

TRIGONOMETRY LESSON 11

PART 1 MULTIPLICATION & DIVISION IDENTITIES

Answers

$$\begin{aligned} 1) \quad & \cot x \tan x \\ & = \left(\frac{\cos x}{\sin x}\right) \left(\frac{\sin x}{\cos x}\right) \\ & = 1 \end{aligned}$$

$$\begin{aligned} 2) \quad & \csc x \cos x \\ & = \left(\frac{1}{\sin x}\right) \cos x \\ & = \frac{\cos x}{\sin x} \\ & = \cot x \end{aligned}$$

$$\begin{aligned} 3) \quad & \frac{\sin x}{\tan x} \\ & = \frac{\sin x}{\frac{\sin x}{\cos x}} \\ & = \sin x \times \frac{\cos x}{\sin x} \\ & = \cos x \end{aligned}$$

$$\begin{aligned} 4) \quad & \frac{1}{\cot x \cos x \tan x} \\ & = \frac{1}{\frac{\cos x}{\sin x} \cos x \frac{\sin x}{\cos x}} \\ & = \frac{1}{\cos x} \\ & = \sec x \end{aligned}$$

$$\begin{aligned} 5) \quad & \frac{\tan x}{\csc x} \\ & = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\sin x}} \\ & = \frac{\sin x}{\cos x} \times \frac{\sin x}{1} \\ & = \frac{\sin^2 x}{\cos x} \end{aligned}$$

$$\begin{aligned} 6) \quad & \frac{\tan x}{\sec x} \\ & = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \\ & = \frac{\sin x}{\cos x} \times \frac{\cos x}{1} \\ & = \sin x \end{aligned}$$

$$\begin{aligned} 7) \quad & \frac{\cos^2 x}{\cot x} \\ & = \frac{\cos^2 x}{\frac{\cos x}{\sin x}} \\ & = \cos^2 x \times \frac{\sin x}{\cos x} \\ & = \cos x \sin x \end{aligned}$$

$$\begin{aligned} 8) \quad & \frac{\sec x \csc x}{\cot x} \\ & = \frac{\frac{1}{\cos x} \frac{1}{\sin x}}{\frac{\cos x}{\sin x}} \\ & = \frac{1}{\cos x \sin x} \times \frac{\sin x}{\cos x} \\ & = \frac{1}{\cos^2 x} \\ & = \sec^2 x \end{aligned}$$

$$\begin{aligned} 9) \quad & \frac{\sec x \csc x}{\csc^2 x} \\ & = \frac{\sec x}{\csc x} \\ & = \frac{1}{\frac{1}{\sin x}} \\ & = \frac{1}{\cos x} \times \frac{\sin x}{1} \\ & = \frac{\sin x}{\cos x} \\ & = \tan x \end{aligned}$$

$$\begin{aligned} 10) \quad & \frac{\tan^2 x \cos x}{2 \sec x} \\ & = \frac{\frac{\sin^2 x}{\cos^2 x} \cos x}{\frac{2}{\cos x}} \\ & = \frac{\sin^2 x}{\cos x} \times \frac{\cos x}{2} \\ & = \frac{1}{2} \sin^2 x \end{aligned}$$

