1 Trigonometric Functions

Some of the most important concepts in the study of trigonometry deal with the idea of similar right triangles. Informally speaking, two figures in the plane are *similar* if they have the same shape but not necessarily the same size. In the case of two similar right triangles, each has a right angle and two acute angles of corresponding measures. Furthermore, their corresponding sides are proportional. These facts allow us to solve many types of problems in astronomy, navigation, architecture, and other fields.

The depiction of Oliver Wendell Holmes' "Chambered Nautilus" shell can be approximated geometrically by a sequence of similar triangles. It can be found at http://www.mathwright.com/librarya/spiral/spiral5.htm.

In Section 1.2 we investigate *similar triangles* and their properties.

1.1 Angles

Basic Terminology ▪ Degree Measure ▪ Standard Position ▪ Coterminal Angles

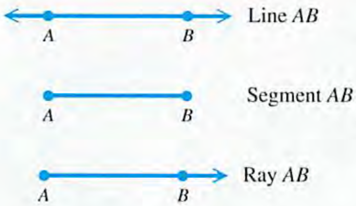


Figure 1

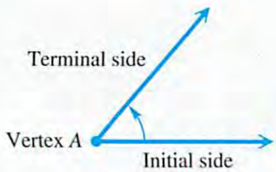


Figure 2

Basic Terminology

Two distinct points A and B determine a line called **line AB.** The portion of the line between A and B, including points A and B themselves, is **line segment AB,** or simply **segment AB.** The portion of line AB that starts at A and continues through B, and on past B, is called **ray AB.** Point A is the **endpoint of the ray.** See Figure 1.

In trigonometry, an **angle** consists of two rays in a plane with a common endpoint, or two line segments with a common endpoint. These two rays (or segments) are called the **sides** of the angle, and the common endpoint is called the **vertex** of the angle. Associated with an angle is its measure, generated by a rotation about the vertex. See Figure 2. This measure is determined by rotating a ray starting at one side of the angle, called the **initial side,** to the position of the other side, called the **terminal side.** ***A counterclockwise rotation generates a positive measure, while a clockwise rotation generates a negative measure.*** The rotation can consist of more than one complete revolution.

Figure 3 shows two angles, one **positive** and one **negative.**

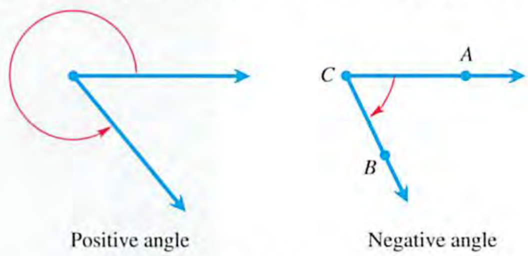
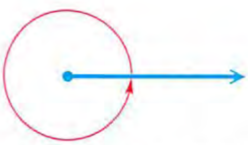


Figure 3

An angle can be named by using the name of its vertex. For example, the angle on the right in Figure 3 can be called angle C. Alternatively, an angle can be named using three letters, with the vertex letter in the middle. Thus, the angle on the right also could be named angle ACB or angle BCA.

Degree Measure

The most common unit for measuring angles is the **degree.** Degree measure was developed by the Babylonians, 4000 yr ago. To use degree measure, we assign 360 degrees to a complete rotation of a ray.* In Figure 4, notice that the terminal side of the angle corresponds to its initial side when it makes a complete rotation. One degree, written $1°$, represents $\frac{1}{360}$ of a rotation. Therefore, $90°$ represents $\frac{90}{360} = \frac{1}{4}$ of a complete rotation, and $180°$ represents $\frac{180}{360} = \frac{1}{2}$ of a complete rotation. An angle measuring between $0°$ and $90°$ is called an **acute angle.** An angle measuring exactly $90°$ is a **right angle.** The symbol ⌉ is often used at the vertex of a right angle to denote the $90°$ measure. An angle measuring more than $90°$ but less than $180°$ is an **obtuse angle,** and an angle of exactly $180°$ is a **straight angle.**



A complete rotation of a ray gives an angle whose measure is $360°$. $\frac{1}{360}$ of a complete rotation gives an angle whose measure is $1°$.

Figure 4

*The Babylonians were the first to subdivide the circumference of a circle into 360 parts. There are various theories as to why the number 360 was chosen. One is that it is approximately the number of days in a year, and it has many divisors, which makes it convenient to work with.

In Figure 5, we use the **Greek letter θ (theta)*** to name each angle.

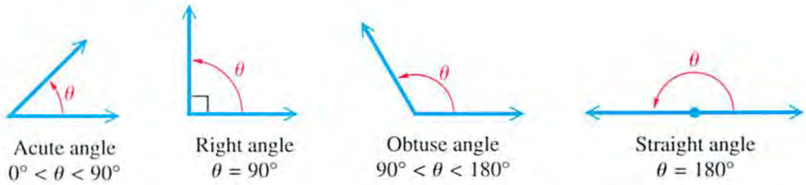


Figure 5

If the sum of the measures of two positive angles is 90°, the angles are called **complementary** and the angles are **complements** of each other. Two positive angles with measures whose sum is 180° are **supplementary,** and the angles are **supplements.**

▶ EXAMPLE 1 FINDING THE COMPLEMENT AND THE SUPPLEMENT OF AN ANGLE

For an angle measuring 40°, find the measure of its **(a)** complement and **(b)** supplement.

Solution

(a) To find the measure of its complement, subtract the measure of the angle from 90°.

$$90^\circ - 40^\circ = 50^\circ \quad \text{Complement of } 40^\circ$$

(b) To find the measure of its supplement, subtract the measure of the angle from 180°.

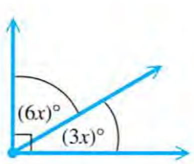
$$180^\circ - 40^\circ = 140^\circ \quad \text{Supplement of } 40^\circ$$

NOW TRY EXERCISE 1. ◀

▶ EXAMPLE 2 FINDING MEASURES OF COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Find the measure of each marked angle in Figure 6.

Solution



(a)

Figure 6

(a) In Figure 6(a), since the two angles form a right angle, they are complementary angles. Thus,

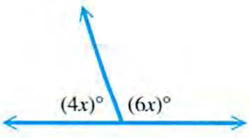
$$6x + 3x = 90$$

$$9x = 90 \quad \text{Combine terms.}$$

Don't stop here. → $x = 10.$ Divide by 9. (Appendix A)

Be sure to determine the measure of each angle by substituting 10 for x. The two angles have measures of $6(10) = 60^\circ$ and $3(10) = 30^\circ$.

*In addition to θ (theta), other Greek letters such as α (alpha) and β (beta) are often used.



(b)
Figure 6

(b) The angles in Figure 6(b) are supplementary, so their sum must be 180°. Therefore,

$$4x + 6x = 180$$
$$10x = 180$$
$$x = 18.$$

These angle measures are $4(18) = 72°$ and $6(18) = 108°$.

NOW TRY EXERCISES 13 AND 15.

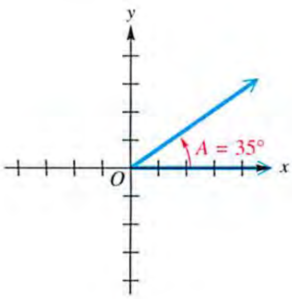


Figure 7

The measure of angle A in Figure 7 is 35°. This measure is often expressed by saying that ***m*(angle *A*)** is 35°, where $m(\text{angle } A)$ is read **"the measure of angle *A*."** It is convenient, however, to abbreviate the symbolism $m(\text{angle } A) = 35°$ as $A = 35°$.

Traditionally, portions of a degree have been measured with minutes and seconds. One **minute,** written **1′**, is $\frac{1}{60}$ of a degree.

$$\mathbf{1' = \left(\frac{1}{60}\right)^\circ \quad \text{or} \quad 60' = 1^\circ}$$

One **second, 1″,** is $\frac{1}{60}$ of a minute.

$$\mathbf{1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ \quad \text{or} \quad 60'' = 1'}$$

The measure 12° 42′ 38″ represents 12 degrees, 42 minutes, 38 seconds.

▶ EXAMPLE 3 CALCULATING WITH DEGREES, MINUTES, AND SECONDS

Perform each calculation.

(a) 51° 29′ + 32° 46′ **(b)** 90° − 73° 12′

Solution

(a)

$$\begin{array}{r} 51^\circ\, 29' \\ +\ 32^\circ\, 46' \\ \hline 83^\circ\, 75' \end{array}$$

Add degrees and minutes separately.

The sum 83° 75′ can be rewritten as

$$83^\circ\, 75' = 83^\circ + 1^\circ\, 15' = 84^\circ\, 15'. \qquad 75' = 60' + 15' = 1^\circ\, 15'$$

(b)

$$\begin{array}{r} 89^\circ\, 60' \\ -\ 73^\circ\, 12' \\ \hline 16^\circ\, 48' \end{array}$$

Write 90° as 89° 60′.

NOW TRY EXERCISES 33 AND 37.

Because calculators are now so prevalent, angles are commonly measured in decimal degrees. For example, 12.4238° represents

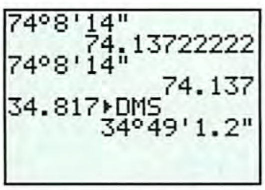
$$12.4238° = 12\frac{4238}{10{,}000}°.$$

▶ EXAMPLE 4 **CONVERTING BETWEEN DECIMAL DEGREES AND DEGREES, MINUTES, AND SECONDS**

(a) Convert 74° 8′ 14″ to decimal degrees to the nearest thousandth.

(b) Convert 34.817° to degrees, minutes, and seconds.

Solution



A graphing calculator performs the conversions in Example 4 as shown above. The ▶DMS option is found in the ANGLE Menu of the TI-83/84 Plus calculator.

(a) $74°\,8'\,14'' = 74° + \frac{8}{60}° + \frac{14}{3600}°$ $\quad 1' = \frac{1}{60}°$ and $1'' = \frac{1}{3600}°$

$\approx 74° + .1333° + .0039°$

$\approx 74.137°$ $\quad$ Add; round to the nearest thousandth.

(b) $34.817° = 34° + .817°$

$= 34° + .817(60')$ $\quad 1° = 60'$

$= 34° + 49.02'$

$= 34° + 49' + .02'$

$= 34° + 49' + .02(60'')$ $\quad 1' = 60''$

$= 34° + 49' + 1.2''$

$= 34°\,49'\,1.2''$

NOW TRY EXERCISES 53 AND 63. ◀

Standard Position

An angle is in **standard position** if its vertex is at the origin and its initial side lies on the positive x-axis. The angles in Figures 8(a) and 8(b) are in standard position. An angle in standard position is said to lie in the quadrant in which its terminal side lies. An acute angle is in quadrant I (Figure 8(a)) and an obtuse angle is in quadrant II (Figure 8(b)). Figure 8(c) shows ranges of angle measures for each quadrant when $0° < \theta < 360°$.

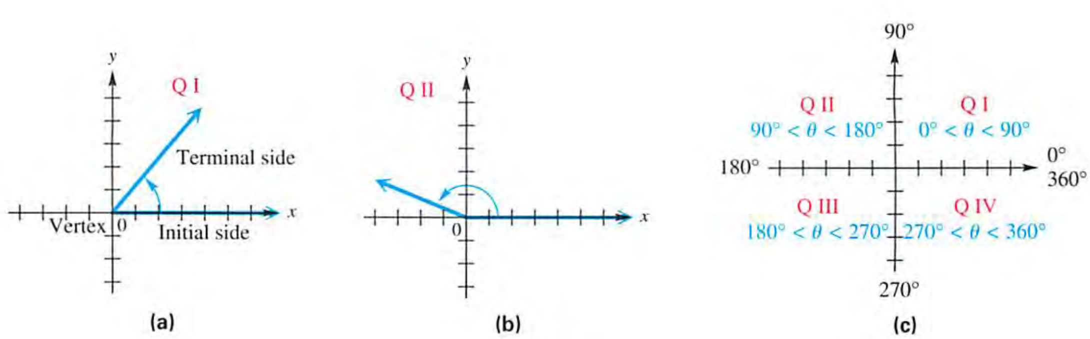


Figure 8

QUADRANTAL ANGLES

Angles in standard position whose terminal sides lie on the x-axis or y-axis, such as angles with measures 90°, 180°, 270°, and so on, are called **quadrantal angles.**

Coterminal Angles A complete rotation of a ray results in an angle measuring 360°. By continuing the rotation, angles of measure larger than 360° can be produced. The angles in Figure 9 with measures 60° and 420° have the same initial side and the same terminal side, but different amounts of rotation. Such angles are called **coterminal angles;** ***their measures differ by a multiple of* 360°.** As shown in Figure 10, angles with measures 110° and 830° are coterminal.

