Name Key	Date	Period
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12.1 Small Investment, Big Reward Exponential Functions

Vocabulary

Define each term in your own words.

1. exponential function

2. half-life

A geometric sequence written in function notation. It gets its name because the variable is in the exponent.

The amount of time it takes a substance to decay to half of its original amount.

Problem Set

Write the explicit formula for each geometric sequence. Then, use the equation to determine the 10^{th} term. Round answers to the nearest thousandth, if necessary.

3.							
	1	2	3	4	5	6	10
	5	15	45	135 405		1,215	98,415

$a_{10} = 5 \cdot 3^{10-1}$	
$= 5 \cdot 3^9$	
$= 5 \cdot 19,683$	

4.							
	1	2	3	4	5	6	10
	200	100	50	25	12.6	6.25	0.391

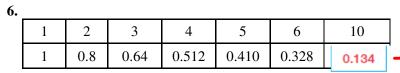
	$a_{10} = 200(0.5)^{10-1}$
\longrightarrow	$= 200(0.5)^9$
	≈ 200(0.001953)

 $a_n = 200 \cdot (0.5)^{n-1}$

 ≈ 0.134

5.							
	1	2	3	4	5	6	10
	1	1.25	1.563	1.953	2.441	3.052	7.451





≈ 7.45 1	a_n	= 1 ·	0.8^{n-1}
	a ₁₀	= 1 ·	0.810-1
		= 1 ·	0.89

7.							
	1	2	3	4	5	6	10
	0.4	0.8	1.6	3.2	6.4	12.8	204.8

$a_n = 0.4 \cdot 2^{n-1}$	
$a_{10} = 0.4 \cdot 2^{10-1}$	
$= 0.4 \cdot 2^9$	
= 0.4 · 512	

8.							
	1	2	3	4	5	6	10
	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	1 729

4.8
$a_n = 27 \cdot \left(\frac{1}{3}\right)^{n-1}$
$a_{10} = 27 \left(\frac{1}{3}\right)^{10-1}$
$= 27 \left(\frac{1}{3}\right)^9$
≈ 27(0.00005)
≈ 0.0014
1

Write an exponential function to represent each geometric sequence. Evaluate the function for the given value of n. Round to the nearest thousandth, if necessary.

9.
$$a_n = 4 \cdot 2.5^{n-1}$$
 $f(n) = 4 \cdot 2.5^{n-1}$
 $n = 10$ $= 4 \cdot 2.5^n \cdot \left| \frac{5}{2} \right|^{-1}$
 $= 4 \cdot 2.5^n \cdot \frac{2}{5}$
 $f(n) = 1.6 \cdot 2.5^n$
 $f(10) = 1.6 \cdot 2.5^{10}$
 $\approx 1.6 \cdot 9536.743$
 $\approx 15,258.789$

10.
$$a_n = 0.3 \cdot 8^{n-1}$$

 $n = 3$ $f(n) = 0.3 \cdot 8^{n-1}$
 $= 0.3 \cdot 8^n \cdot 8^{-1}$
 $= 0.3 \cdot 8^n \cdot \frac{1}{8}$
 $= 0.0375 \cdot 8^n$
 $f(3) = 0.0375 \cdot 8^3$
 $= 0.0375 \cdot 512$
 $= 19.2$

11.
$$a_n = 150 \cdot 0.8^{n-1}$$

 $n = 2$
 $f(n) = 150 \cdot 0.8^{n-1}$
 $= 150 \cdot 0.8^n \cdot 0.8^{-1}$
 $= 150 \cdot 0.8^n \cdot \frac{10}{8}$
 $f(n) = 187.5 \cdot 0.8^n$
 $f(n) = 150 \cdot 0.8^n \cdot 0.8^{-1}$
 $= 150 \cdot 0.8^n \cdot 0.8^n$
 $= 187.5 \cdot 0.8^n$
 $= 187.5 \cdot 0.64$
 $= 120$

12.
$$a_n = 0.05 \cdot 1.25^{n-1}$$
 $n = 24$

$$= .05(1.25)^{n} \cdot (1.25)^{n}$$

$$= .05(1.25)^{n} \cdot (1.$$

14.
$$a_n = 1,000 \cdot 0.5^{n-1}$$
 $f(n) = 1000 (.5)^n$. $(.5)^n$ $= 1000 (.5)^n$.

Write an exponential function A(t), where t represents elapsed time, to represent each half-life situation. Then, use the function to complete each table. Round as necessary.

15.							
	Elapsed Time (hours)	0	2	4	6	8	20
	Drug in Bloodstream (mg)	120	60	30	15	7.5	0.1172
	Number of Half-Life Cycles	0	1	2	3	4	10

$$A(t) = 120 \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$A(20) = 120 \left(\frac{1}{2}\right)^{\frac{20}{2}}$$

$$= 120 \left(\frac{1}{2}\right)^{10}$$

$$= 120(0.00098)$$

$$= 0.1172$$

16.

Elapsed Time (minutes)	0	5	10	15	20	100
Bacteria Subject to Growth Inhibitor	800	400	200	100	50	0.000763
Number of Half-Life Cycles	0	1	2	3	4	20

$$A(t) = 800 \left(\frac{1}{2}\right)^{\frac{1}{5}}$$

$$A(100) = 800 \left(\frac{1}{2}\right)^{\frac{100}{5}}$$

$$= 800 \left(\frac{1}{2}\right)^{20} \approx 800(0.0000095) \approx 0.000763$$