TRIGONOMETRY LESSON 11

PART II ADDITION & SUBTRACTION IDENTITIES

$$\begin{aligned}
 1) \quad \sec x - \sin x &= \frac{1}{\cos x} - \frac{\sin x}{1} \\
 &= \frac{1}{\cos x} - \left(\frac{\sin x}{1}\right) \frac{\cos x}{\cos x} \\
 &= \frac{1}{\cos x} - \frac{\sin x \cos x}{\cos x} \\
 &= \frac{1 - \sin x \cos x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \sin x + \tan x \sin x &= \sin x + \left(\frac{\sin x}{\cos x}\right) \sin x \\
 &= \frac{\sin x}{1} + \frac{\sin^2 x}{\cos x} \\
 &= \left(\frac{\sin x}{1}\right) \frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x} \\
 &= \frac{\sin x \cos x}{\cos x} + \frac{\sin^2 x}{\cos x} \\
 &= \frac{\sin x \cos x + \sin^2 x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \sec^2 x + \cot x &= \frac{1}{\cos^2 x} + \frac{\cos x}{\sin x} \\
 &= \frac{1}{\cos^2 x} \left(\frac{\sin x}{\sin x}\right) + \frac{\cos x}{\sin x} \left(\frac{\cos^2 x}{\cos^2 x}\right) \\
 &= \frac{\sin x}{\cos^2 x \sin x} + \frac{\cos^3 x}{\cos^2 x \sin x} \\
 &= \frac{\sin x + \cos^3 x}{\cos^2 x \sin x}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \csc^2 x - \tan x &= \frac{1}{\sin^2 x} - \frac{\sin x}{\cos x} \\
 &= \frac{1}{\sin^2 x} \left(\frac{\cos x}{\cos x}\right) - \frac{\sin x}{\cos x} \left(\frac{\sin^2 x}{\sin^2 x}\right) \\
 &= \frac{\cos x}{\sin^2 x \cos x} - \frac{\sin^3 x}{\sin^2 x \cos x} \\
 &= \frac{\cos x - \sin^3 x}{\sin^2 x \cos x}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad \csc x - \sec x &= \frac{1}{\sin x} - \frac{1}{\cos x} \\
 &= \left(\frac{1}{\sin x}\right) \frac{\cos x}{\cos x} - \left(\frac{1}{\cos x}\right) \frac{\sin x}{\sin x} \\
 &= \frac{\cos x}{\sin x \cos x} - \frac{\sin x}{\sin x \cos x} \\
 &= \frac{\cos x - \sin x}{\sin x \cos x}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad \sec x - \tan x &= \frac{1}{\cos x} - \frac{\sin x}{\cos x} \\
 &= \frac{1 - \sin x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad \cos x + \tan x &= \cos x + \frac{\sin x}{\cos x} \\
 &= \frac{\cos x \left(\frac{\cos x}{\cos x}\right) + \sin x}{\cos x} \\
 &= \frac{\cos^2 x + \sin x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad \cot x + \sin x &= \frac{\cos x}{\sin x} + \sin x \\
 &= \frac{\cos x}{\sin x} + \left(\frac{\sin x}{1}\right) \frac{\sin x}{\sin x} \\
 &= \frac{\cos x}{\sin x} + \left(\frac{\sin x}{1}\right) \frac{\sin x}{\sin x} \\
 &= \frac{\cos x}{\sin x} + \frac{\sin^2 x}{\sin x} \\
 &= \frac{\cos x + \sin^2 x}{\sin x}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad 1 + \tan x &= 1 + \frac{\sin x}{\cos x} \\
 &= \left(\frac{1}{1}\right) \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x} \\
 &= \frac{\cos x + \sin x}{\cos x} \\
 &= \frac{\cos x + \sin x}{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad \csc x + 1 &= \frac{1}{\sin x} + \frac{\sin x}{\sin x} \\
 &= \frac{1 + \sin x}{\sin x}
 \end{aligned}$$