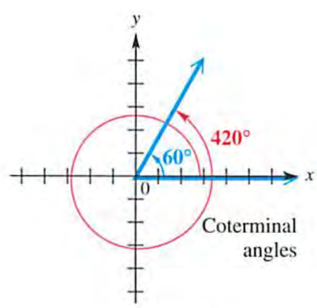


Figure 9

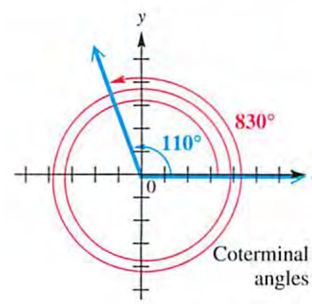


Figure 10

▶ EXAMPLE 5 FINDING MEASURES OF COTERMINAL ANGLES

Find the angles of least possible positive measure coterminal with each angle.

(a) 908° **(b)** −75° **(c)** −800°

Solution

(a) Add or subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°. Since

$$908^\circ - 2 \cdot 360^\circ = 188^\circ,$$

an angle of 188° is coterminal with an angle of 908°. See Figure 11.

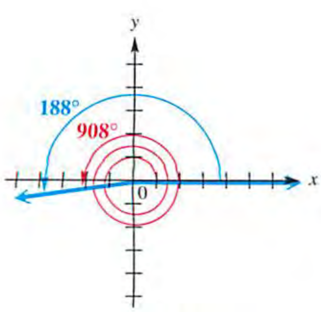


Figure 11

(b) Use a rotation of $360^\circ + (-75^\circ) = 285^\circ$. See Figure 12.

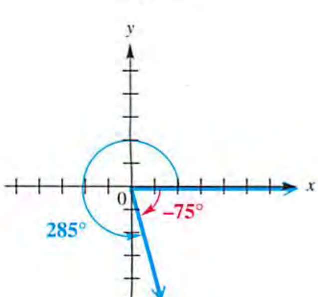


Figure 12

(c) The least integer multiple of 360° greater than 800° is

$$360^\circ \cdot 3 = 1080^\circ.$$

Add 1080° to −800° to obtain

$$1080^\circ + (-800^\circ) = 280^\circ.$$

NOW TRY EXERCISES 73, 83, AND 87. ◀

Sometimes it is necessary to find an expression that will generate all angles coterminal with a given angle. For example, we can obtain any angle coterminal with 60° by adding an appropriate integer multiple of 360° to 60°. Let n represent any integer; then the expression

$$60° + \boldsymbol{n \cdot 360°} \quad \text{Angles coterminal with } 60°$$

represents all such coterminal angles. The table shows a few possibilities.

Value of n	Angle Coterminal with 60°
2	$60° + 2 \cdot 360° = 780°$
1	$60° + 1 \cdot 360° = 420°$
0	$60° + 0 \cdot 360° = 60°$ (the angle itself)
−1	$60° + (-1) \cdot 360° = -300°$

Examples of Coterminal Quadrantal Angles

Quadrantal Angle θ	Coterminal with θ
0°	±360°, ±720°
90°	−630°, −270°, 450°
180°	−180°, 540°, 900°
270°	−450°, −90°, 630°

The table in the margin shows some examples of coterminal quadrantal angles.

▶ EXAMPLE 6 ANALYZING THE REVOLUTIONS OF A CD PLAYER

CAV (Constant Angular Velocity) CD players always spin at the same speed. Suppose a CAV player makes 480 revolutions per min. Through how many degrees will a point on the edge of a CD move in 2 sec?

Solution The player revolves 480 times in 1 min or $\frac{480}{60}$ times = 8 times per sec (since 60 sec = 1 min). In 2 sec, the player will revolve $2 \cdot 8 = 16$ times. Each revolution is 360°, so a point on the edge of the CD will revolve $16 \cdot 360° = 5760°$ in 2 sec.

NOW TRY EXERCISE 127. ◀

1.1 Exercises

Find **(a)** *the complement and* **(b)** *the supplement of an angle with the given measure. See Examples 1 and 3.*

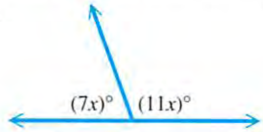
1. 30° **2.** 60° **3.** 45° **4.** 18°

5. 54° **6.** 89° **7.** 1° **8.** 10°

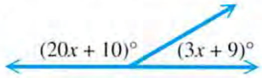
9. 14° 20′ **10.** 39° 50′ **11.** 20° 10′ 30″ **12.** 50° 40′ 50″

Find the measure of each unknown angle in Exercises 13–22. See Example 2.

13.



14.



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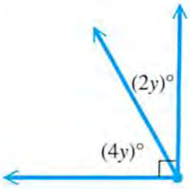

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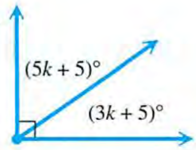

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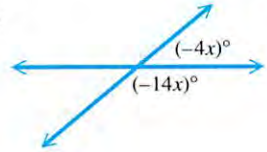

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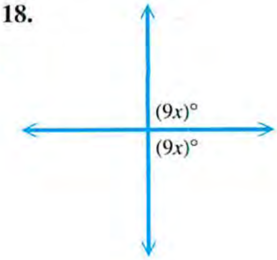


19. supplementary angles with measures $10x + 7$ and $7x + 3$ degrees

20. supplementary angles with measures $6x - 4$ and $8x - 12$ degrees

21. complementary angles with measures $9x + 6$ and $3x$ degrees

22. complementary angles with measures $3x - 5$ and $6x - 40$ degrees

23. *Concept Check* What is the measure of an angle that is its own complement?

24. *Concept Check* What is the measure of an angle that is its own supplement?

Find the measure of the smaller angle formed by the hands of a clock at the following times.

25.

26.

27. 3:15

28. 9:00

Concept Check *Answer each question.*

29. If an angle measures $x°$, how can we represent its complement?

30. If an angle measures $x°$, how can we represent its supplement?

31. If a positive angle has measure $x°$ between 0° and 60°, how can we represent the first negative angle coterminal with it?

32. If a negative angle has measure $x°$ between 0° and −60°, how can we represent the first positive angle coterminal with it?

Perform each calculation. See Example 3.

33. $62° 18' + 21° 41'$

34. $75° 15' + 83° 32'$

35. $71° 18' - 47° 29'$

36. $47° 23' - 73° 48'$

37. $90° - 51° 28'$

38. $90° - 17° 13'$

39. $180° - 119° 26'$

40. $180° - 124° 51'$

41. $26° 20' + 18° 17' - 14° 10'$

42. $55° 30' + 12° 44' - 8° 15'$

43. $90° - 72° 58' 11''$

44. $90° - 36° 18' 47''$

Convert each angle measure to decimal degrees. If applicable, round to the nearest thousandth of a degree. See Example 4(a).

45. $35° 30'$

46. $82° 30'$

47. $112° 15'$

48. $133° 45'$

49. $-60° 12'$

50. $-70° 48'$

51. $20° 54' 00''$

52. $38° 42' 00''$

53. $91° 35' 54''$

54. $34° 51' 35''$

55. $274° 18' 59''$

56. $165° 51' 9''$

Convert each angle measure to degrees, minutes, and seconds. See Example 4(b).

57. $39.25°$ **58.** $46.75°$ **59.** $126.76°$ **60.** $174.255°$

61. $-18.515°$ **62.** $-25.485°$ **63.** $31.4296°$ **64.** $59.0854°$

65. $89.9004°$ **66.** $102.3771°$ **67.** $178.5994°$ **68.** $122.6853°$

Find the angle of least positive measure (not equal to the given measure) coterminal with each angle. See Example 5.

69. $32°$ **70.** $86°$ **71.** $26° 30'$ **72.** $58° 40'$

73. $-40°$ **74.** $-98°$ **75.** $-125°$ **76.** $-203°$

77. $361°$ **78.** $541°$ **79.** $-361°$ **80.** $-541°$

81. $539°$ **82.** $699°$ **83.** $850°$ **84.** $1000°$

85. $5280°$ **86.** $8440°$ **87.** $-5280°$ **88.** $-8440°$

Give two positive and two negative angles that are coterminal with the given quadrantal angle.

89. $90°$ **90.** $180°$ **91.** $0°$ **92.** $270°$

Give an expression that generates all angles coterminal with each angle. Let n represent any integer.

93. $30°$ **94.** $45°$ **95.** $135°$ **96.** $225°$

97. $-90°$ **98.** $-180°$ **99.** $0°$ **100.** $360°$

101. Explain why the answers to Exercises 99 and 100 give the same set of angles.

102. ***Concept Check*** Which two of the following are not coterminal with $r°$?

A. $360° + r°$ **B.** $r° - 360°$ **C.** $360° - r°$ **D.** $r° + 180°$

Concept Check *Sketch each angle in standard position. Draw an arrow representing the correct amount of rotation. Find the measure of two other angles, one positive and one negative, that are coterminal with the given angle. Give the quadrant of each angle, if applicable.*

103. $75°$ **104.** $89°$ **105.** $174°$ **106.** $234°$

107. $300°$ **108.** $512°$ **109.** $-61°$ **110.** $-159°$

111. $90°$ **112.** $180°$ **113.** $-90°$ **114.** $-180°$

Concept Check *Locate each point in a coordinate system. Draw a ray from the origin through the given point. Indicate with an arrow the angle in standard position having least positive measure. Then find the distance r from the origin to the point, using the distance formula of* **Appendix B.**

115. $(-3, -3)$ **116.** $(4, -4)$ **117.** $(-3, -5)$ **118.** $(-5, 2)$

119. $(\sqrt{2}, -\sqrt{2})$ **120.** $(-2\sqrt{2}, 2\sqrt{2})$ **121.** $(-1, \sqrt{3})$ **122.** $(\sqrt{3}, 1)$

123. $(-2, 2\sqrt{3})$ **124.** $(4\sqrt{3}, -4)$ **125.** $(0, -4)$ **126.** $(0, 2)$

Solve each problem. See Example 6.

127. ***Revolutions of a Turntable*** A turntable in a shop makes 45 revolutions per min. How many revolutions does it make per second?

128. ***Revolutions of a Windmill*** A windmill makes 90 revolutions per min. How many revolutions does it make per second?

129. ***Rotating Tire*** A tire is rotating 600 times per min. Through how many degrees does a point on the edge of the tire move in $\frac{1}{2}$ sec?

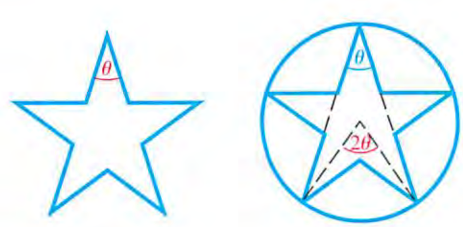
130. ***Rotating Airplane Propeller*** An airplane propeller rotates 1000 times per min. Find the number of degrees that a point on the edge of the propeller will rotate in 1 sec.

131. ***Rotating Pulley*** A pulley rotates through 75° in 1 min. How many rotations does the pulley make in an hour?

132. ***Surveying*** One student in a surveying class measures an angle as 74.25°, while another student measures the same angle as 74° 20′. Find the difference between these measurements, both to the nearest minute and to the nearest hundredth of a degree.

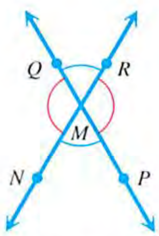
133. ***Viewing Field of a Telescope*** Due to Earth's rotation, celestial objects like the moon and the stars appear to move across the sky, rising in the east and setting in the west. As a result, if a telescope on Earth remains stationary while viewing a celestial object, the object will slowly move outside the viewing field of the telescope. For this reason, a motor is often attached to telescopes so that the telescope rotates at the same rate as Earth. Determine how long it should take the motor to turn the telescope through an angle of 1 min in a direction perpendicular to Earth's axis.

134. ***Angle Measure of a Star on the American Flag*** Determine the measure of the angle in each point of the five-pointed star appearing on the American flag. (*Hint:* Inscribe the star in a circle, and use the following theorem from geometry: *An angle whose vertex lies on the circumference of a circle is equal to half the central angle that cuts off the same arc.* See the figure.)



1.2 Angle Relationships and Similar Triangles

Geometric Properties ▪ Triangles



Vertical angles

Figure 13

Geometric Properties In Figure 13, we extended the sides of angle *NMP* to form another angle, *RMQ*. The pair of angles *NMP* and *RMQ* are called **vertical angles.** Another pair of vertical angles, *NMQ* and *PMR*, are also formed. Vertical angles have the following important property.

