Name Key Date _____ Period _____

12.4 Skills Practice: I Feel the Earth Move
Logarithmic Functions

Vocabulary
Write the term that best completes each sentence.

logarithm	logarithmic function	common logarithm	natural logarithm

1. The logarithm of a number for a given base is the exponent to which the base must be raised in order to produce that number.
2. A natural logarithm is a logarithm with base *e*, and is usually written as *ln*.
3. A logarithmic function is a function involving a logarithm.
4. A common logarithm is a logarithm with a base 10 and is usually written without a base specified.

Problem Set
Write each exponential equation as a corresponding logarithmic equation.

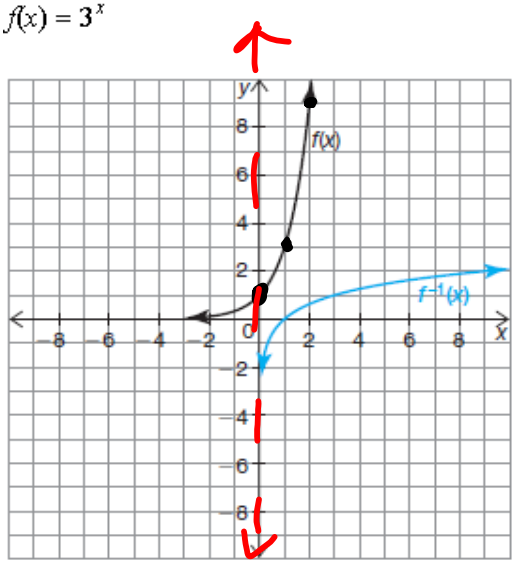
5. $3^2 = 9$ $\log_3(9) = 2$
6. $5^4 = 625$ $\log_5(625) = 4$
7. $4^{-3} = \frac{1}{64}$ $\log_4\left(\frac{1}{64}\right) = -3$
8. $10^{-5} = \frac{1}{100,000}$ $\log\left(\frac{1}{100,000}\right) = -5$
9. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$ $\log_{\frac{1}{2}}\left(\frac{1}{32}\right) = 5$
10. $\left(\frac{1}{11}\right)^{-2} = 121$ $\log_{\frac{1}{11}}(121) = -2$

Write each logarithmic equation as a corresponding exponential equation.

11. $\log_7\left(\frac{1}{49}\right) = -2$ $7^{-2} = \frac{1}{49}$
12. $\log_{\frac{1}{3}}\left(\frac{1}{729}\right) = 6$ $\left(\frac{1}{3}\right)^6 = \frac{1}{729}$
13. $\log_2(128) = 7$ $2^7 = 128$
14. $\log_6\left(\frac{1}{1296}\right) = -4$ $6^{-4} = \frac{1}{1296}$
15. $\log_{\frac{1}{5}}\left(\frac{1}{125}\right) = 3$ $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$
16. $\log_9(729) = 3$ $9^3 = 729$

Graph the inverse of each exponential function $f(x)$. Then, describe the domain, range, asymptotes, and end behavior of the inverse.

17. $f(x) = 3^x$



Domain: $x > 0$ or $(0, \infty)$

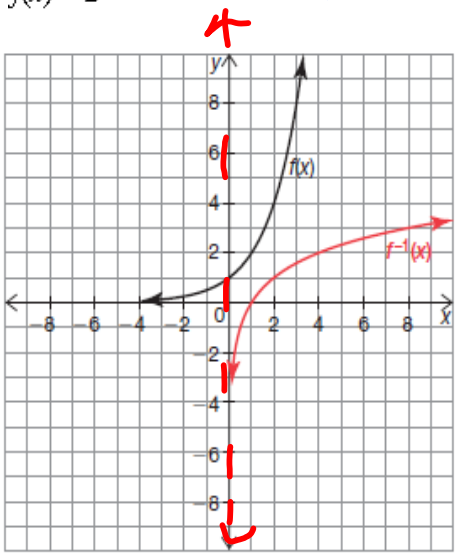
Range: All real numbers or $(-\infty, \infty)$

Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptotic behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