TRIGONOMETRY LESSON 11

PART III THREE SPECIAL IDENTITIES

$$\begin{aligned}
 1) \quad & \sec x - \tan x \sin x \\
 &= \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right) \sin x \\
 &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\
 &= \frac{1 - \sin^2 x}{\cos x} \\
 &= \frac{\cos^2 x}{\cos x} \\
 &= \cos x
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \cos x + \tan x \sin x \\
 &= \cos x + \left(\frac{\sin x}{\cos x}\right) \sin x \\
 &= \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} \\
 &= \left(\frac{\cos x}{1}\right) \frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x} \\
 &= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \\
 &= \frac{1}{\cos x} \\
 &= \sec x
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \tan x + \cot x \\
 &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\
 &= \left(\frac{\sin x}{\cos x}\right) \frac{\sin x}{\sin x} + \left(\frac{\cos x}{\sin x}\right) \frac{\cos x}{\cos x} \\
 &= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} \\
 &= \frac{1}{\sin x \cos x} \\
 &= \sec x \csc x
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & 1 + \tan^2 x \\
 &= 1 + \frac{\sin^2 x}{\cos^2 x} \\
 &= \left(\frac{1}{1}\right) \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \\
 &= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} \\
 &= \frac{1}{\cos^2 x} \\
 &= \sec^2 x
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \sec x - \cos x \\
 &= \frac{1}{\cos x} - \cos x \\
 &= \frac{1}{\cos x} - \left(\frac{\cos x}{1}\right) \frac{\cos x}{\cos x} \\
 &= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \\
 &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \frac{\sin x}{\cos x} \sin x \\
 &= \tan x \sin x
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & \sin x + \cot x \cos x \\
 &= \sin x + \left(\frac{\cos x}{\sin x}\right) \cos x \\
 &= \sin x + \frac{\cos^2 x}{\sin x} \\
 &= \left(\frac{\sin x}{1}\right) \frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x} \\
 &= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} \\
 &= \frac{1}{\sin x} \\
 &= \csc x
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & \sec^2 x - 1 \\
 &= \frac{1}{\cos^2 x} - 1 \\
 &= \frac{1}{\cos^2 x} - \left(\frac{1}{1}\right) \frac{\cos^2 x}{\cos^2 x} \\
 &= \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \\
 &= \frac{1 - \cos^2 x}{\cos^2 x} \\
 &= \frac{\sin^2 x}{\cos^2 x} \\
 &= \sin^2 x \left(\frac{1}{\cos^2 x}\right) \\
 &= \sin^2 x \sec^2 x
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & 1 - \csc^2 x \\
 &= 1 - \frac{1}{\sin^2 x} \\
 &= \left(\frac{1}{1}\right) \frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} \\
 &= \frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} \\
 &= \frac{\sin^2 x - 1}{\sin^2 x} \\
 &= \frac{-\cos^2 x}{\sin^2 x} \\
 &= -\cot^2 x
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & \csc x - \sin x \\
 &= \frac{1}{\sin x} - \sin x \\
 &= \frac{1}{\sin x} - \left(\frac{\sin x}{1}\right) \frac{\sin x}{\sin x} \\
 &= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \\
 &= \frac{1 - \sin^2 x}{\sin x} \\
 &= \frac{\cos^2 x}{\sin x} \\
 &= \left(\frac{\cos x}{\sin x}\right) \cos x \\
 &= \cot x \cos x
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & 1 - \sec^2 x \\
 &= 1 - \frac{1}{\cos^2 x} \\
 &= \left(\frac{1}{1}\right) \frac{\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \\
 &= \frac{\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} \\
 &= \frac{\cos^2 x - 1}{\cos^2 x} \\
 &= \frac{-\sin^2 x}{\cos^2 x} \\
 &= -\tan^2 x
 \end{aligned}$$

