Write an exponential function with the given characteristics.

12. increasing over $(-\infty, \infty)$

    Answers will vary.

    reference point $\left(-1, \dfrac{1}{9}\right)$ $f(x) = 9^x$

13. decreasing over $(-\infty, \infty)$

    Answers will vary.

    $f(x) = \left(\dfrac{2}{3}\right)^x$

    reference point $\left(1, \dfrac{2}{3}\right)$

14. increasing over $(-\infty, \infty)$

    Answers will vary.

    reference point $\left(-3, \dfrac{1}{8}\right)$ $f(x) = 2^x$

15. End behavior $\lim_{x \to -\infty,} f(x) = \infty$

    and

    $\lim_{x \to \infty,} f(x) = 0$ Answers will vary.

    $f(x) = \left(\dfrac{2}{3}\right)^x$

    reference point $\left(-4, \dfrac{81}{16}\right)$

Use the formula for compound interest to determine the amount of money in each account after interest is accrued.

$$I(t) = P\left(1 + \frac{r}{n}\right)^{rt}$$

16. An investor deposits $1,000 in an account that promises 5% interest calculated at the end of each year. How much will be in the account after seven years?

    There will be $1,407.10 in the account after seven years.

    $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

    $A(7) = 1,000\left(1 + \frac{0.05}{1}\right)^{1 \cdot 7}$

    $= 1,000(1.05)^7$

    $\approx 1,407.10$

17. At the start of the school year, Fairview High School deposits PTA dues in an account that offers 3.5% compound interest at the end of a year. If $2500 is collected in PTA dues, how much money will the school have at the start of the next school year?

    The school will have $2,587.50 at the start of the next school year.

    $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

    $A(1) = 2,500\left(1 + \frac{0.035}{1}\right)^{1 \cdot 1}$

    $= 2,500(1.035)^1$

    $= 2,587.50$

18. Kyle put $300 of his birthday money in the bank. The bank compounds interest twice a year at 4%. How much money will Kyle have after three years?

    Kyle will have $337.85 after three years.

    $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

    $A(3) = 300\left(1 + \frac{0.04}{2}\right)^{2 \cdot 3}$

    $= 300(1.02)^6$

    $\approx 300(1.126)$

    ≈ 337.85

19. An investing group has $50,000 to invest. They put the money in an account that compounds interest monthly at a rate 6%. How much money will the group have at the end of 10 years?

    The group will have $90,969.84 at the end of 10 years.

    $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

    $A(10) = 50,000\left(1 + \frac{0.06}{12}\right)^{12 \cdot 10}$

    $= 50,000(1.005)^{120}$

    $\approx 50,000(1.82)$

    $\approx 90,969.84$

20. Interest is compounded quarterly at Money Bank at a rate of 5.5%. A new client opens an account with $7200. How much money will be in the account at the end of six years?

    There will be $9,992.48 in the account after six years.

    $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

    $A(6) = 7,200\left(1 + \frac{0.055}{4}\right)^{4 \cdot 6}$

    $= 7,200(1.01375)^{24}$

    $\approx 7,200(1.3878)$

    $\approx 9,992.48$

21. Sasha wants to earn the maximum interest on her money. She decides to deposit $50 in two different banks for 90 days (3 months) to compare them before she deposits all of her money. She finds a bank that compounds interest daily at 2.2% and another bank that compounds interest monthly at 4.8%. Which bank will earn her more money?

She earns $50.28 from the first bank and $50.60 from the second bank. The bank that compounds interest monthly with a 4.8% interest rate will earn Sasha more money.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$A(0.25) = 50\left(1 + \frac{0.022}{365}\right)^{0.25 \cdot 365}$$

$$\approx 50(1.00006)^{90}$$

$$\approx 50(1.0055)$$

$$\approx 50.28$$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$A(0.25) = 50\left(1 + \frac{0.048}{12}\right)^{0.25 \cdot 12}$$

$$= 50(1.004)^3$$

$$\approx 50(1.012)$$

$$\approx 50.60$$

Use the formula for population growth to predict the population of each city. $N(t) = N_0(e)^{rt}$

22. The population of Austin, Texas is growing 3.9% per year. If the population in 2010 was approximately 790,000, what is the predicted population for 2015?

The population of Austin, Texas will be about 960,096 in 2015.

$$N(t) = N_0 e^{rt}$$

$$N(5) = 790{,}000 e^{(0.039 \cdot 5)}$$

$$= 790{,}000 e^{0.195}$$

$$\approx 960{,}096$$

23. The population of Boston, Massachusetts is growing at a rate of 1.8%. The population in 2013 was approximately 636,500. What is the predicted population for 2025?

The predicted population of Boston, Massachusetts for 2025 is 789,962.

$$N(t) = N_0 e^{rt}$$

$$N(12) = 636{,}500 e^{(0.018 \cdot 12)}$$

$$= 636{,}500 e^{0.216}$$

$$\approx 789{,}962$$

24. The population of Charlotte, North Carolina in 2013 was approximately 775,000. If the rate of growth is about 3.2%, what is an approximation of Charlotte's population in 2000?

In 2000, the population of Charlotte, North Carolina was approximately 511,252 people.

$$N(t) = N_0 e^{rt}$$

$$N(-13) = 775{,}000 e^{(0.032 \cdot -13)}$$

$$= 775{,}000 e^{-0.416}$$

$$\approx 511{,}252$$

25. The population of Beijing, China in 2012 was approximately 20,690,000 and is growing at a rate of about 5.5%. What is an approximation of Beijing's population in 1980?

In 1980, the population of Beijing, China was approximately 3,559,608.

$$N(t) = N_0 e^{rt}$$

$$N(-32) = 20{,}690{,}000 e^{(0.055 \cdot -32)}$$

$$= 20{,}690{,}000 e^{-1.76}$$

$$\approx 3{,}559{,}608$$

26. The population of Detroit, Michigan is decreasing at a rate of about 0.75%. Detroit's population in 2013 was approximately 700,000. What is the predicted population for 2015?

The predicted population for Detroit, Michigan in 2015 is 689,578.

$$N(t) = N_0 e^{rt}$$

$$N(2) = 700{,}000 e^{(-0.0075 \cdot 2)}$$

$$= 700{,}000 e^{-0.015}$$

$$\approx 689{,}578$$

27. The population of Berlin, Germany was about 3,290,000 in 2011. Its population is declining at a rate of about 0.2%. What is the predicted population for 2050?

In 2050, the population for Berlin, Germany will be about 3,043,133 people.

$$N(t) = N_0 e^{rt}$$

$$N(39) = 3{,}290{,}000 e^{(-0.002 \cdot 39)}$$

$$= 3{,}290{,}000 e^{-0.078}$$

$$\approx 3{,}043{,}133$$