More Than Meets the Eye

Transformations of Logarithmic Functions

12.5

LEARNING GOALS

In this lesson, you will:

- Dilate, reflect, and translate logarithmic functions using reference points.
- Investigate graphs of logarithmic functions through intercepts, asymptotes, intervals of increase and decrease, and end behavior.

'Practice makes perfect'!

When you practice the same motion over and over, you are building up procedural memory in your brain that instructs your muscles to perform a task. The more often your muscles receive those same instructions, the more quickly and efficiently they are able to carry them out until they become like second nature to you. Athletes use this idea of "muscle memory" when conditioning their bodies to perform, and musicians use it to train their fingers to hit the correct keys or strings accurately.

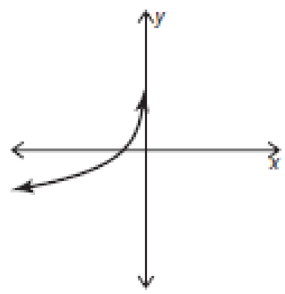
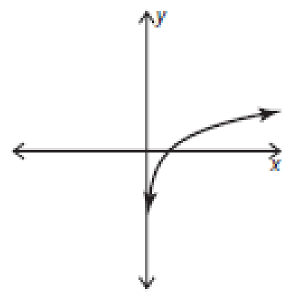
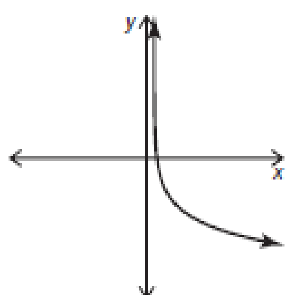
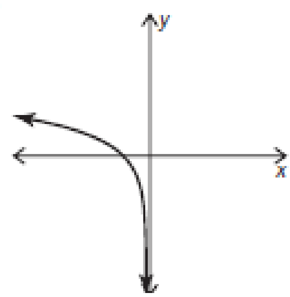
The best way to train your body and mind when learning a new skill is to practice it slowly at first to be sure that your technique is perfect, and then repeat that same quality practice as often as possible. Break the skill or information up into pieces and work on it, a piece at a time, until it is committed to memory. Once you've learned all of the parts, continue to practice, practice, practice until your new skill becomes an ingrained habit.

What skills do you like to practice?

PROBLEM 1 Don't Flip Out! It's Just a Reflection

1. The two tables show four logarithmic functions and four logarithmic graphs. Match the logarithmic function to its corresponding graph, and write the function under the graph it represents.

Logarithmic Functions	
$f(x) = \log_2(x)$	$g(x) = -\log_2(x)$
$h(x) = \log_2(-x)$	$j(x) = -\log_2(-x)$

Logarithmic Graphs	
A.  $j(x) = -\log_2(-x)$	B.  $f(x) = \log_2(x)$
C.  $g(x) = -\log_2(x)$	D.  $h(x) = \log_2(-x)$

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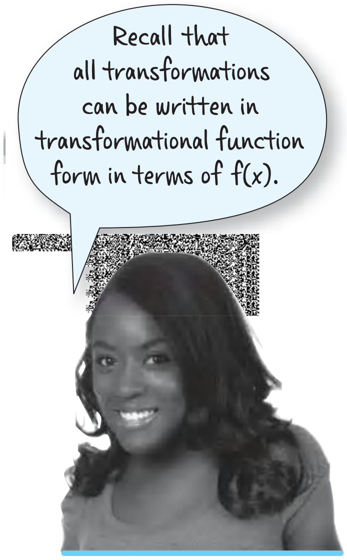
2. Analyze the graphs of $f(x)$, $g(x)$, $h(x)$, and $j(x)$. Write an equation for each function $g(x)$, $h(x)$, and $j(x)$ in terms of $f(x)$. Describe each transformation on $f(x)$.



$g(x) = -\log x$
$h(x) = \log(-x)$
$j(x) = -\log(-x)$

Logarithmic Function	Transformation on $f(x)$
$g(x) = -f(x)$	Reflection across the x axis
$h(x) = f(-x)$	Reflection across the y axis
$j(x) = -f(-x)$	Reflection across the x axis and y axis

3. Analyze the key characteristics of each logarithmic function. What similarities exist among the four functions?

$(-\infty, \infty)$

For the four logarithmic functions, the range is the set of all real numbers. Each function has a vertical asymptote at $x = 0$.

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4. What generalizations can you make about the effects of these transformations on the domain and range of a logarithmic function?

The domain is the set of all positive numbers for the logarithm of x. The domain for the logarithm of the opposite of x is the set of all negative numbers. The domain will never equal zero, regardless of how the graph of the logarithmic function is reflected, because there is a vertical asymptote at $x = 0$.