VERTICAL ANGLES

Vertical angles have equal measures.

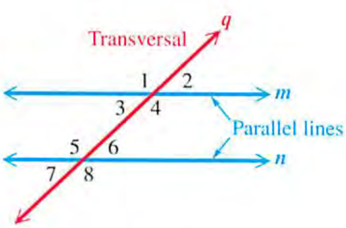
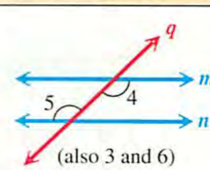
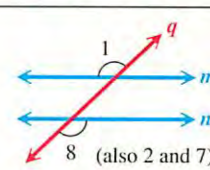
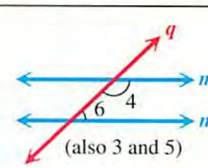
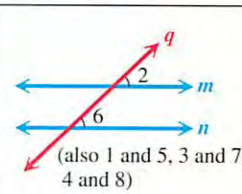


Figure 14

Parallel lines are lines that lie in the same plane and do not intersect. Figure 14 shows parallel lines m and n. When a line q intersects two parallel lines, q is called a **transversal.** In Figure 14, the transversal intersecting the parallel lines forms eight angles, indicated by numbers.

We learn in geometry that the degree measures of angles 1 through 8 in Figure 14 possess some special properties. The following chart gives the names of these angles and rules about their measures.

Name	Sketch	Rule
Alternate interior angles		Angle measures are equal.
Alternate exterior angles		Angle measures are equal.
Interior angles on same side of transversal		Angle measures add to 180°.
Corresponding angles		Angle measures are equal.

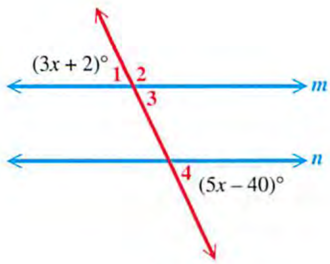


Figure 15

▶ EXAMPLE 1 FINDING ANGLE MEASURES

Find the measures of angles 1, 2, 3, and 4 in Figure 15, given that lines m and n are parallel.

Solution Angles 1 and 4 are alternate exterior angles, so they are equal.

$$3x + 2 = 5x - 40$$

$$42 = 2x \quad \text{Subtract } 3x\text{; add 40. (Appendix A)}$$

$$21 = x \quad \text{Divide by 2.}$$

Angle 1 has measure

$$3x + 2 = 3 \cdot 21 + 2 = 65°, \quad \text{Substitute 21 for } x.$$

and angle 4 has measure

$$5x - 40 = 5 \cdot 21 - 40 = 65°. \quad \text{Substitute 21 for } x.$$

Angle 2 is the supplement of a 65° angle, so it has measure

$$180° - 65° = 115°.$$

Angle 3 is a vertical angle to angle 1, so its measure is 65°. (There are other ways to determine these measures.)

NOW TRY EXERCISES 3 AND 11. ◀

(a)

(b)

Figure 16

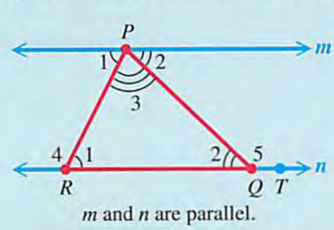
Triangles An important property of triangles, first proved by Greek geometers, deals with the sum of the measures of the angles of any triangle.

ANGLE SUM OF A TRIANGLE

***The sum of the measures of the angles of any triangle is* 180°.**

While it is not an actual proof, we give a rather convincing argument for the truth of this statement, using any size triangle cut from a piece of paper. Tear each corner from the triangle, as suggested in Figure 16(a). You should be able to rearrange the pieces so that the three angles form a straight angle, which has measure 180°, as shown in Figure 16(b).

CONNECTIONS Use this figure to discuss why the measures of the angles of a triangle must add up to the same sum as the measure of a straight angle.



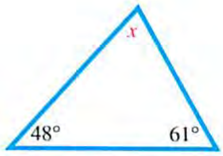


Figure 17

▶ EXAMPLE 2 APPLYING THE ANGLE SUM OF A TRIANGLE PROPERTY

The measures of two of the angles of a triangle are 48° and 61°. (See Figure 17.) Find the measure of the third angle, x.

Solution

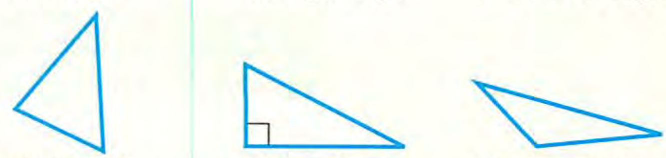
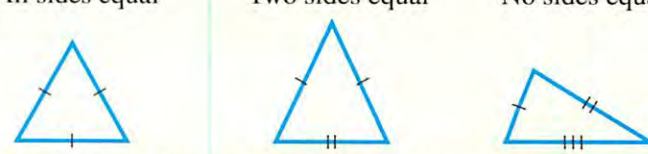
$$\begin{aligned} 48^\circ + 61^\circ + x &= 180^\circ && \text{The sum of the angles is } 180^\circ. \\ 109^\circ + x &= 180^\circ && \text{Add.} \\ x &= 71^\circ && \text{Subtract } 109^\circ. \end{aligned}$$

The third angle of the triangle measures 71°.

NOW TRY EXERCISES 5 AND 15. ◀

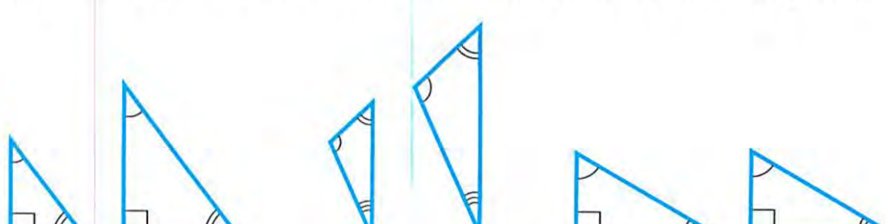
We classify triangles according to angles and sides, as shown below.

TYPES OF TRIANGLES

	All acute	One right angle	One obtuse angle
Angles			
	Acute triangle	**Right triangle**	**Obtuse triangle**
	All sides equal	Two sides equal	No sides equal
Sides			
	Equilateral triangle	**Isosceles triangle**	**Scalene triangle**

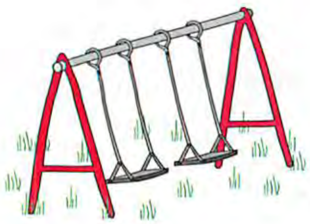
NOW TRY EXERCISES 25, 27, AND 31. ◀

Similar triangles are triangles of exactly the same shape but not necessarily the same size. Figure 18 shows three pairs of similar triangles. The two triangles in Figure 18(c) not only have the same shape but also the same size. Triangles that are both the same size and the same shape are called **congruent triangles.** If two triangles are congruent, then it is possible to pick one of them up and place it on top of the other so that they coincide. ***If two triangles are congruent, then they must be similar. However, two similar triangles need not be congruent.***



(a) (b) (c)

Figure 18



As shown in the figure, the triangular supports for a child's swing set are congruent (and thus similar) triangles, machine-produced with exactly the same dimensions each time. These supports are just one example of similar triangles. The supports of a long bridge, all the same shape but decreasing in size toward the center of the bridge, are examples of similar (but not congruent) triangles.

Suppose a correspondence between two triangles *ABC* and *DEF* is set up as shown in Figure 19.

Angle *A* corresponds to angle *D*.
Angle *B* corresponds to angle *E*.
Angle *C* corresponds to angle *F*.
Side *AB* corresponds to side *DE*.
Side *BC* corresponds to side *EF*.
Side *AC* corresponds to side *DF*.

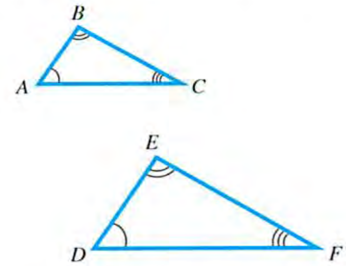


Figure 19

The small arcs found at the angles in Figure 19 denote the corresponding angles in the triangles.

CONDITIONS FOR SIMILAR TRIANGLES

For triangle *ABC* to be similar to triangle *DEF*, the following conditions must hold.

1. Corresponding angles must have the same measure.
2. Corresponding sides must be proportional. (That is, the ratios of their corresponding sides must be equal.)

NOW TRY EXERCISE 41.

▶ EXAMPLE 3 FINDING ANGLE MEASURES IN SIMILAR TRIANGLES

In Figure 20, triangles *ABC* and *NMP* are similar. Find the measures of angles *B* and *C*.

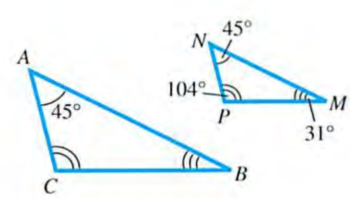


Figure 20

Solution Since the triangles are similar, corresponding angles have the same measure. Since *C* corresponds to *P* and *P* measures 104°, angle *C* also measures 104°. Since angles *B* and *M* correspond, *B* measures 31°.

NOW TRY EXERCISE 47.

▶ EXAMPLE 4 FINDING SIDE LENGTHS IN SIMILAR TRIANGLES

Given that triangle *ABC* and triangle *DFE* in Figure 21 are similar, find the lengths of the unknown sides of triangle *DFE*.

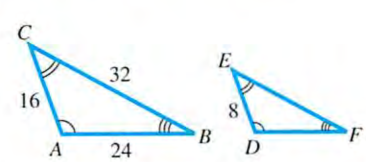


Figure 21

Solution Similar triangles have corresponding sides in proportion. Use this fact to find the unknown side lengths in triangle *DFE*.

Side DF of triangle DFE corresponds to side AB of triangle ABC, and sides DE and AC correspond. This leads to the proportion

$$\frac{8}{16} = \frac{DF}{24}.$$

Recall this property of proportions from algebra.

$$\textbf{If} \quad \frac{a}{b} = \frac{c}{d}, \quad \textbf{then} \quad ad = bc.$$

We use this property to solve the equation for DF.

$$\frac{8}{16} = \frac{DF}{24}$$
$$8 \cdot 24 = 16 \cdot DF$$
$$192 = 16 \cdot DF \quad \text{Multiply.}$$
$$12 = DF \quad \text{Divide by 16.}$$

Side DF has length 12.

Side EF corresponds to CB. This leads to another proportion.

$$\frac{8}{16} = \frac{EF}{32}$$
$$8 \cdot 32 = 16 \cdot EF$$
$$16 = EF \quad \text{Solve for } EF.$$

Side EF has length 16.

NOW TRY EXERCISE 53.

▶ EXAMPLE 5 FINDING THE HEIGHT OF A FLAGPOLE

Firefighters at the Morganza Fire Station need to measure the height of the station flagpole. They find that at the instant when the shadow of the station is 18 m long, the shadow of the flagpole is 99 ft long. The station is 10 m high. Find the height of the flagpole.

Solution Figure 22 shows the information given in the problem. The two triangles are similar, so corresponding sides are in proportion.

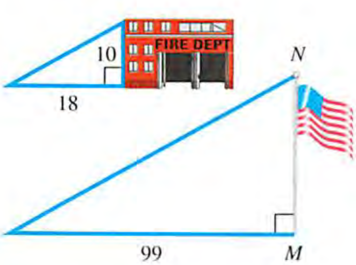


Figure 22

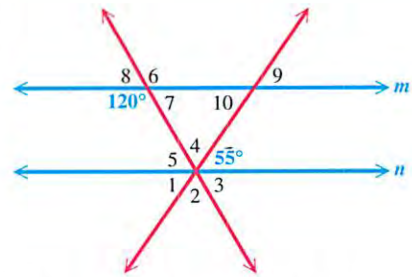
$$\frac{MN}{10} = \frac{99}{18}$$
$$\frac{MN}{10} = \frac{11}{2} \quad \text{Lowest terms}$$
$$MN \cdot 2 = 10 \cdot 11$$
$$MN = 55 \quad \text{Solve for } MN.$$

The flagpole is 55 ft high.

NOW TRY EXERCISE 57.

1.2 Exercises

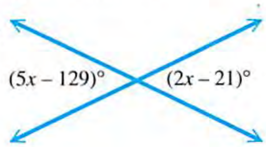
1. *Concept Check* Use the given figure to find the measures of the numbered angles, given that lines m and n are parallel.



