LESSON 13.2 Skills Practice

Mad Props
Properties of Logarithms

Name Key

Rewrite each logarithmic expression in **expanded** form using the properties of logarithms.

1. $\log_3(5x)$

$\log_3(5x) = \log_3 5 + \log_3 x$

Product

2. $\log_5\left(\frac{a}{b}\right)$

$\log_5\left(\frac{a}{b}\right) = \log_5 a - \log_5 b$

Quotient

3. $\log_7(n^4)$

$\log_7(n^4) = 4\log_7 n$

Power

4. $\log\left(\frac{x}{7}\right)$

$\log\left(\frac{x}{7}\right) = \log x - \log 7$

Quotient

5. $\log_2(mn)$

$\log_2(mn) = \log_2 m + \log_2 n$

Product

6. $\log(p^q)$

$\log(p^q) = q\log p$

Power

7. $\ln(x^2)$

$\ln(x^2) = 2\ln x$

Power

8. $\ln\left(\frac{c}{3}\right)$

$\ln\left(\frac{c}{3}\right) = \ln c - \ln 3$

Quotient

9. $\log_3(7x^2)$

$\log_3(7x^2) = \log_3 7 + 2\log_3 x$

Product & Power

10. $\ln(2x^3y^2)$

$\ln(2x^3y^2) = \ln 2 + 3\ln x + 2\ln y$

Product + Power

11. $\log\left(\frac{xy}{5}\right)$

$\log\left(\frac{xy}{5}\right) = \log x + \log y - \log 5$

Product & Quotient

12. $\log_7\left(\frac{3x^4}{y}\right)$

$\log_7\left(\frac{3x^4}{y}\right) = \log_7 3 + 4\log_7 x - \log_7 y$

Product, Power + Quotient

13. $\ln\left(\frac{x}{7y}\right)$

$\ln\left(\frac{x}{7y}\right) = \ln x - \ln 7 - \ln y$

14. $\log_5\left(\frac{7x^2}{y^3}\right)$

$\log_5\left(\frac{7x^2}{y^3}\right) = \log_5 7 + 2\log_5 x - 3\log_5 y$

15. $\log(xyz)$

$\log(xyz) = \log x + \log y + \log z$

16. $\ln\left(\frac{x+1}{(y+3)^2}\right)$

$\ln\left(\frac{x+1}{(y+3)^2}\right)$

$= \ln(x+1) - 2\ln(y+3)$

Rewrite each logarithmic expression as a **single logarithm**.

17. $\log x - 2\log y$

$\log x - 2\log y = \boxed{\log\left(\frac{x}{y^2}\right)}$

18. $3\log_4 x + \log_4 y - \log_4 z$

$3\log_4 x + \log_4 y - \log_4 z = \boxed{\log_4\left(\frac{x^3y}{z}\right)}$

19. $6\log_2 x - 2\log_2 x$

$6\log_2 x - 2\log_2 x = \log_2\left(\frac{x^6}{x^2}\right) = \boxed{\log_2(x^4)}$

20. $\log 3 + 2\log 7 - \log 6$

$\log 3 + 2\log 7 - \log 6 = \log\left(\frac{3(7)^2}{6}\right)$

$\boxed{= \log\left(\frac{147}{6}\right) = \log 24.5 = 1.3892}$

21. $\log x + 3\log y - \frac{1}{2}\log z$

$\log x + \cancel{3}\log y^3 - \cancel{\frac{1}{2}}\log z^{1/2} = \boxed{\log\left(\frac{xy^3}{z^{\frac{1}{2}}}\right)}$ or $\log\left(\frac{xy^3}{\sqrt{z}}\right)$

22. $7\log_3 x - (2\log_3 x + 5\log_3 y)$

$7\log_3 x - (2\log_3 x + 5\log_3 y)$

$= \log_3\left(\frac{x^7}{x^2y^5}\right) = \boxed{\log_3\left(\frac{x^5}{y^5}\right)}$