TRIGONOMETRY LESSON 11

PART IV COMPOUND FRACTIONS & SPECIAL IDENTITIES

$$\begin{aligned}
 1) \quad & \frac{\sec x}{\cot x + \tan x} \\
 &= \frac{1}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \\
 &= \frac{1}{\frac{(\cos x)\cos x + (\sin x)\sin x}{\sin x \cos x}} \\
 &= \frac{1}{\cos^2 x + \sin^2 x} \\
 &= \frac{1}{\cos^2 x + \sin^2 x} \times \frac{\sin x \cos x}{\sin x \cos x} \\
 &= \frac{1}{1} \times \frac{\sin x \cos x}{\sin x \cos x} \\
 &= \sin x
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \frac{\sin x + \cos x}{\sec x + \csc x} \\
 &= \frac{\sin x + \cos x}{\frac{1}{\cos x} + \frac{1}{\sin x}} \\
 &= \frac{\sin x + \cos x}{\frac{(\cos x)\sin x + (\sin x)\cos x}{\cos x \sin x}} \\
 &= \frac{(\frac{1}{\cos x})\sin x + (\frac{1}{\sin x})\cos x}{\sin x + \cos x} \\
 &= \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\sin x + \cos x} \\
 &= \frac{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}}{\sin x + \cos x} \\
 &= \frac{1}{\cos x \sin x} \times \frac{\cos x \sin x}{\sin x + \cos x} \\
 &= \cos x \sin x
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{\sin x + \tan x}{\cos x + 1} \\
 &= \frac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1} \\
 &= \frac{(\frac{\sin x}{1})\cos x + \frac{\sin x}{\cos x}}{\cos x + 1} \\
 &= \frac{\sin x \cos x + \sin x}{\cos x + 1} \\
 &= \frac{\sin x \cos x + \sin x}{\cos x + 1} \times \frac{1}{\cos x + 1} \\
 &= \frac{\sin x (\cos x + 1)}{\cos x + 1} \times \frac{1}{\cos x + 1} \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \frac{\tan x - \sin x}{\tan x \sin x} \\
 &= \frac{\frac{\sin x}{\cos x} - \sin x}{\left(\frac{\sin x}{\cos x}\right)\sin x} \\
 &= \frac{\frac{\sin x}{\cos x} - \left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}} \\
 &= \frac{\frac{\sin x}{\cos x} - \frac{\sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}} \\
 &= \frac{\frac{\sin x - \sin x \cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}} \\
 &= \frac{\sin x - \sin x \cos x}{\sin^2 x} \times \frac{\cos x}{\cos x} \\
 &= \frac{\sin x(1 - \cos x)}{\cos x} \times \frac{\cos x}{\sin^2 x} \\
 &= \frac{1 - \cos x}{\sin x}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \frac{\cos x - \csc x}{\sin x - \sec x} \\
 &= \frac{\cos x - \frac{1}{\sin x}}{\sin x - \frac{1}{\cos x}} \\
 &= \frac{\left(\frac{\cos x}{1}\right)\sin x - \frac{1}{\sin x}}{\left(\frac{\sin x}{1}\right)\cos x - \frac{1}{\cos x}} \\
 &= \frac{\cos x \sin x - 1}{\sin x \cos x - 1} \\
 &= \frac{\cos x \sin x - 1}{\cos x \sin x - 1} \times \frac{\cos x}{\cos x} \\
 &= \frac{\cos x}{\sin x} \\
 &= \cot x
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & \frac{1 + \cos x}{\tan x + \sin x} \\
 &= \frac{1 + \cos x}{\frac{\sin x}{\cos x} + \sin x} \\
 &= \frac{1 + \cos x}{\frac{\sin x}{\cos x} + \left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}} \\
 &= \frac{1 + \cos x}{\frac{\sin x + \sin x \cos x}{\cos x}} \\
 &= \frac{1 + \cos x}{\sin x + \sin x \cos x} \times \frac{\cos x}{\cos x} \\
 &= \frac{1 + \cos x}{\sin x(1 + \cos x)} \times \frac{\cos x}{\cos x} \\
 &= \frac{\cos x}{\sin x} \\
 &= \cot x
 \end{aligned}$$

TRIGONOMETRY LESSON 11

PART IV COMPOUND FRACTIONS & SPECIAL IDENTITIES

$$\begin{aligned}
 7) \quad & \frac{1 + \tan^2 x}{1 + \cot^2 x} \\
 &= \frac{\sec^2 x}{\csc^2 x} \\
 &= \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} \\
 &= \frac{1}{\cos^2 x} \cdot \sin^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} \\
 &= \tan^2 x
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & \frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} \\
 &= \cos^2 x + \sin^2 x \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & \frac{1 + \tan x}{1 + \cot x} \\
 &= \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}} \\
 &= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} \\
 &= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & \frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} \\
 &= \frac{\cos x (\sec x + 1)}{\sec x - 1 (\sec x + 1)} + \frac{\cos x (\sec x - 1)}{\sec x + 1 (\sec x - 1)} \\
 &= \frac{\cos x \sec x + \cos x}{(\sec x - 1)(\sec x + 1)} + \frac{\cos x \sec x - \cos x}{(\sec x + 1)(\sec x - 1)} \\
 &= \frac{\cos x \sec x + \cos x + \cos x \sec x - \cos x}{(\sec x - 1)(\sec x + 1)} \\
 &= \frac{2 \cos x \sec x}{(\sec x - 1)(\sec x + 1)} \\
 &= \frac{2 \cos x \sec x}{(\sec^2 x - 1)} \\
 &= \frac{2 \cos x \sec x}{\tan^2 x} \\
 &= \frac{2 \cos x \cdot \frac{1}{\cos x}}{\tan^2 x} \\
 &= \frac{2}{\tan^2 x} \\
 &= 2 \cot^2 x
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & \frac{\tan x}{1 + \tan x} \\
 &= \frac{\frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} \\
 &= \frac{\frac{\sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} \\
 &= \frac{\sin x}{\cos x + \sin x} \\
 &= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\cos x + \sin x} \\
 &= \frac{\sin x}{\cos x + \sin x}
 \end{aligned}$$