2. Consider Figure 14. If the measure of one of the angles is known, can the measures of the remaining seven angles be determined? Explain.

Find the measure of each marked angle. In Exercises 11–14, m and n are parallel. See Examples 1 and 2.

3.



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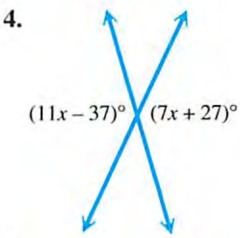

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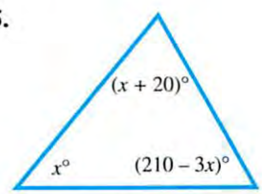

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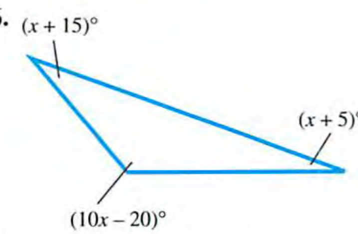

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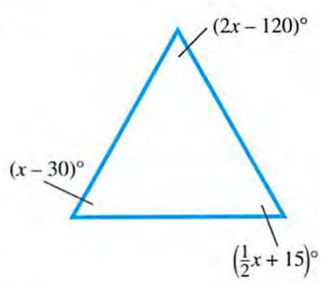

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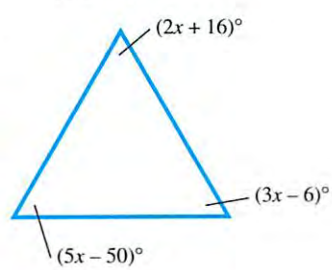

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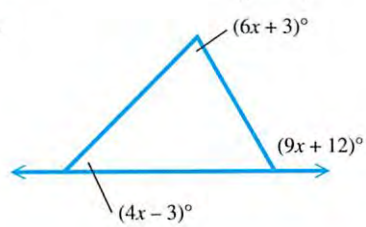

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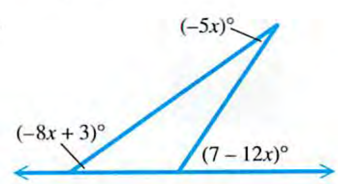

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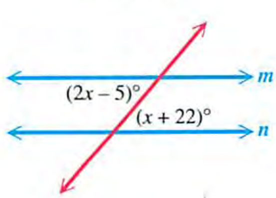

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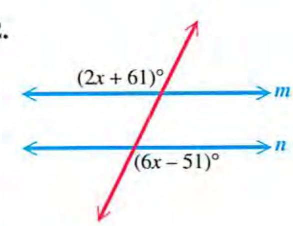

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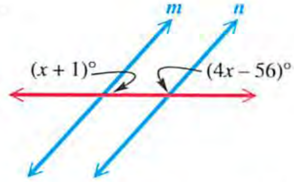

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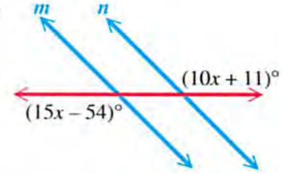


The measures of two angles of a triangle are given. Find the measure of the third angle. See Example 2.

15. 37°, 52° **16.** 29°, 104° **17.** 147° 12′, 30° 19′

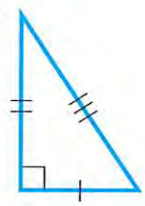
18. 136° 50′, 41° 38′ **19.** 74.2°, 80.4° **20.** 29.6°, 49.7°

21. 51° 20′ 14″, 106° 10′ 12″ **22.** 17° 41′ 13″, 96° 12′ 10″

23. Can a triangle have angles of measures 85° and 100°? Explain.

24. Can a triangle have two obtuse angles? Explain.

Concept Check *Classify each triangle in Exercises 25–36 as* acute, right, *or* obtuse. *Also classify each as* equilateral, isosceles, *or* scalene.

25.


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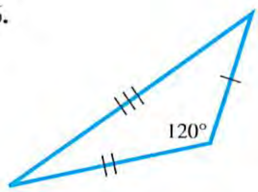

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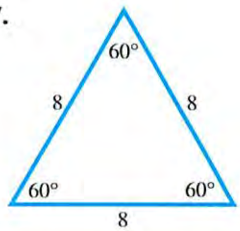

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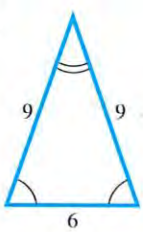

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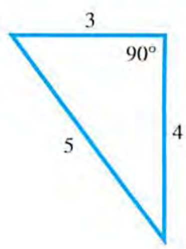

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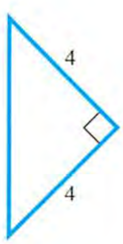

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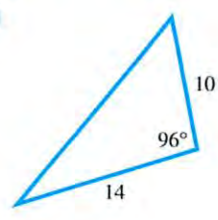

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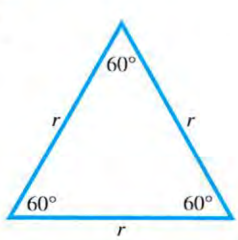

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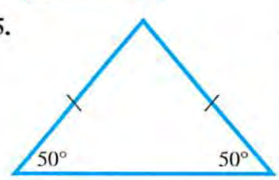

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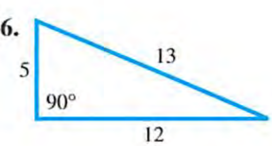


37. Write a definition of *isosceles right triangle.*

38. Explain why the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

39. Must all equilateral triangles be similar? Explain.

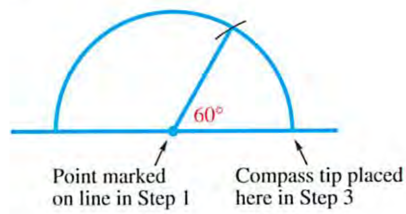
40. ***Carpentry Technique*** The following technique is used by carpenters to draw a 60° angle with a straightedge and compass. Explain why this technique works. (*Source:* Hamilton, J. E. and M. S. Hamilton, *Math to Build On,* Construction Trades Press, 1993.)

Step 1 Draw a straight line segment, and mark a point near the center of the line.

Step 2 Place the compass tip on the marked point, and draw a semicircle.

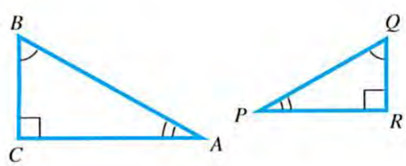
Step 3 Without changing the setting of the compass, place the tip of the compass at the right intersection of the line and the semicircle and then mark a small arc across the semicircle.

Step 4 Draw a line segment from the marked point on the line to the point where the arc crosses the semicircle. This line will make a 60° angle with the original line.



Concept Check *Name the corresponding angles and the corresponding sides of each pair of similar triangles.*

41.



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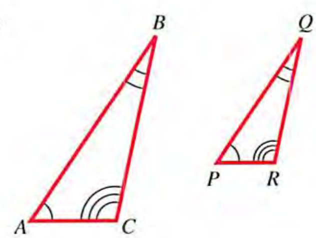
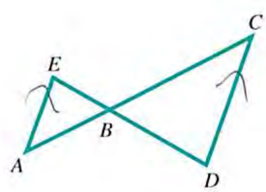
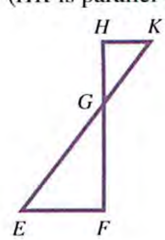


43. (*EA* is parallel to *CD*.)

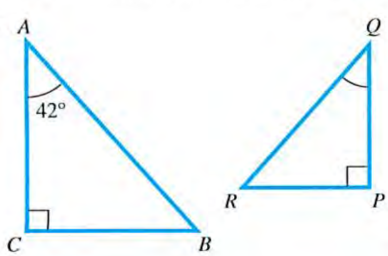


44. (*HK* is parallel to *EF*.)



Find all unknown angle measures in each pair of similar triangles. See Example 3.

45.



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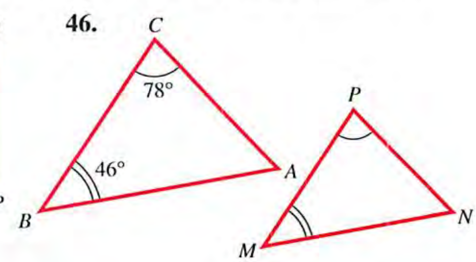

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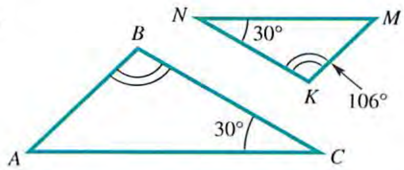

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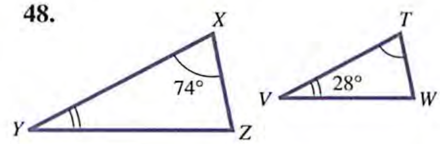

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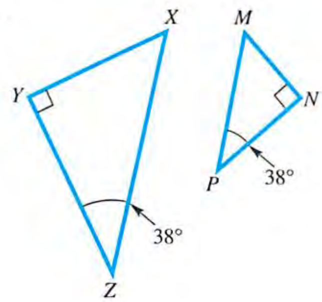

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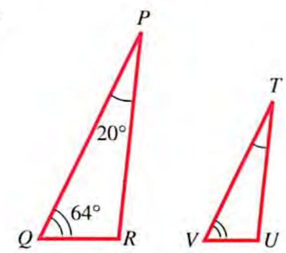
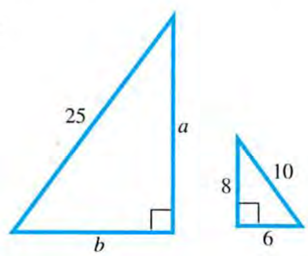


Find the unknown side lengths labeled with a variable in each pair of similar triangles. See Example 4.

51.



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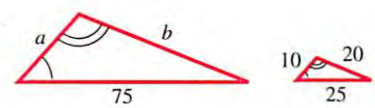

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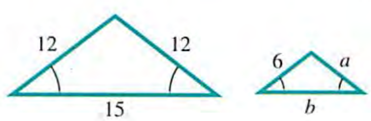
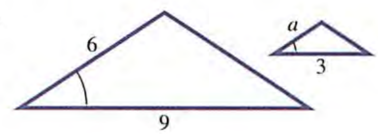


54.



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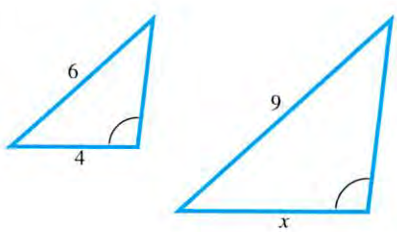

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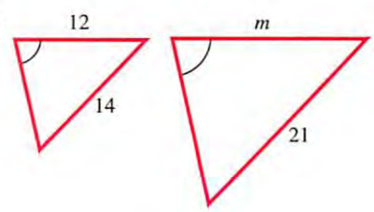


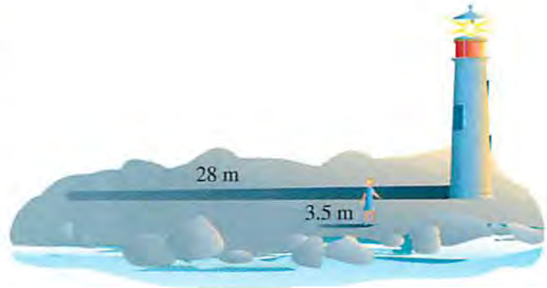
Solve each problem. See Example 5.

57. ***Height of a Tree*** A tree casts a shadow 45 m long. At the same time, the shadow cast by a vertical 2-m stick is 3 m long. Find the height of the tree.

58. ***Height of a Lookout Tower*** A forest fire lookout tower casts a shadow 180 ft long at the same time that the shadow of a 9-ft truck is 15 ft long. Find the height of the tower.

59. ***Lengths of Sides of a Triangle*** On a photograph of a triangular piece of land, the lengths of the three sides are 4 cm, 5 cm, and 7 cm, respectively. The shortest side of the actual piece of land is 400 m long. Find the lengths of the other two sides.

60. ***Height of a Lighthouse*** The Biloxi lighthouse in the figure casts a shadow 28 m long at 7 P.M. At the same time, the shadow of the lighthouse keeper, who is 1.75 m tall, is 3.5 m long. How tall is the lighthouse?



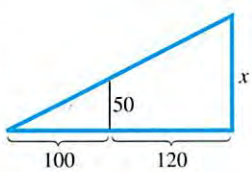
Not to scale

61. ***Height of a Building*** A house is 15 ft tall. Its shadow is 40 ft long at the same time the shadow of a nearby building is 300 ft long. Find the height of the building.