23. $2\ln(2x+3) - 4\ln(y-2)$

$2\ln(2x+3) - 4\ln(y-2) = \boxed{\ln\left(\frac{(2x+3)^2}{(y-2)^4}\right)}$

24. $\ln(x-7) - 2(\ln x + \ln y)$

$\ln(x-7) - 2(\ln x + \ln y) = \boxed{\ln\left(\frac{x-7}{x^2y^2}\right)}$

25. Use the properties of logarithms to rewrite each logarithmic expression in **expanded form**.

a. $\log_3(ab^2c^{\cancel{3}\,2})$

$\log_3(ab^2c^3) = \log_3 a + \log_3(b^2) + \log_3(c^3)$

$= \log_3 a + 2\log_3 b + \cancel{3}\,2\log_3 c$

b. $\log\left(\frac{x^3}{5y^2}\right)$

$\log\left(\frac{x^3}{5y^2}\right) = \log(x^3) - \log 5 - \log(y^2)$

$= 3\log x - \log 5 - 2\log y$ or

$3\log x - (\log 5 + 2\log y)$

c. $\log_2(6mn^4)$

$\log_2(6mn^4) = \log_2 6 + \log_2 m + \log_2(n^4)$

$= \log_2 6 + \log_2 m + 4\log_2 n$

d. $\ln\left(\frac{2x}{y^{10}}\right)$

$\ln\left(\frac{2x}{y^{10}}\right) = \ln 2 + \ln x - \ln(y^{10})$

$= \ln 2 + \ln x - 10\ln y$

26. Use the properties of logarithms to rewrite each logarithmic expression as a single logarithm.

a. $2\log_5 3 - \log_5 y$

$2\log_5 3 - \log_5 y = \log_5(3^2) - \log_5 y$

$\boxed{= \log_5\left(\frac{9}{y}\right)}$

b. $7\ln x + \ln 8 - 3\ln y$

$7\ln x + \ln 8 - 3\ln y = \ln(x^7) + \ln 8 - \ln(y^3)$

$\boxed{= \ln\left(\frac{8x^7}{y^3}\right)}$

c. $2(\log 5 + \log m) - \log(m^3)$

$2(\log 5 + \log m) - \log(m^3) = 2\log 5 + 2\log m - 3\log m$

$= \log(5^2) - \log m$

$\boxed{= \log\left(\frac{25}{m}\right)}$

d. $8\log_2 x - 3(\log_2 y + 2\log_2 x)$

$8\log_2 x - 3(\log_2 y + 2\log_2 x) = 8\log_2 x - 3\log_2 y - 6\log_2 x$

$= 2\log_2 x - 3\log_2 y$

$= \log_2(x^2) - \log_2(y^3)$

$\boxed{= \log_2\left(\frac{x^2}{y^3}\right)}$

27. An earthquake's magnitude, M, can be determined using the formula $M = \log\left(\frac{I}{10^{-4}}\right)$, where I represents the intensity of the earthquake. Rewrite the logarithmic expression in the formula in **expanded form**.

$M = \log\left(\frac{I}{10^{-4}}\right)$

$= \log I - \log 10^{-4}$

$= \log I + 4\log 10$

$\boxed{= \log I + 4}$

or $M = \log(I\,10^4)$

$= \log I + 4\log 10 = \boxed{\log I + 4}$

28. The loudness, L, of a sound, in decibels, can be determined using the formula $L = 10\log\left(\frac{I}{10^{-12}}\right)$, where I represents the intensity of the sound. Rewrite the logarithmic expression in the formula in **expanded form**.

$L = 10\log\left(\frac{I}{10^{-12}}\right)$

$= 10(\log I - \log 10^{-12})$

$= 10\log I + 120\log 10$

$\boxed{= 10\log I + 120}$

OR

$L = 10\log(I\,10^{12})$

$10\log I + 10\log 10^{12}$

$10\log I + 120\log 10$