19.

Elapsed Time (Days)	0	6	12	18	24	42
Rat Population Exposed to Virus	5000	2500	1250	625	313	39
Number of Half-Life Cycles	0	1	2	3	4	7

$A(t) = 5,000\left(\frac{1}{2}\right)^{\frac{t}{6}}$

$A(42) = 5,000\left(\frac{1}{2}\right)^{\frac{42}{6}}$

$= 5,000\left(\frac{1}{2}\right)^{7}$ $\approx 5,000(0.0078)$ ≈ 39.063

20.

Elapsed Time (Hours)	0	2	4	6	8	16
Participants in Tennis Tournament	256	128	64	32	16	1
Number of Half-Life Cycles	0	1	2	3	4	8

$A(t) = 256\left(\frac{1}{2}\right)^{\frac{t}{2}}$

$A(16) = 256\left(\frac{1}{2}\right)^{\frac{16}{2}}$

$= 256\left(\frac{1}{2}\right)^{8}$

$\approx 256(0.0039)$

$= 1$