62. ***Height of a Carving of Lincoln*** Assume that Lincoln was $6\frac{1}{3}$ ft tall and his head $\frac{3}{4}$ ft long. Knowing that the carved head of Lincoln at Mt. Rushmore is 60 ft tall, find how tall his entire body would be if it were carved into the mountain.

In each diagram, there are two similar triangles. Find the unknown measurement. (Hint: In the sketch for Exercise 63, the side of length 100 *in the small triangle corresponds to the side of the length* $100 + 120 = 220$ *in the large triangle.)*

63.



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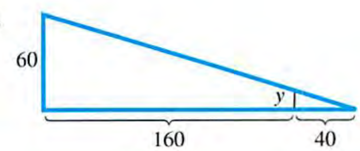

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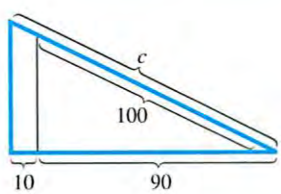

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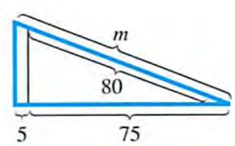
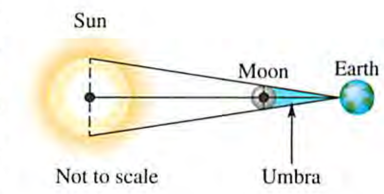


Solve each problem.

67. ***Lengths of Sides of a Quadrilateral*** Two quadrilaterals (four-sided figures) are similar. The lengths of the three shortest sides of the first quadrilateral are 18 cm, 24 cm, and 32 cm. The lengths of the two longest sides of the second quadrilateral are 48 cm and 60 cm. Find the unknown lengths of the sides of these two figures.

68. ***Distance Between Two Cities*** By drawing lines on a map, a triangle can be formed by the cities of Phoenix, Tucson, and Yuma. On the map, the distance between Phoenix and Tucson is 8 cm, the distance between Phoenix and Yuma is 12 cm, and the distance between Tucson and Yuma is 17 cm. The actual straight-line distance from Phoenix to Yuma is 230.0 km. Find the distances between the other pairs of cities to the nearest tenth of a kilometer.

69. *Solar Eclipse on Earth* The sun has a diameter of about 865,000 mi with a maximum distance from Earth's surface of about 94,500,000 mi. The moon has a smaller diameter of 2159 mi. For a total solar eclipse to occur, the moon must pass between Earth and the sun. The moon must also be close enough to Earth for the moon's **umbra** (shadow) to reach the surface of Earth. (*Source:* Karttunen, H., P. Kröger, H. Oja, M. Putannen, and K. Donners, Editors, *Fundamental Astronomy,* Fourth Edition, Springer-Verlag, 2003.)



(a) Calculate the maximum distance that the moon can be from Earth and still have a total solar eclipse occur. (*Hint:* Use similar triangles.)

(b) The closest approach of the moon to Earth's surface was 225,745 mi and the farthest was 251,978 mi. (*Source: World Almanac and Book of Facts.*) Can a total solar eclipse occur every time the moon is between Earth and the sun?

70. *Solar Eclipse on Neptune* (Refer to Exercise 69.) The sun's distance from Neptune is approximately 2,800,000,000 mi (2.8 billion mi). The largest moon of Neptune is Triton, with a diameter of approximately 1680 mi. (*Source: World Almanac and Book of Facts.*)

(a) Calculate the maximum distance that Triton can be from Neptune for a total eclipse of the sun to occur on Neptune. (*Hint:* Use similar triangles.)

(b) Triton is approximately 220,000 mi from Neptune. Is it possible for Triton to cause a total eclipse on Neptune?

71. *Solar Eclipse on Mars* (Refer to Exercise 69.) The sun's distance from the surface of Mars is approximately 142,000,000 mi. One of Mars' two moons, Phobos, has a maximum diameter of 17.4 mi. (*Source:* Zeilik, M., S. Gregory, and E. Smith, *Introductory Astronomy and Astrophysics,* Second Edition, Saunders College Publishers, 1998.)

(a) Calculate the maximum distance that the moon Phobos can be from Mars for a total eclipse of the sun to occur on Mars.

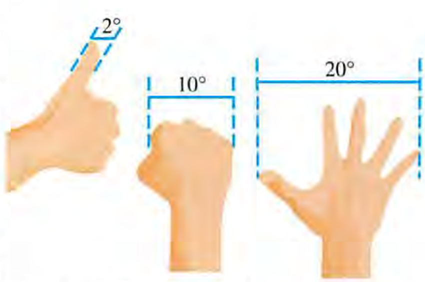
(b) Phobos is approximately 5800 mi from Mars. Is it possible for Phobos to cause a total eclipse on Mars?

72. *Solar Eclipse on Jupiter* (Refer to Exercise 69.) The sun's distance from the surface of Jupiter is approximately 484,000,000 mi. One of Jupiter's moons, Ganymede, has a diameter of 3270 mi. (*Source:* Wright, J. W., General Editor, *The Universal Almanac,* Andrews and McMeel, 1997.)

(a) Calculate the maximum distance that the moon Ganymede can be from Jupiter for a total eclipse of the sun to occur on Jupiter.

(b) Ganymede is approximately 665,000 mi from Jupiter. Is it possible for Ganymede to cause a total eclipse on Jupiter?

73. *Sizes and Distances in the Sky* Astronomers use degrees, minutes, and seconds to measure sizes and distances in the sky along an arc from the horizon to the zenith point directly overhead. An adult observer on Earth can judge distances in the sky using his or her hand at arm's length. An outstretched hand will be about 20 arc degrees wide from the tip of the thumb to the tip of the little finger. A clenched fist at arm's length measures about 10 arc degrees, and a thumb corresponds to about 2 arc degrees. (*Source:* Levy, D. H., *Skywatching,* The Nature Company, 1994.)

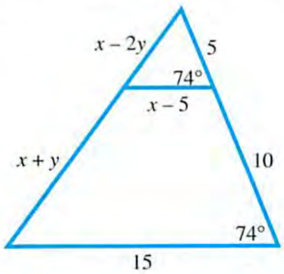


(a) The apparent size of the moon is about 31 arc minutes. What part of your thumb would cover the moon?

(b) If an outstretched hand plus a fist cover the distance between two bright stars, about how far apart in arc degrees are the stars?

In each figure, two similar triangles are present. Find the value of each variable.

74.



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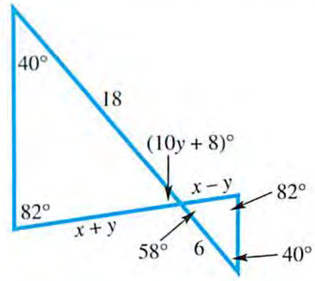
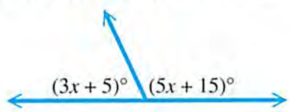


CHAPTER 1 ▶ Quiz (Sections 1.1–1.2)

1. For an angle measuring 19°, give the measure of its **(a)** complement and **(b)** supplement.

Find the measure of each unknown angle.

2.



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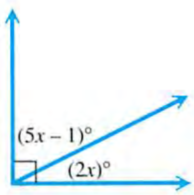

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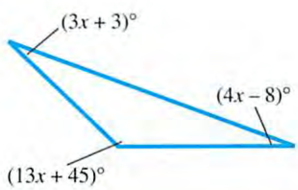

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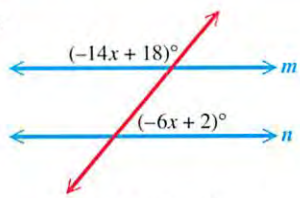


m and *n* are parallel.

6. Perform each indicated conversion.

(a) 77° 12′ 09″ to decimal degrees **(b)** 22.0250° to degrees, minutes, seconds

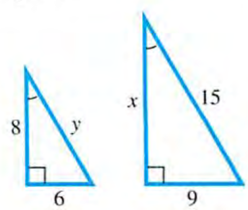
7. Find the angle of least positive measure (not equal to the given angle) coterminal with each angle.

(a) 410° **(b)** −60° **(c)** 890° **(d)** 57°

8. ***Rotating Flywheel*** A flywheel rotates 300 times per min. Through how many degrees does a point on the edge of the flywheel move in 1 sec?

9. ***Length of a Shadow*** If a vertical antenna 45 ft tall casts a shadow 15 ft long, how long would the shadow of a 30-ft pole be at the same time and place?

10. Find the unknown side lengths x and y in this pair of similar triangles.



1.3 Trigonometric Functions

Trigonometric Functions ▪ Quadrantal Angles

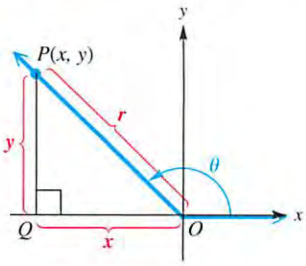


Figure 23

Trigonometric Functions

The study of trigonometry covers the six **trigonometric functions** defined here. To define these functions, we start with an angle θ in standard position, and choose any point P having coordinates (x, y) on the terminal side of angle θ. (The point P must not be the vertex of the angle.) See Figure 23. A perpendicular from P to the x-axis at point Q determines a right triangle, having vertices at O, P, and Q. We find the distance r from $P(x, y)$ to the origin, $(0, 0)$, using the distance formula.

$$r = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} \quad \text{(Appendix B)}$$

Notice that $r > 0$ since this is the undirected distance.

The six trigonometric functions of angle θ are **sine, cosine, tangent, cotangent, secant,** and **cosecant.** In the following definitions, we use the customary abbreviations for the names of these functions: **sin, cos, tan, cot, sec,** and **csc.**

TRIGONOMETRIC FUNCTIONS

Let (x, y) be a point other than the origin on the terminal side of an angle θ in standard position. The distance from the point to the origin is $r = \sqrt{x^2 + y^2}$. The six trigonometric functions of θ are defined as follows.

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc \theta = \frac{r}{y} \quad (y \neq 0) \qquad \sec \theta = \frac{r}{x} \quad (x \neq 0) \qquad \cot \theta = \frac{x}{y} \quad (y \neq 0)$$

▶ EXAMPLE 1 FINDING FUNCTION VALUES OF AN ANGLE

The terminal side of an angle θ in standard position passes through the point $(8, 15)$. Find the values of the six trigonometric functions of angle θ.

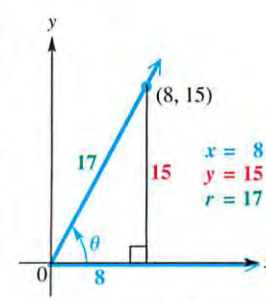


Figure 24

Solution Figure 24 shows angle θ and the triangle formed by dropping a perpendicular from the point $(8, 15)$ to the x-axis. The point $(8, 15)$ is 8 units to the right of the y-axis and 15 units above the x-axis, so $x = 8$ and $y = 15$. Since $r = \sqrt{x^2 + y^2}$,

$$r = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17.$$

We can now find the values of the six trigonometric functions of angle θ.

$$\sin \theta = \frac{y}{r} = \frac{15}{17} \qquad \cos \theta = \frac{x}{r} = \frac{8}{17} \qquad \tan \theta = \frac{y}{x} = \frac{15}{8}$$

$$\csc \theta = \frac{r}{y} = \frac{17}{15} \qquad \sec \theta = \frac{r}{x} = \frac{17}{8} \qquad \cot \theta = \frac{x}{y} = \frac{8}{15}$$

NOW TRY EXERCISE 5. ◀

▶ EXAMPLE 2 FINDING FUNCTION VALUES OF AN ANGLE

The terminal side of an angle θ in standard position passes through the point $(-3, -4)$. Find the values of the six trigonometric functions of angle θ.

