12.5 More Than Meets the Eye
Transformations of Logarithmic Functions

Analyze the graphs of \( f(x) \) and \( g(x) \). Describe the transformations performed on the graph of \( f(x) \) to produce the graph of the transformed function \( g(x) \). Then, write an equation for \( g(x) \).

1. The graph of \( g(x) \) was horizontally translated right 4 units to produce the graph of \( g(x) \).
   \[ g(x) = \log_2(x - 4) \]

2. The graph of \( g(x) \) was vertically translated up 3 units to produce the graph of \( g(x) \).
   \[ g(x) = \log_2(x) + 3 \]

3. The graph of \( g(x) \) was horizontally translated left 2 units and vertically translated down 5 units to produce the graph of \( g(x) \).
   \[ g(x) = \log_2(x + 2) - 5 \]

4. The graph of \( g(x) \) was reflected over the \( x \)-axis and horizontally translated up 3 units to produce the graph of \( g(x) \).
   \[ g(x) = -\log_2(x + 5) + 3 \]

5. The graph of \( g(x) \) was reflected over the \( x \)-axis and horizontally translated right 3 units to produce the graph of \( g(x) \).
   \[ g(x) = -\log_2(x - 3) \]

6. Hint: This one has dilation.

The graph of \( g(x) \) was stretched vertically by a factor of 6 to produce the graph of \( g(x) \).
   \[ g(x) = 6 \log_2(x) \]
The graph of \( f(x) = \log(x) \) is shown. Use the graph of \( f(x) \) to sketch the transformed function \( m(x) \) on the coordinate plane. Then, state the domain, range, transformed function and asymptotes of \( m(x) \).

8. \( m(x) = f(x - 4) \)
   - Domain of \( m(x) \): \((4, \infty)\)
   - Range of \( m(x) \): \((0, \infty)\)
   - Asymptote of \( m(x) \): \( x = 4 \)
   - Transformed function: \( m(x) = \log(x-4) \)
   - End Behavior \( m(x) \): \( \lim_{x \to \infty} m(x) = \infty \)
   - Asymptotic Behavior \( m(x) \): \( \lim_{x \to 4^-} m(x) = -\infty \)

9. \( m(x) = f(x + 1) - 3 \)
   - Domain of \( m(x) \): \((-1, \infty)\)
   - Range of \( m(x) \): \((0, \infty)\)
   - Asymptote of \( m(x) \): \( x = -1 \)
   - Transformed function: \( m(x) = \log(x+1) - 3 \)
   - End Behavior \( m(x) \): \( \lim_{x \to \infty} m(x) = \infty \)
   - Asymptotic Behavior \( m(x) \): \( \lim_{x \to -1^+} m(x) = -\infty \)

10. \( m(x) = f(-x) + 5 \)
    - Domain of \( m(x) \): \((-\infty, \infty)\)
    - Range of \( m(x) \): \((-\infty, \infty)\)
    - Asymptote of \( m(x) \):
    - Transformed function: \( m(x) = \log(-x) + 5 \)
    - End Behavior \( m(x) \): \( \lim_{x \to \infty} m(x) = \infty \)
    - Asymptotic Behavior \( m(x) \): \( \lim_{x \to 0^+} m(x) = -\infty \)

11. \( m(x) = -f(x - 2) + 2 \)
    - Domain of \( m(x) \): \((2, \infty)\)
    - Range of \( m(x) \): \((0, \infty)\)
    - Asymptote of \( m(x) \): \( x = 2 \)
    - Transformed function: \( m(x) = -\log(x-2) + 2 \)
    - End Behavior \( m(x) \): \( \lim_{x \to \infty} m(x) = -\infty \)
    - Asymptotic Behavior \( m(x) \): \( \lim_{x \to 2^+} m(x) = -\infty \)

12. \( m(x) = f(x + 5) \)
    - Domain of \( m(x) \): \((-5, \infty)\)
    - Range of \( m(x) \): \((-\infty, \infty)\)
    - Asymptote of \( m(x) \): \( x = -5 \)
    - Transformed function: \( m(x) = \log(x + 5) \)
    - End Behavior \( m(x) \): \( \lim_{x \to \infty} m(x) = \infty \)
    - Asymptotic Behavior \( m(x) \): \( \lim_{x \to -5^+} m(x) = -\infty \)
Write a transformed logarithmic function, \( c(x) \), in terms of \( f(x) = \log_2(x) \) with the characteristics given.

13. vertical asymptote at \( x = 6 \)  
    \[ c(x) = f(x - 6) \]  

14. domain of \( (-\infty, 0) \)  
    \[ c(x) = f(-x) \]

15. \( x \rightarrow y \)  

<table>
<thead>
<tr>
<th>Reference Points on ( f(x) )</th>
<th>( \rightarrow )</th>
<th>Corresponding Points on ( c(x) )</th>
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<tbody>
<tr>
<td>( \frac{1}{2}, -1 )</td>
<td>( \rightarrow )</td>
<td>( \left( \frac{1}{2}, -3 \right) )</td>
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<td>( 1, 0 )</td>
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<td>( (1, 0) )</td>
</tr>
<tr>
<td>( 2, 1 )</td>
<td>( \rightarrow )</td>
<td>( (2, 3) )</td>
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\[ c(x) = 3f(x) \]

Consider the function \( y = f(x) \) and the transformed function \( g(x) \). Write an equation for \( g^{-1}(x) \) in terms of \( f^{-1}(x) \).

19. \( g(x) = f(x) + 3 \)  
    \[ g^{-1}(x) = f^{-1}(x - 3) \]

20. \( g(x) = f(x - 2) \)

21. \( g(x) = f(x + 6) \)

22. \( g(x) = f(x) - 5 \)

23. \( g(x) = f(x + 1) - 3 \)

24. \( g(x) = f(x - 4) + 2 \)