

LESSON 13.2 Skills Practice

Mad Props
Properties of Logarithms

Name Key

Rewrite each logarithmic expression in **expanded** form using the properties of logarithms.

1. $\log_3(5x)$

$\log_3(5x) = \log_3 5 + \log_3 x$

2. $\log_5\left(\frac{a}{b}\right)$

$\log_5\left(\frac{a}{b}\right) = \log_5 a - \log_5 b$

3. $\log_7(n^4)$

$\log_7(n^4) = 4 \log_7 n$

4. $\log\left(\frac{x}{7}\right)$

$\log\left(\frac{x}{7}\right) = \log x - \log 7$

5. $\log_2(mn)$

$\log_2(mn) = \log_2 m + \log_2 n$

6. $\log(p^q)$

$\log(p^q) = q \log p$

7. $\ln(x^2)$

$\ln(x^2) = 2 \ln x$

8. $\ln\left(\frac{c}{3}\right)$

$\ln\left(\frac{c}{3}\right) = \ln c - \ln 3$

9. $\log_3(7x^2)$

$\log_3(7x^2) = \log_3 7 + 2 \log_3 x$

10. $\ln(2x^3y^2)$

$\ln(2x^3y^2) = \ln 2 + 3 \ln x + 2 \ln y$

11. $\log\left(\frac{xy}{5}\right)$

$\log\left(\frac{xy}{5}\right) = \log x + \log y - \log 5$

12. $\log_7\left(\frac{3x^4}{y}\right)$

$\log_7\left(\frac{3x^4}{y}\right) = \log_7 3 + 4 \log_7 x - \log_7 y$

13. $\ln\left(\frac{x}{7y}\right)$

[Redacted]

14. $\log_5\left(\frac{7x^2}{y^3}\right)$

[Redacted]

15. $\log(xyz)$

[Redacted]

16. $\ln\left(\frac{x+1}{(y+3)^2}\right)$

[Redacted]

Rewrite each logarithmic expression as a **single logarithm**.

17. $\log x - 2 \log y$

$$\log x - 2 \log y = \log \left(\frac{x}{y^2} \right)$$

18. $3 \log_4 x + \log_4 y - \log_4 z$

$$3 \log_4 x + \log_4 y - \log_4 z = \log_4 \left(\frac{x^3 y}{z} \right)$$

19. $6 \log_2 x - 2 \log_2 x$

$$6 \log_2 x - 2 \log_2 x = \log_2 \left(\frac{x^6}{x^2} \right) = \log_2 (x^4)$$

20. $\log 3 + 2 \log 7 - \log 6$

$$\begin{aligned} \log 3 + 2 \log 7 - \log 6 &= \log \left(\frac{3(7)^2}{6} \right) \\ &= \log \left(\frac{147}{6} \right) = \log 24.5 = 1.3892 \end{aligned}$$

21. $\log x + 3 \log y - \frac{1}{2} \log z$

$$\log x + 3 \log y - \frac{1}{2} \log z = \log \left(\frac{xy^3}{z^{\frac{1}{2}}} \right)$$

22. $7 \log_3 x - (2 \log_3 x + 5 \log_3 y)$

$$\begin{aligned} 7 \log_3 x - (2 \log_3 x + 5 \log_3 y) \\ = \log_3 \left(\frac{x^7}{x^2 y^5} \right) = \log_3 \left(\frac{x^5}{y^5} \right) \end{aligned}$$

23. $2 \ln (2x + 3) - 4 \ln (y - 2)$

$$2 \ln (2x + 3) - 4 \ln (y - 2) = \ln \left(\frac{(2x + 3)^2}{(y - 2)^4} \right)$$

24. $\ln (x - 7) - 2(\ln x + \ln y)$

$$\ln (x - 7) - 2(\ln x + \ln y) = \ln \left(\frac{x - 7}{x^2 y^2} \right)$$

25. Use the properties of logarithms to rewrite each logarithmic expression in **expanded form**.

a. $\log_3 (ab^2c^3)$

$$\begin{aligned} \log_3 (ab^2c^3) &= \log_3 a + \log_3 (b^2) + \log_3 (c^3) \\ &= \log_3 a + 2 \log_3 b + 3 \log_3 c \end{aligned}$$

b. $\log \left(\frac{x^3}{5y^2} \right)$

$$\begin{aligned} \log \left(\frac{x^3}{5y^2} \right) &= \log (x^3) - \log 5 - \log (y^2) \\ &= 3 \log x - \log 5 - 2 \log y \end{aligned}$$

c. $\log_2 (6mn^4)$

$$\begin{aligned} \log_2 (6mn^4) &= \log_2 6 + \log_2 m + \log_2 (n^4) \\ &= \log_2 6 + \log_2 m + 4 \log_2 n \end{aligned}$$

d. $\ln \left(\frac{2x}{y^{10}} \right)$

$$\begin{aligned} \ln \left(\frac{2x}{y^{10}} \right) &= \ln 2 + \ln x - \ln (y^{10}) \\ &= \ln 2 + \ln x - 10 \ln y \end{aligned}$$

26. Use the properties of logarithms to rewrite each logarithmic expression as a single logarithm.

a. $2 \log_5 3 - \log_5 y$

$$\begin{aligned} 2 \log_5 3 - \log_5 y &= \log_5 (3^2) - \log_5 y \\ &= \log_5 \left(\frac{9}{y} \right) \end{aligned}$$

b. $7 \ln x + \ln 8 - 3 \ln y$

$$\begin{aligned} 7 \ln x + \ln 8 - 3 \ln y &= \ln (x^7) + \ln 8 - \ln (y^3) \\ &= \ln \left(\frac{8x^7}{y^3} \right) \end{aligned}$$

c. $2(\log 5 + \log m) - \log (m^3)$

$$\begin{aligned} 2(\log 5 + \log m) - \log (m^3) &= 2 \log 5 + 2 \log m - 3 \log m \\ &= \log (5^2) - \log m \\ &= \log \left(\frac{25}{m} \right) \end{aligned}$$

d. $8 \log_2 x - 3(\log_2 y + 2 \log_2 x)$

$$\begin{aligned} 8 \log_2 x - 3(\log_2 y + 2 \log_2 x) &= 8 \log_2 x - 3 \log_2 y - 6 \log_2 x \\ &= 2 \log_2 x - 3 \log_2 y \\ &= \log_2 (x^2) - \log_2 (y^3) \end{aligned}$$

$$= \log_2 \left(\frac{x^2}{y^3} \right)$$

27. An earthquake's magnitude, M , can be determined using the formula $M = \log \left[\frac{I}{10^{-4}} \right]$, where I represents the intensity of the earthquake. Rewrite the logarithmic expression in the formula in **expanded form**.

$$\begin{aligned} M &= \log \left(\frac{I}{10^{-4}} \right) = \log I + 4 \log 10 \\ &= \log I - \log 10^{-4} = \log I + 4 \end{aligned}$$

or $M = \log (I 10^4)$
 $= \log I + 4 \log 10 = \log I + 4$

28. The loudness, L , of a sound, in decibels, can be determined using the formula $L = 10 \log \left[\frac{I}{10^{-12}} \right]$, where I represents the intensity of the sound. Rewrite the logarithmic expression in the formula in **expanded form**.

$$\begin{aligned} L &= 10 \log \left(\frac{I}{10^{-12}} \right) = 10 \log I + 120 \log 10 \\ &= 10(\log I - \log 10^{-12}) = 10 \log I + 120 \end{aligned}$$

OR $L = 10 \log (I 10^{12})$
 $= 10 \log I + 10 \log 10^{12}$
 $= 10 \log I + 120 \log 10$