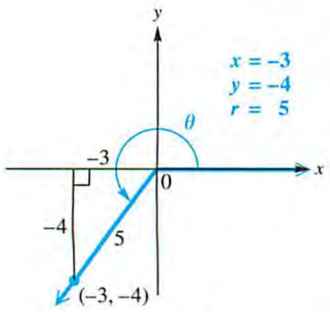


Figure 25

Solution As shown in Figure 25, $x = -3$ and $y = -4$. The value of r is

$$r = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5. \quad \text{Remember that } r > 0.$$

Then use the definitions of the trigonometric functions.

$$\sin\theta = \frac{-4}{5} = -\frac{4}{5} \qquad \cos\theta = \frac{-3}{5} = -\frac{3}{5} \qquad \tan\theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc\theta = \frac{5}{-4} = -\frac{5}{4} \qquad \sec\theta = \frac{5}{-3} = -\frac{5}{3} \qquad \cot\theta = \frac{-3}{-4} = \frac{3}{4}$$

NOW TRY EXERCISE 19. ◀

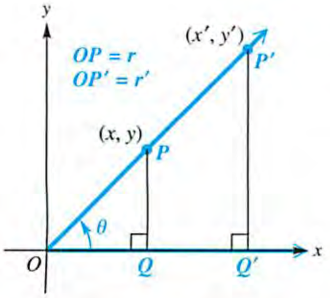


Figure 26

We can find the six trigonometric functions using *any* point other than the origin on the terminal side of an angle. To see why any point may be used, refer to Figure 26, which shows an angle θ and two distinct points on its terminal side. Point P has coordinates (x, y), and point P' (read **"P-prime"**) has coordinates (x', y'). Let r be the length of the hypotenuse of triangle OPQ, and let r' be the length of the hypotenuse of triangle $OP'Q'$. Since corresponding sides of similar triangles are proportional,

$$\frac{y}{r} = \frac{y'}{r'}, \quad \text{(Section 1.2)}$$

so $\sin\theta = \frac{y}{r}$ is the same no matter which point is used to find it. A similar result holds for the other five trigonometric functions.

We can also find the trigonometric function values of an angle if we know the equation of the line coinciding with the terminal ray. Recall from algebra that the graph of the equation

$$Ax + By = 0 \quad \text{(Appendix B)}$$

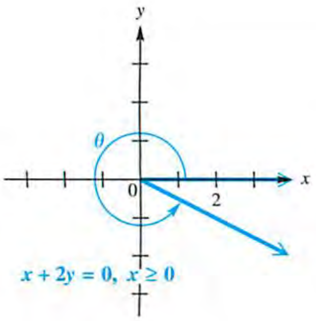


Figure 27

is a line that passes through the origin. If we restrict x to have only nonpositive or only nonnegative values, we obtain as the graph a ray with endpoint at the origin. For example, the graph of $x + 2y = 0$, $x \geq 0$, shown in Figure 27, is a ray that can serve as the terminal side of an angle θ in standard position. By choosing a point on the ray, we can find the trigonometric function values of the angle.

▶ EXAMPLE 3 FINDING FUNCTION VALUES OF AN ANGLE

Find the six trigonometric function values of the angle θ in standard position, if the terminal side of θ is defined by $x + 2y = 0$, $x \geq 0$.

Solution The angle is shown in Figure 28 on the next page. We can use *any* point except $(0, 0)$ on the terminal side of θ to find the trigonometric function values. We choose $x = 2$ and find the corresponding y-value.

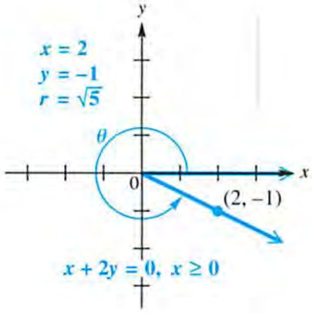


Figure 28

$$\begin{aligned} x + 2y &= 0, x \geq 0 \\ 2 + 2y &= 0 && \text{Let } x = 2. \\ 2y &= -2 && \text{Subtract 2. (Appendix A)} \\ y &= -1 && \text{Divide by 2.} \end{aligned}$$

The point $(2, -1)$ lies on the terminal side, and the corresponding value of r is $r = \sqrt{2^2 + (-1)^2} = \sqrt{5}$. Now we use the definitions of the trigonometric functions.

$$\sin \theta = \frac{y}{r} = \frac{-1}{\sqrt{5}} = \frac{-1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$, which equals 1, to rationalize the denominators.

$$\cos \theta = \frac{x}{r} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{2} = -\frac{1}{2}$$

$$\csc \theta = \frac{r}{y} = \frac{\sqrt{5}}{-1} = -\sqrt{5} \qquad \sec \theta = \frac{r}{x} = \frac{\sqrt{5}}{2} \qquad \cot \theta = \frac{x}{y} = \frac{2}{-1} = -2$$

NOW TRY EXERCISE 45.

Recall that when the equation of a line is written in slope-intercept form $y = mx + b$, the coefficient of x is the slope of the line. In Example 3, the equation $x + 2y = 0$ can be written as $y = -\frac{1}{2}x$, so the slope is $-\frac{1}{2}$. Notice that $\tan \theta = -\frac{1}{2}$. ***In general, it is true that $m = \tan \theta$.***

Note The trigonometric function values we found in Examples 1–3 are *exact.* If we were to use a calculator to approximate these values, the decimal results would not be acceptable if exact values were required.

Quadrantal Angles

If the terminal side of an angle in standard position lies along the y-axis, any point on this terminal side has x-coordinate 0. Similarly, an angle with terminal side on the x-axis has y-coordinate 0 for any point on the terminal side. Since the values of x and y appear in the denominators of some trigonometric functions, and since a fraction is undefined if its denominator is 0, some trigonometric function values of quadrantal angles (i.e., those with terminal side on an axis) are undefined.

When determining trigonometric function values of quadrantal angles, Figure 29 can help find the ratios. Because *any* point on the terminal side can be used, it is convenient to choose the point one unit from the origin, with $r = 1$. (In **Chapter 3** we extend this idea to the *unit circle*.)

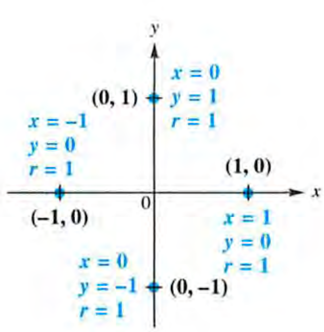


Figure 29

To find the function values of a quadrantal angle, determine the position of the terminal side, choose the one of these four points that lies on this terminal side, and then use the definitions involving x, y, and r.

▶ EXAMPLE 4 FINDING FUNCTION VALUES OF QUADRANTAL ANGLES

Find the values of the six trigonometric functions for each angle.

(a) an angle of 90°

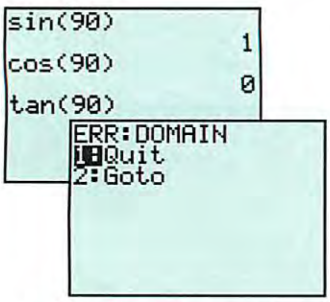
(b) an angle θ in standard position with terminal side through $(-3, 0)$

Solution

(a) The sketch in Figure 30 shows that the terminal side passes through $(0, 1)$. So $x = 0$, $y = 1$, and $r = 1$. Thus,

$$\sin 90^\circ = \frac{1}{1} = 1 \qquad \cos 90^\circ = \frac{0}{1} = 0 \qquad \tan 90^\circ = \frac{1}{0} \ \text{(undefined)}$$

$$\csc 90^\circ = \frac{1}{1} = 1 \qquad \sec 90^\circ = \frac{1}{0} \ \text{(undefined)} \qquad \cot 90^\circ = \frac{0}{1} = 0.$$



A calculator in degree mode returns the correct values for sin 90° and cos 90°. The second screen shows an ERROR message for tan 90°, because 90° is not in the domain of the tangent function.

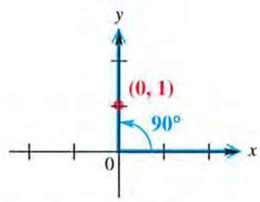


Figure 30

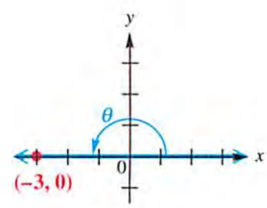


Figure 31

(b) Figure 31 shows the angle. Here, $x = -3$, $y = 0$, and $r = 3$, so the trigonometric functions have the following values.

$$\sin \theta = \frac{0}{3} = 0 \qquad \cos \theta = \frac{-3}{3} = -1 \qquad \tan \theta = \frac{0}{-3} = 0$$

$$\csc \theta = \frac{3}{0} \ \text{(undefined)} \qquad \sec \theta = \frac{3}{-3} = -1 \qquad \cot \theta = \frac{-3}{0} \ \text{(undefined)}$$

(Verify that these values can also be found by using the point $(-1, 0)$.)

NOW TRY EXERCISES 13, 57, 59, 63, AND 65. ◀

The conditions under which the trigonometric function values of quadrantal angles are undefined are summarized here.

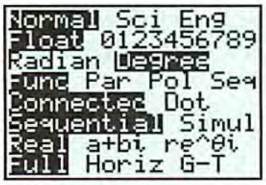
UNDEFINED FUNCTION VALUES

If the terminal side of a quadrantal angle lies along the y-axis, then the tangent and secant functions are undefined. If it lies along the x-axis, then the cotangent and cosecant functions are undefined.

The function values of the most commonly used quadrantal angles, 0°, 90°, 180°, 270°, and 360°, are summarized in the table on the next page. They can be determined when needed by using Figure 29 and the method of Example 4(a).

Function Values of Quadrantal Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	0	1	0	Undefined	1	Undefined
90°	1	0	Undefined	0	Undefined	1
180°	0	−1	0	Undefined	−1	Undefined
270°	−1	0	Undefined	0	Undefined	−1
360°	0	1	0	Undefined	1	Undefined



TI-83 Plus

TI-84 Plus

Figure 32

The values given in this table can be found with a calculator that has trigonometric function keys. ***Make sure the calculator is set in degree mode.***

▶ **Caution** ***One of the most common errors involving calculators in trigonometry occurs when the calculator is set for radian measure, rather than degree measure.*** (Radian measure of angles is discussed in **Chapter 3.**) Be sure you know how to set your calculator in degree mode. See Figure 32, which illustrates degree mode for TI-83/84 Plus calculators.

1.3 Exercises

Concept Check *Sketch an angle θ in standard position such that θ has the least possible positive measure, and the given point is on the terminal side of θ. Then find the values of the six trigonometric functions for each angle. Rationalize denominators when applicable. See Examples 1, 2, and 4.*

1. $(5, -12)$ **2.** $(-12, -5)$ **3.** $(-3, 4)$ **4.** $(-4, -3)$

5. $(-8, 15)$ **6.** $(15, -8)$ **7.** $(7, -24)$ **8.** $(-24, -7)$

9. $(0, 2)$ **10.** $(0, 5)$ **11.** $(-4, 0)$ **12.** $(-5, 0)$

13. $(0, -4)$ **14.** $(0, -3)$ **15.** $(1, \sqrt{3})$ **16.** $(-1, \sqrt{3})$

17. $(\sqrt{2}, \sqrt{2})$ **18.** $(-\sqrt{2}, -\sqrt{2})$ **19.** $(-2\sqrt{3}, -2)$ **20.** $(-2\sqrt{3}, 2)$

21. For any nonquadrantal angle θ, $\sin\theta$ and $\csc\theta$ will have the same sign. Explain why.

22. *Concept Check* How is the value of r interpreted geometrically in the definitions of the sine, cosine, secant, and cosecant functions?

23. *Concept Check* If $\cot\theta$ is undefined, what is the value of $\tan\theta$?

24. *Concept Check* If the terminal side of an angle θ is in quadrant III, what is the sign of each of the trigonometric function values of θ?

Concept Check *Suppose that the point (x, y) is in the indicated quadrant. Decide whether the given ratio is positive or negative. Recall that $r = \sqrt{x^2 + y^2}$. (Hint: Drawing a sketch may help.)*

25. II, $\frac{x}{r}$ **26.** III, $\frac{y}{r}$ **27.** IV, $\frac{y}{x}$ **28.** IV, $\frac{x}{y}$

29. II, $\frac{y}{r}$ **30.** III, $\frac{x}{r}$ **31.** IV, $\frac{x}{r}$ **32.** IV, $\frac{y}{r}$

33. II, $\frac{x}{y}$ **34.** II, $\frac{y}{x}$ **35.** III, $\frac{y}{x}$ **36.** III, $\frac{x}{y}$

37. IV, $\frac{x}{y}$
38. IV, $\frac{y}{x}$
39. I, $\frac{x}{y}$
40. I, $\frac{y}{x}$
41. I, $\frac{y}{r}$
42. I, $\frac{x}{r}$
43. I, $\frac{r}{x}$
44. I, $\frac{r}{y}$

In Exercises 45–54, an equation of the terminal side of an angle θ in standard position is given with a restriction on x. Sketch the least positive such angle θ, and find the values of the six trigonometric functions of θ. See Example 3.