18. $f(x) = 2^x$



Domain: $x > 0$ or $(0, \infty)$

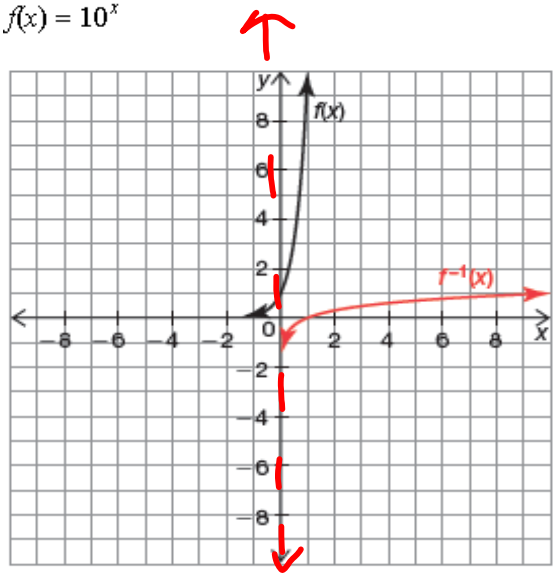
Range: All real numbers or $(-\infty, \infty)$

Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptotic behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

19. $f(x) = 10^x$



Domain: $x > 0$ $(0, \infty)$

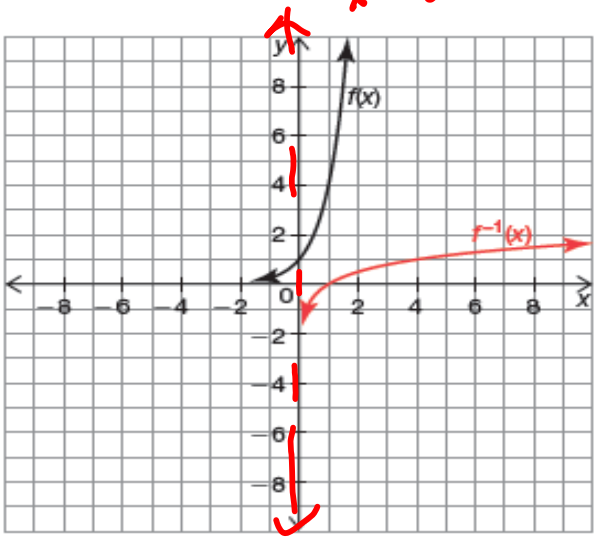
Range: All real numbers $(-\infty, \infty)$

Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptotic behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

20. $f(x) = 4^x$



Domain: $x > 0$ or $(0, \infty)$

Range: All real numbers $(-\infty, \infty)$

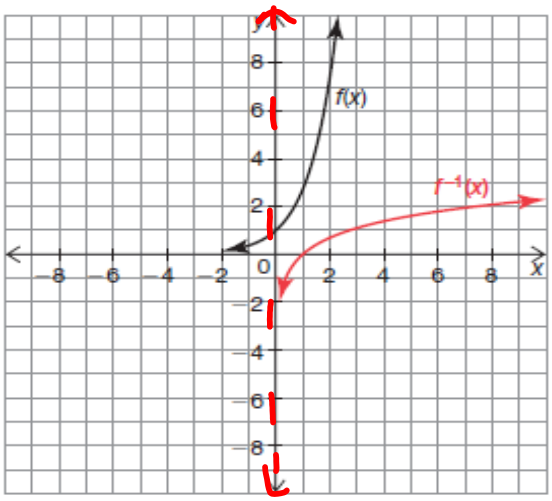
Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptotic behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

21. $f(x) = e^x$

$f(x) = e^x$



Domain: $x > 0$ or $(0, \infty)$

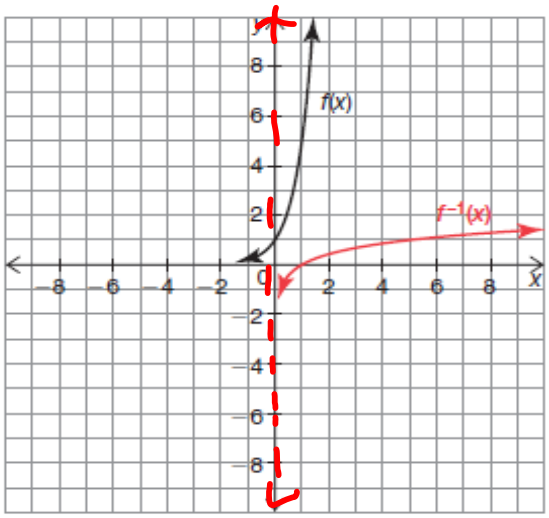
Range: All real numbers or $(-\infty, \infty)$

Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = +\infty$

Asymptotic behavior $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

22. $f(x) = 5^x$



Domain: $x > 0$ or $(0, \infty)$

Range: All real numbers or $(-\infty, \infty)$

Asymptotes: $x = 0$

End behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptotic behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

Solve each logarithmic equation.

23. $-2 = \log_9\left(\frac{1}{b}\right)$

$\frac{1}{b} = 9^{-2}$

$\frac{1}{b} = \frac{1}{81}$

$b = 81$

24. $-0.903 \approx x \cdot \log(0.5)$

$-0.903 \approx x \cdot -0.301$

$\frac{-0.903}{-0.301} = x$

$3 = x$

25. $\frac{1}{2} = \log_n(3)$

$n^{\left[\frac{1}{2}\right]} = 3$

$n = 3^2$

$n = 9$

26. $2.398 \approx \log b$

$10^{2.398} \approx b$

$250 \approx b$

27. $0.058 \approx \ln z$

$e^{0.058} \approx z$

$1.06 \approx z$

28. $-1.349 = \frac{1}{2}\log\left(\frac{g}{1000}\right)$

$2 \cdot (-1.349) = \log\left(\frac{g}{1000}\right)$

$-2.698 = \log\left(\frac{g}{1000}\right)$

$10^{-2.698} = \frac{g}{1000}$

$0.002 \approx \frac{g}{1000}$

$2 \approx g$

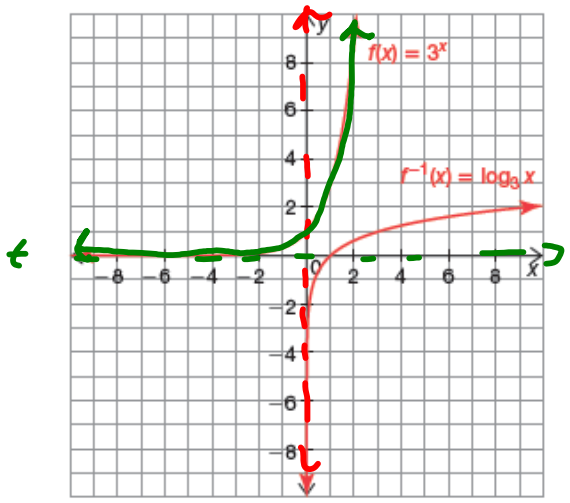
I Feel the Earth Move
Logarithmic Functions

1. Given: $f(x) = 3^x$.

   a. Write the function $f^{-1}(x)$, the inverse of $f(x) = 3^x$.

   The inverse of $f(x) = 3^x$ is $f^{-1}(x) = \log_3 x$.

   b. Graph and label the functions $f(x)$ and $f^{-1}(x)$ on the coordinate plane.



   c. Describe how to calculate $f^{-1}(3)$ without a calculator. Then, calculate $f^{-1}(3)$, $f^{-1}(9)$, and $f^{-1}(27)$.

   I know that $f^{-1}(3) = \log_3 3$, which can be written in exponential form as $3^y = 3$. Therefore, y must be equal to 1 and $f^{-1}(3) = 1$.

$f^{-1}(3) = \log_3 3$

$= 1$

$f^{-1}(9) = \log_3 9$

$= 2$

$f^{-1}(27) = \log_3 27$

$= 3$

   d. Determine the domain, range, asymptotes, intercepts, end behavior, and intervals of increase and decrease for $f^{-1}(x)$.

asymptote behavior

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

Intercept: $(1, 0)$

End Behavior: $\lim_{x \to \infty} f^{-1}(x) = \infty$

Asymptote Behavior: $\lim_{x \to 0^+} f^{-1}(x) = -\infty$

Increasing on the Interval $(0, \infty)$