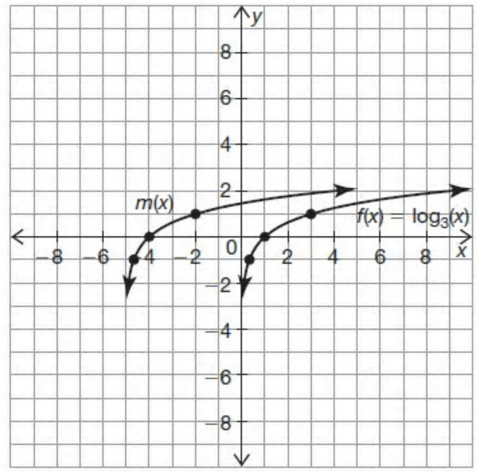
The range will always be the set of all real numbers for any basic logarithmic function, regardless of reflection.

PROBLEM 2 You've Got Some Moves!

1. Analyze the graphs of $f(x)$ and the transformed function. Describe the transformations produced on $f(x)$ to create the transformed function. Then, write an equation for the transformed function in terms of $f(x)$. For each set of points shown on $f(x)$, the corresponding points are shown on the transformed function.

   a. Describe the transformation:

   horizontally translate left 5 units



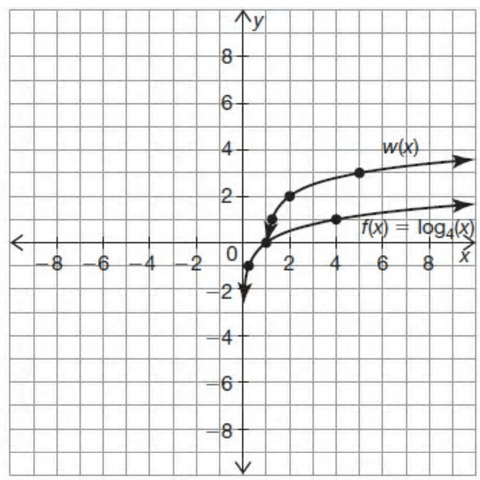
   Transformed function form:

   $m(x) = \log_3(x+5)$

   $m(x) =$ $f(x + 5)$

   b. Describe the transformation:

   horizontally translate right 1 unit,
   vertically translate up 2 units



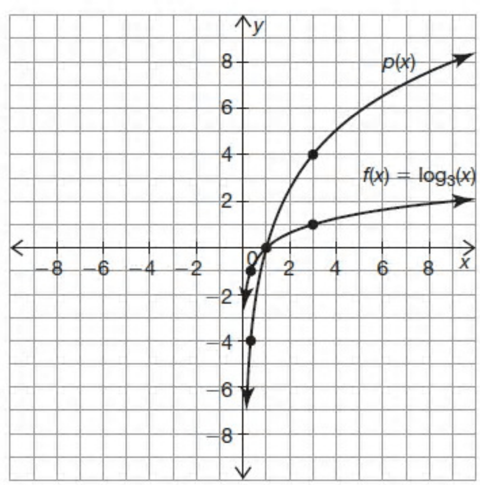
   Transformed function form:

   $W(x) = \log_4(x-1) + 2$

   $w(x) =$ $f(x - 1) + 2$

   c. Describe the transformation:

   vertical dilation by a factor of 4
   Stretch



   Transformed function form:

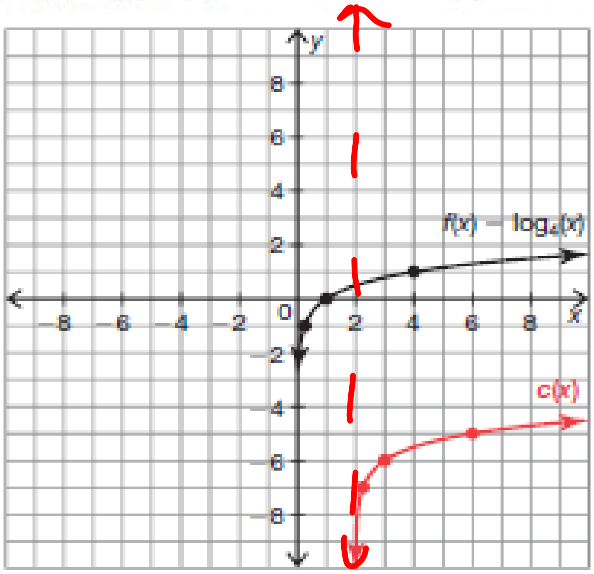
   $p(x) = 4\log_3(x)$

   $p(x) =$ $4f(x)$

2. The graph of a basic logarithmic function *f(x)* is shown. Graph each transformation of *f(x)*. State the domain, range, and asymptotes of your graph.