45. $2x + y = 0, x \geq 0$
46. $3x + 5y = 0, x \geq 0$
47. $-6x - y = 0, x \leq 0$
48. $-5x - 3y = 0, x \leq 0$
49. $-4x + 7y = 0, x \leq 0$
50. $6x - 5y = 0, x \geq 0$
51. $x + y = 0, x \geq 0$
52. $x - y = 0, x \geq 0$
53. $-\sqrt{3}x + y = 0, x \leq 0$
54. $\sqrt{3}x + y = 0, x \leq 0$

To work Exercises 55–72, begin by reproducing the graph in Figure 29 on page 25. Keep in mind that for each of the four points labeled in the figure, $r = 1$. For each quadrantal angle, identify the appropriate values of x, y, and r to find the indicated function value. If it is undefined, say so. See Example 4.

55. $\cos 90°$
56. $\sin 90°$
57. $\tan 180°$
58. $\cot 90°$
59. $\sec 180°$
60. $\csc 270°$
61. $\sin(-270°)$
62. $\cos(-90°)$
63. $\cot 540°$
64. $\tan 450°$
65. $\csc(-450°)$
66. $\sec(-540°)$
67. $\sin 1800°$
68. $\cos 1800°$
69. $\csc 1800°$
70. $\cot 1800°$
71. $\sec 1800°$
72. $\tan 1800°$

Use the trigonometric function values of quadrantal angles given in this section to evaluate each expression. An expression such as $\cot^2 90°$ *means* $(\cot 90°)^2$, *which is equal to* $0^2 = 0$.

73. $\cos 90° + 3 \sin 270°$
74. $\tan 0° - 6 \sin 90°$
75. $3 \sec 180° - 5 \tan 360°$
76. $4 \csc 270° + 3 \cos 180°$
77. $\tan 360° + 4 \sin 180° + 5 \cos^2 180°$
78. $2 \sec 0° + 4 \cot^2 90° + \cos 360°$
79. $\sin^2 180° + \cos^2 180°$
80. $\sin^2 360° + \cos^2 360°$
81. $\sec^2 180° - 3 \sin^2 360° + \cos 180°$
82. $5 \sin^2 90° + 2 \cos^2 270° - \tan 360°$
83. $-2 \sin^4 0° + 3 \tan^2 0°$
84. $-3 \sin^4 90° + 4 \cos^3 180°$

If n is an integer, $n \cdot 180°$ *represents an integer multiple of* $180°$, $(2n + 1) \cdot 90°$ *represents an odd integer multiple of* $90°$, *and so on. Decide whether each expression is equal to* 0, 1, -1, *or is* undefined.

85. $\cos[(2n + 1) \cdot 90°]$
86. $\sin[n \cdot 180°]$
87. $\tan[n \cdot 180°]$
88. $\tan[(2n + 1) \cdot 90°]$
89. $\sin[270° + n \cdot 360°]$
90. $\cot[n \cdot 180°]$

91. $\cot[(2n + 1) \cdot 90°]$

92. $\cos[n \cdot 360°]$

93. $\sec[(2n + 1) \cdot 90°]$

94. $\csc[n \cdot 180°]$

Concept Check *In later chapters we will study trigonometric functions of angles other than quadrantal angles, such as 15°, 30°, 60°, 75°, and so on. To prepare for some important concepts, provide conjectures in Exercises 95–98. Be sure that your calculator is in degree mode.*

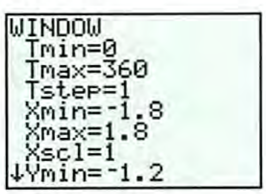
95. The angles 15° and 75° are complementary. With your calculator determine sin 15° and cos 75°. Make a conjecture about the sines and cosines of complementary angles, and test your hypothesis with other pairs of complementary angles. (*Note:* This relationship will be discussed in detail in **Section 2.1.**)

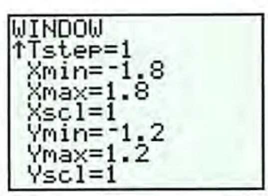
96. The angles 25° and 65° are complementary. With your calculator determine tan 25° and cot 65°. Make a conjecture about the tangents and cotangents of complementary angles, and test your hypothesis with other pairs of complementary angles. (*Note:* This relationship will be discussed in detail in **Section 2.1.**)

97. With your calculator determine sin 10° and sin(−10°). Make a conjecture about the sines of an angle and its negative, and test your hypothesis with other angles. (*Note:* This relationship will be discussed in detail in **Section 5.1.**)

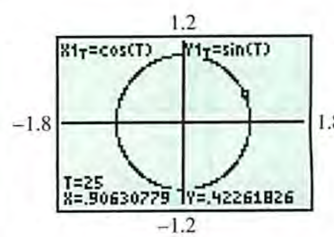
98. With your calculator determine cos 20° and cos(−20°). Make a conjecture about the cosines of an angle and its negative, and test your hypothesis with other angles. (*Note:* This relationship will be discussed in detail in **Section 5.1.**)

In Exercises 99–104, set your graphing calculator in parametric and degree modes. Set the window and functions (see the third screen) as shown here, and graph. A circle of radius 1 *will appear on the screen. Trace to move a short distance around the circle. In the screen, the point on the circle corresponds to an angle* T = 25°. *Since* $r = 1$, cos 25° *is* X = .90630779, *and* sin 25° *is* Y = .42261826.





This screen is a continuation of the previous one.



99. Use the right- and left-arrow keys to move to the point corresponding to 20°. What are cos 20° and sin 20°?

100. For what angle T, 0° ≤ T ≤ 90°, is cos T ≈ .766?

101. For what angle T, 0° ≤ T ≤ 90°, is sin T ≈ .574?

102. For what angle T, 0° ≤ T ≤ 90°, does cos T = sin T?

103. As T increases from 0° to 90°, does the cosine increase or decrease? What about the sine?

104. As T increases from 90° to 180°, does the cosine increase or decrease? What about the sine?

1.4 Using the Definitions of the Trigonometric Functions

Reciprocal Identities ▪ Signs and Ranges of Function Values ▪ Pythagorean Identities ▪ Quotient Identities

Identities are equations that are true for all values of the variables for which all expressions are defined. Identities are studied in more detail in **Chapter 5.**

$$(x + y)^2 = x^2 + 2xy + y^2 \qquad 2(x + 3) = 2x + 6 \qquad \text{Identities (Appendix A)}$$

Reciprocal Identities

Recall the definition of a reciprocal: the **reciprocal** of the nonzero number x is $\frac{1}{x}$. For example, the reciprocal of 2 is $\frac{1}{2}$, and the reciprocal of $\frac{8}{11}$ is $\frac{11}{8}$. There is no reciprocal for 0. Scientific calculators have a reciprocal key, usually labeled $\boxed{1/x}$ or $\boxed{x^{-1}}$. Using this key gives the reciprocal of any nonzero number entered in the display.

The definitions of the trigonometric functions in the previous section on page 23 were written so that functions in the same column are reciprocals of each other. Since $\sin\theta = \frac{y}{r}$ and $\csc\theta = \frac{r}{y}$,

$$\sin\theta = \frac{1}{\csc\theta} \quad \text{and} \quad \csc\theta = \frac{1}{\sin\theta}, \quad \text{provided } \sin\theta \neq 0.$$

Also, $\cos\theta$ and $\sec\theta$ are reciprocals, as are $\tan\theta$ and $\cot\theta$. The **reciprocal identities** hold for any angle θ that does not lead to a 0 denominator.

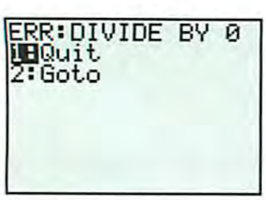
RECIPROCAL IDENTITIES

For all angles θ for which both functions are defined,

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}.$$

(a)



(b)

Figure 33

The screen in Figure 33(a) shows how to find csc 90°, sec 180°, and csc(−270°), using the appropriate reciprocal identities and the reciprocal key of a graphing calculator in degree mode. Attempting to find sec 90° by entering $\frac{1}{\cos 90°}$ produces an ERROR message, indicating the reciprocal is undefined. See Figure 33(b). Compare these results with the ones found in the table of quadrantal angle function values in **Section 1.3.** ■

▶ **Caution** ***Be sure not to use the inverse trigonometric function keys to find reciprocal function values.*** For example,

$$\sin^{-1}(90°) \neq \frac{1}{\sin(90°)}$$

Inverse trigonometric functions are covered in **Section 2.3.**

▶ **Note** Identities can be written in different forms. For example,

$$\sin\theta = \frac{1}{\csc\theta} \quad \text{can be written} \quad \csc\theta = \frac{1}{\sin\theta}, \quad \text{or} \quad (\sin\theta)(\csc\theta) = 1.$$

▶ EXAMPLE 1 USING THE RECIPROCAL IDENTITIES

Find each function value.

(a) $\cos\theta$, given that $\sec\theta = \frac{5}{3}$ **(b)** $\sin\theta$, given that $\csc\theta = -\frac{\sqrt{12}}{2}$

Solution

(a) Since $\cos\theta$ is the reciprocal of $\sec\theta$,

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{5}{3}} = 1 \div \frac{5}{3} = 1 \cdot \frac{3}{5} = \frac{3}{5}.$$

Simplify the complex fraction.

(b) $$\sin\theta = \frac{1}{-\frac{\sqrt{12}}{2}}$$ $\sin\theta = \frac{1}{\csc\theta}$

$$= -\frac{2}{\sqrt{12}}$$ Simplify the complex fraction as in part (a).

$$= -\frac{2}{2\sqrt{3}}$$ $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$$= -\frac{1}{\sqrt{3}}$$

We are multiplying by $1 = \frac{\sqrt{3}}{\sqrt{3}}$.

$$= -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$ Rationalize the denominator.

NOW TRY EXERCISES 1 AND 9. ◀

Signs and Ranges of Function Values

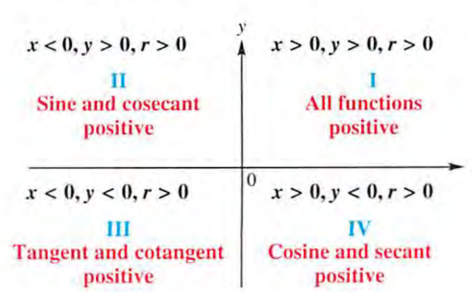
In the definitions of the trigonometric functions, r is the distance from the origin to the point (x, y). This distance is undirected, so $r > 0$. If we choose a point (x, y) in quadrant I, then both x and y will be positive, and the values of all six functions will be positive.

A point (x, y) in quadrant II has $x < 0$ and $y > 0$. This makes the values of sine and cosecant positive for quadrant II angles, while the other four functions take on negative values. Similar results can be obtained for the other quadrants.

This important information is summarized here.

Signs of Function Values

θ in Quadrant	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−



▶ EXAMPLE 2 DETERMINING SIGNS OF FUNCTIONS OF NONQUADRANTAL ANGLES

Determine the signs of the trigonometric functions of an angle in standard position with the given measure.

(a) $87°$ **(b)** $300°$ **(c)** $-200°$

Solution

(a) An angle of $87°$ is in the first quadrant, with x, y, and r all positive, so all of its trigonometric function values are positive.

(b) A $300°$ angle is in quadrant IV, so the cosine and secant are positive, while the sine, cosecant, tangent, and cotangent are negative.

(c) A $-200°$ angle is in quadrant II. The sine and cosecant are positive, and all other function values are negative.

NOW TRY EXERCISES 19, 21, AND 25. ◀

▶ **Note** Because numbers that are reciprocals will always have the same sign, knowing the sign of a function value will automatically determine the sign of the reciprocal function value.

▶ EXAMPLE 3 IDENTIFYING THE QUADRANT OF AN ANGLE

Identify the quadrant (or possible quadrants) of an angle θ that satisfies the given conditions.

(a) $\sin\theta > 0, \tan\theta < 0$ **(b)** $\cos\theta < 0, \sec\theta < 0$

Solution

(a) Since $\sin\theta > 0$ in quadrants I and II and $\tan\theta < 0$ in quadrants II and IV, both conditions are met only in quadrant II.

(b) The cosine and secant functions are both negative in quadrants II and III, so in this case θ could be in either of these two quadrants.

NOW TRY EXERCISES 35 AND 41. ◀

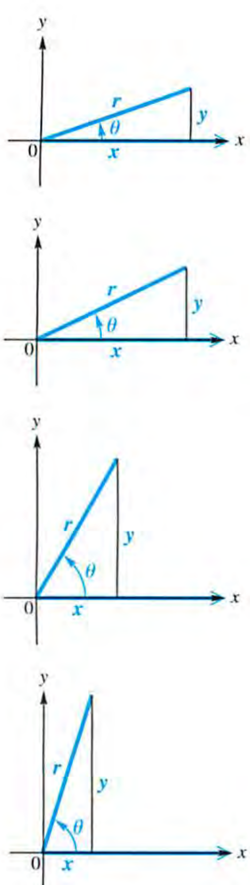


Figure 34

Figure 34 shows an angle θ as it increases in measure from near $0°$ toward $90°$. In each case, the value of r is the same. As the measure of the angle increases, y increases but never exceeds r, so $y \leq r$. Dividing both sides by the positive number r gives $\frac{y}{r} \leq 1$.