$$\begin{aligned}
 12) \quad & \frac{\sin^2 x}{1 - \sin x} + \frac{\sin^2 x}{1 + \sin x} \\
 &= \frac{\sin^2 x (1 + \sin x)}{1 - \sin x (1 + \sin x)} + \frac{\sin^2 x (1 - \sin x)}{1 + \sin x (1 - \sin x)} \\
 &= \frac{\sin^2 x + \sin^3 x}{(1 - \sin x)(1 + \sin x)} + \frac{\sin^2 x - \sin^3 x}{(1 + \sin x)(1 - \sin x)} \\
 &= \frac{\sin^2 x + \sin^3 x + \sin^2 x - \sin^3 x}{(1 - \sin x)(1 + \sin x)} \\
 &= \frac{2 \sin^2 x}{(1 - \sin x)(1 + \sin x)} \\
 &= \frac{2 \sin^2 x}{(1 - \sin^2 x)} \\
 &= \frac{2 \sin^2 x}{\cos^2 x} \\
 &= 2 \tan^2 x
 \end{aligned}$$

TRIGONOMETRY LESSON 11

PART V OTHER PROOFS

$$\begin{aligned}
 1) \quad & \frac{3 \tan x}{1 + \tan^2 x} \\
 &= \frac{3 \tan x}{\sec^2 x} \\
 &= \frac{3 \sin x}{\cos^2 x} \\
 &= \frac{3 \sin x}{\cos x} \times \frac{\cos x}{\cos x} \\
 &= 3 \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{1}{1 + \cot^2 x} \\
 &= \frac{1}{\csc^2 x} \\
 &= \frac{1}{\frac{1}{\sin^2 x}} \\
 &= 1 \times \frac{\sin^2 x}{1} \\
 &= \sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \sec^2 x - \cos^2 x - \sin^2 x \\
 &= \sec^2 x - (\cos^2 x + \sin^2 x) \\
 &= \sec^2 x - 1 \\
 &= \tan^2 x
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & (\sin x + \cos x)^2 + (\sin x - \cos x)^2 \\
 &= \sin^2 x + 2 \sin x \cos x + \cos^2 x + \sin^2 x - 2 \sin x \cos x + \cos^2 x \\
 &= 2 \sin^2 x + 2 \cos^2 x \\
 &= 2(\sin^2 x + \cos^2 x) \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & (1 + \sin x)^2 + \cos^2 x \\
 &= 1 + 2 \sin x + \sin^2 x + \cos^2 x \\
 &= 1 + 2 \sin x + 1 \\
 &= 2 + 2 \sin x \\
 &= 2(1 + \sin x)
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & \sin^4 x - \cos^4 x \\
 &= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) \\
 &= (\sin^2 x - \cos^2 x) \\
 &= \sin^2 x - (1 - \sin^2 x) \\
 &= \sin^2 x - 1 + \sin^2 x \\
 &= 2 \sin^2 x - 1
 \end{aligned}$$