Write the logarithmic function described by the transformations below:

a. $c(x) = f(x - 2) - 6$



$c(x) = \log_4(x-2) - 6$

Domain of $c(x)$: $(2, \infty)$

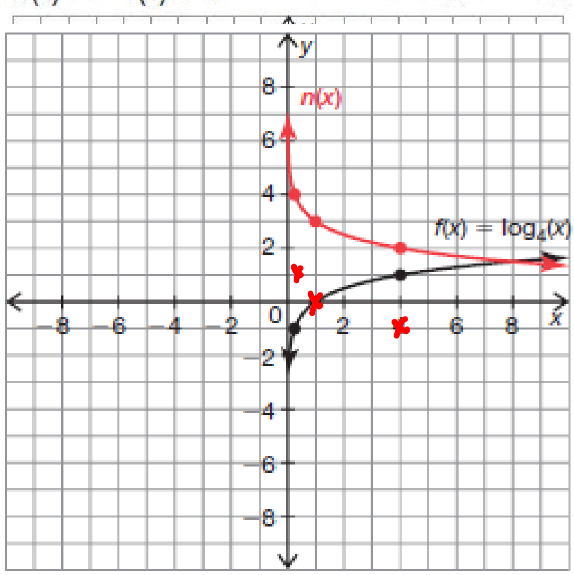
Range of $c(x)$: $(-\infty, \infty)$

Asymptote of $c(x)$: $x = 2$

End Behavior: $\lim_{x\to\infty} c(x) = \infty$

Asymptotic Behavior: $\lim_{x\to 2^+} c(x) = -\infty$

b. $n(x) = -f(x) + 3$



$n(x) = -\log_4(x) + 3$

Domain of $n(x)$: $(0, \infty)$

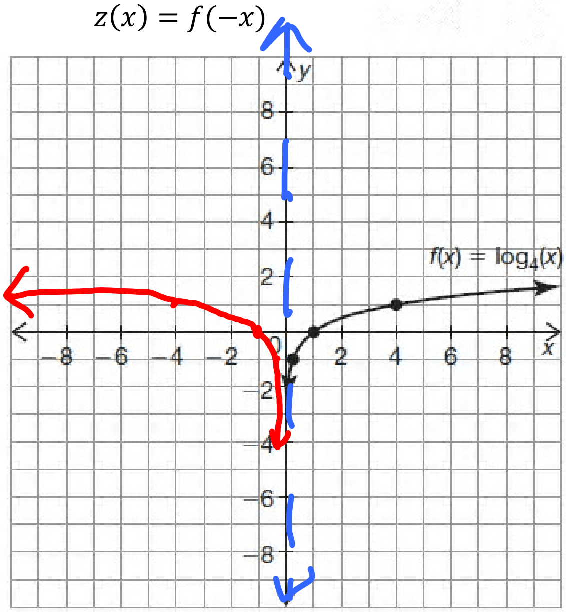
Range of $n(x)$: $(-\infty, \infty)$

Asymptote of $n(x)$: $x = 0$

End Behavior: $\lim_{x\to\infty} n(x) = -\infty$

Asymptotic Behavior: $\lim_{x\to 0^+} n(x) = +\infty$

c. $z(x) = f(-x)$



$z(x) = \log_4(-x)$

Domain of $z(x)$: $(-\infty, 0)$

Range of $z(x)$: $(-\infty, \infty)$

Asymptote of $z(x)$: $x = 0$

End Behavior: $\lim_{x\to -\infty} z(x) = +\infty$

Asymptotic Behavior: $\lim_{x\to 0^-} z(x) = -\infty$

3. Write a transformed logarithmic function in terms of $f(x)$ with the characteristic(s) given. Then, graph the transformed function.

a. $f(x) = \log_2(x)$

vertical asymptote at $x = -3$

[redacted] $\log_2(-x-3)$ o/r

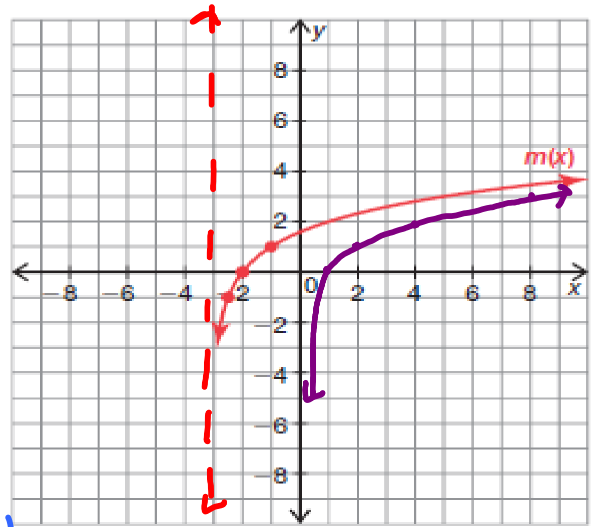
$-\log_2(-(x+3))$

Transformed function form: $m(x) = \log_2(x+3)$

$m(x) =$ $f(x+3)$

or $m(x) = \log_2(-x+5)$

$(-(x+1))$



c. $f(x) = \log_5(x)$

Domain: $(-\infty, 0)$

Transformed function form: $k(x) = \log_5(-x)$

[redacted]

$k(x) =$ $f(-x)$