$$\begin{aligned}
 7) \quad & (\tan x - 1)^2 \\
 &= (\tan x - 1)(\tan x - 1) \\
 &= \tan^2 x - 2 \tan x + 1 \\
 &= \tan^2 x + 1 - 2 \tan x \\
 &= \sec^2 x - 2 \tan x
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & (1 - \sec^2 x)(1 - \sin^2 x) \\
 &= (-\tan^2 x)(\cos^2 x) \\
 &= -\frac{\sin^2 x}{\cos^2 x} \cos^2 x \\
 &= -\sin^2 x
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & \csc x - \csc^3 x \\
 &= \csc x(1 - \csc^2 x) \\
 &= \csc x(-\cot^2 x) \\
 &= \frac{1}{\sin x} \left(-\frac{\cos^2 x}{\sin^2 x} \right) \\
 &= -\frac{\cos^2 x}{\sin^3 x}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & \csc^4 x - 1 \\
 &= (\csc^2 x - 1)(\csc^2 x + 1) \\
 &= \cot^2 x(\csc^2 x + 1) \\
 &= \frac{\cos^2 x}{\sin^2 x} \left(\frac{1}{\sin^2 x} + 1 \right) \\
 &= \frac{\cos^2 x}{\sin^4 x} + \frac{\cos^2 x}{\sin^2 x} \\
 &= \frac{\cos^2 x}{\sin^4 x} + \frac{\cos^2 x}{\sin^2 x} \left(\frac{\sin^2 x}{\sin^2 x} \right) \\
 &= \frac{\cos^2 x}{\sin^4 x} + \frac{\cos^2 x \sin^2 x}{\sin^4 x} \\
 &= \frac{\cos^2 x + \cos^2 x \sin^2 x}{\sin^4 x} \\
 &= \frac{\cos^2 x(1 + \sin^2 x)}{\sin^4 x}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & \tan^2 x - \cot^2 x \\
 &= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} \\
 &= \frac{\sin^2 x}{\cos^2 x} \left(\frac{\sin^2 x}{\sin^2 x} \right) - \frac{\cos^2 x}{\sin^2 x} \left(\frac{\cos^2 x}{\cos^2 x} \right) \\
 &= \frac{\sin^4 x}{\cos^2 x \sin^2 x} - \frac{\cos^4 x}{\sin^2 x \cos^2 x} \\
 &= \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} \\
 &= \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x} \\
 &= \frac{(\sin^2 x - \cos^2 x)}{\cos^2 x \sin^2 x}
 \end{aligned}$$

TRIGONOMETRY LESSON 11

PART VI CONJUGATES

$$\begin{aligned} 1) \quad & \frac{\cos x}{1 - \sin x} \\ &= \frac{\cos x}{1 - \sin x} \left(\frac{1 + \sin x}{1 + \sin x} \right) \\ &= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} \\ &= \frac{\cancel{\cos x}(1 + \sin x)}{\cos^2 x} \\ &= \frac{(1 + \sin x)}{\cos x} \end{aligned}$$

$$\begin{aligned} 2) \quad & \frac{1}{1 - \sin x} \\ &= \frac{1}{1 - \sin x} \left(\frac{1 + \sin x}{1 + \sin x} \right) \\ &= \frac{1 + \sin x}{1 - \sin^2 x} \\ &= \frac{1 + \sin x}{\cos^2 x} \end{aligned}$$

$$\begin{aligned} 3) \quad & \frac{1 - \cos x}{\sin x} \\ &= \frac{1 - \cos x}{\sin x} \left(\frac{1 + \cos x}{1 + \cos x} \right) \\ &= \frac{1 - \cos^2 x}{\sin x(1 + \cos x)} \\ &= \frac{\sin^2 x}{\sin x(1 + \cos x)} \\ &= \frac{\sin x}{1 + \cos x} \end{aligned}$$

$$\begin{aligned} 4) \quad & \frac{1 - \sin x}{\cos x} \\ &= \frac{1 - \sin x}{\cos x} \left(\frac{1 + \sin x}{1 + \sin x} \right) \\ &= \frac{1 - \sin^2 x}{\cos x(1 + \sin x)} \\ &= \frac{\cos^2 x}{\cancel{\cos x}(1 + \sin x)} \\ &= \frac{\cos x}{1 + \sin x} \end{aligned}$$