In a similar way, angles in quadrant IV suggest that

$$-1 \leq \frac{y}{r},$$

so $$-1 \leq \frac{y}{r} \leq 1$$

and $$\mathbf{-1 \leq \sin\theta \leq 1.}$$ $\frac{y}{r} = \sin\theta$ for any angle θ. (Section 1.3)

Similarly, $$\mathbf{-1 \leq \cos\theta \leq 1.}$$

The tangent of an angle is defined as $\frac{y}{x}$. It is possible that $x < y$, $x = y$, or $x > y$. Thus, $\frac{y}{x}$ can take any value, so **tan θ can be any real number, as can cot θ.**

The functions sec θ and csc θ are reciprocals of the functions cos θ and sin θ, respectively, making

$$\sec\theta \le -1 \quad \text{or} \quad \sec\theta \ge 1 \qquad \text{and} \qquad \csc\theta \le -1 \quad \text{or} \quad \csc\theta \ge 1.$$

In summary, the ranges of the trigonometric functions are as follows.

RANGES OF TRIGONOMETRIC FUNCTIONS

Trigonometric Function of θ	Range (Set-Builder Notation)	Range (Interval Notation)
$\sin\theta$, $\cos\theta$	$\{y \mid \lvert y\rvert \le 1\}$	$[-1, 1]$
$\tan\theta$, $\cot\theta$	$\{y \mid y$ is a real number$\}$	$(-\infty, \infty)$
$\sec\theta$, $\csc\theta$	$\{y \mid \lvert y\rvert \ge 1\}$	$(-\infty, -1] \cup [1, \infty)$

▶ EXAMPLE 4 DECIDING WHETHER A VALUE IS IN THE RANGE OF A TRIGONOMETRIC FUNCTION

Decide whether each statement is *possible* or *impossible.*

(a) $\sin\theta = 2.5$ **(b)** $\tan\theta = 110.47$ **(c)** $\sec\theta = .6$

Solution

(a) For any value of θ, $-1 \le \sin\theta \le 1$. Since $2.5 > 1$, it is impossible to find a value of θ with $\sin\theta = 2.5$.

(b) Tangent can take on any real number value. Thus, $\tan\theta = 110.47$ is possible.

(c) Since $\lvert\sec\theta\rvert \ge 1$ for all θ for which the secant is defined, the statement $\sec\theta = .6$ is impossible.

NOW TRY EXERCISES 45, 49, AND 51. ◀

The six trigonometric functions are defined in terms of x, y, and r, where the Pythagorean theorem shows that $r^2 = x^2 + y^2$ and $r > 0$. With these relationships, knowing the value of only one function and the quadrant in which the angle lies makes it possible to find the values of the other trigonometric functions.

▶ EXAMPLE 5 FINDING ALL FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT

Suppose that angle θ is in quadrant II and $\sin\theta = \frac{2}{3}$. Find the values of the other five trigonometric functions.

Solution Choose any point on the terminal side of angle θ. For simplicity, since $\sin\theta = \frac{y}{r}$, choose the point with $r = 3$.

$$\sin\theta = \frac{2}{3} \quad \text{Given value}$$

$$\frac{y}{r} = \frac{2}{3} \quad \text{Substitute } \tfrac{y}{r} \text{ for } \sin\theta.$$

Since $\frac{y}{r} = \frac{2}{3}$ and $r = 3$, then $y = 2$. To find x, use the equation $x^2 + y^2 = r^2$.

$$x^2 + y^2 = r^2$$

$$x^2 + 2^2 = 3^2 \quad \text{Substitute.}$$

$$x^2 + 4 = 9 \quad \text{Apply exponents.}$$

$$x^2 = 5 \quad \text{Subtract 4. (Appendix A)}$$

Remember *both* roots.

$$x = \sqrt{5} \quad \text{or} \quad x = -\sqrt{5} \quad \text{Square root property (Appendix A)}$$

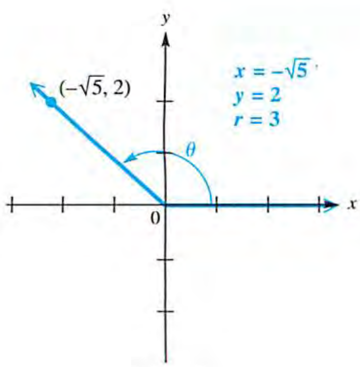


Figure 35

Since θ is in quadrant II, x must be negative, as shown in Figure 35, so $x = -\sqrt{5}$, and the point $(-\sqrt{5}, 2)$ is on the terminal side of θ. Now we can find the values of the remaining trigonometric functions.

$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{5}}{3} = -\frac{\sqrt{5}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{3}{-\sqrt{5}} = -\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{2}{-\sqrt{5}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Remember to rationalize denominators.

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{5}}{2} = -\frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{3}{2}$$

NOW TRY EXERCISE 71.

Pythagorean Identities

We derive three new identities from the relationship $x^2 + y^2 = r^2$.

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2} \quad \text{Divide by } r^2.$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1 \quad \text{Power rule for exponents; } \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1 \quad \cos\theta = \frac{x}{r},\ \sin\theta = \frac{y}{r} \text{ (Section 1.3)}$$

or $$\mathbf{\sin^2\theta + \cos^2\theta = 1}$$

Starting again with $x^2 + y^2 = r^2$ and dividing through by x^2 gives

$$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2} \quad \text{Divide by } x^2.$$

$$1 + \left(\frac{y}{x}\right)^2 = \left(\frac{r}{x}\right)^2 \quad \text{Power rule for exponents}$$

$$1 + (\tan\theta)^2 = (\sec\theta)^2 \quad \tan\theta = \frac{y}{x},\ \sec\theta = \frac{r}{x} \text{ (Section 1.3)}$$

or $$\mathbf{\tan^2\theta + 1 = \sec^2\theta.}$$

Similarly, dividing through by y^2 leads to

$$\mathbf{1 + \cot^2\theta = \csc^2\theta.}$$

These three identities are called the **Pythagorean identities** since the original equation that led to them, $x^2 + y^2 = r^2$, comes from the Pythagorean theorem.

PYTHAGOREAN IDENTITIES

For all angles θ for which the function values are defined,

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad \tan^2 \theta + 1 = \sec^2 \theta \qquad 1 + \cot^2 \theta = \csc^2 \theta.$$

As before, we have given only one form of each identity. However, algebraic transformations produce equivalent identities. For example, by subtracting $\sin^2 \theta$ from both sides of $\sin^2 \theta + \cos^2 \theta = 1$, we get the equivalent identity

$$\cos^2 \theta = 1 - \sin^2 \theta. \quad \text{Alternative form}$$

You should be able to transform these identities quickly and also recognize their equivalent forms.

▼ LOOKING AHEAD TO CALCULUS

The reciprocal, Pythagorean, and quotient identities are used in calculus to find derivatives and integrals of trigonometric functions. A standard technique of integration called **trigonometric substitution** relies on the Pythagorean identities.

Quotient Identities

Consider the quotient of $\sin \theta$ and $\cos \theta$, for $\cos \theta \neq 0$.

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r} \div \frac{x}{r} = \frac{y}{r} \cdot \frac{r}{x} = \frac{y}{x} = \tan \theta$$

Similarly, $\frac{\cos \theta}{\sin \theta} = \cot \theta$, for $\sin \theta \neq 0$. Thus, we have the **quotient identities.**

QUOTIENT IDENTITIES

For all angles θ for which the denominators are not zero,

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \qquad \frac{\cos \theta}{\sin \theta} = \cot \theta.$$

▶ EXAMPLE 6 FINDING OTHER FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT

Find $\sin \theta$ and $\tan \theta$, given that $\cos \theta = -\frac{\sqrt{3}}{4}$ and $\sin \theta > 0$.

Solution Start with $\sin^2 \theta + \cos^2 \theta = 1$.

$$\sin^2 \theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1 \quad \text{Replace } \cos \theta \text{ with } -\tfrac{\sqrt{3}}{4}.$$

$$\sin^2 \theta + \frac{3}{16} = 1 \quad \text{Square } -\tfrac{\sqrt{3}}{4}.$$

$$\sin^2 \theta = \frac{13}{16} \quad \text{Subtract } \tfrac{3}{16}.$$

$$\sin \theta = \pm\frac{\sqrt{13}}{4} \quad \text{Take square roots.}$$

Choose the correct sign here.

$$\sin \theta = \frac{\sqrt{13}}{4} \quad \text{Choose the positive square root since } \sin \theta \text{ is positive.}$$

To find $\tan\theta$, use the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}} = \frac{\sqrt{13}}{4}\left(-\frac{4}{\sqrt{3}}\right) = -\frac{\sqrt{13}}{\sqrt{3}}$$

$$= -\frac{\sqrt{13}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{39}}{3} \quad \text{Rationalize the denominator.}$$

NOW TRY EXERCISE 75.

▶ **Caution** ***In problems like those in Examples 5 and 6, be careful to choose the correct sign when square roots are taken.***

▶ EXAMPLE 7 FINDING OTHER FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT

Find $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{4}{3}$ and θ is in quadrant III.

Solution Since θ is in quadrant III, $\sin\theta$ and $\cos\theta$ will both be negative. It is tempting to say that since $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan\theta = \frac{4}{3}$, then $\sin\theta = -4$ and $\cos\theta = -3$. This is *incorrect,* however, since both $\sin\theta$ and $\cos\theta$ must be in the interval $[-1, 1]$.

We use the Pythagorean identity $\tan^2\theta + 1 = \sec^2\theta$ to find $\sec\theta$, and then the reciprocal identity $\cos\theta = \frac{1}{\sec\theta}$ to find $\cos\theta$.

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta \quad \tan\theta = \tfrac{4}{3}$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\frac{25}{9} = \sec^2\theta$$

Be careful to choose the correct sign here.

$$-\frac{5}{3} = \sec\theta \quad \text{Choose the negative square root since } \sec\theta \text{ is negative when } \theta \text{ is in quadrant III.}$$

$$-\frac{3}{5} = \cos\theta \quad \text{Secant and cosine are reciprocals.}$$

Since $\sin^2\theta = 1 - \cos^2\theta$,

$$\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2 \quad \cos\theta = -\tfrac{3}{5}$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$\sin^2\theta = \frac{16}{25}$$

Again, be careful.

$$\sin\theta = -\frac{4}{5}. \quad \text{Choose the negative square root.}$$

NOW TRY EXERCISE 73.

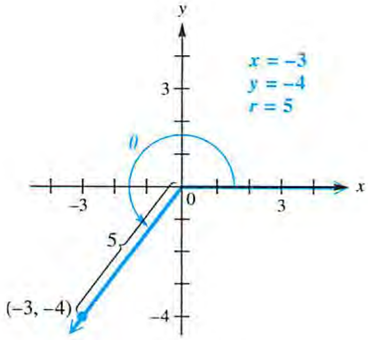


Figure 36

▶ **Note** Example 7 can also be worked by drawing θ in standard position in quadrant III, finding r to be 5, and then using the definitions of $\sin \theta$ and $\cos \theta$ in terms of x, y, and r. See Figure 36.

When using this method, be sure to choose the correct signs for x and y. This is analogous to choosing the correct signs after applying the Pythagorean identities. Always check to be sure that the signs of the functions correspond to those found in the table at the bottom of page 31.

1.4 Exercises

Use the appropriate reciprocal identity to find each function value. Rationalize denominators when applicable. See Example 1.

1. $\sec \theta$, given that $\cos \theta = \frac{2}{3}$

2. $\sec \theta$, given that $\cos \theta = \frac{5}{8}$

3. $\csc \theta$, given that $\sin \theta = -\frac{3}{7}$

4. $\csc \theta$, given that $\sin \theta = -\frac{8}{43}$

5. $\cot \theta$, given that $\tan \theta = 5$

6. $\cot \theta$, given that $\tan \theta = 18$

7. $\cos \theta$, given that $\sec \theta = -\frac{5}{2}$

8. $\cos \theta$, given that $\sec \theta = -\frac{11}{7}$

9. $\sin \theta$, given that $\csc \theta = \frac{\sqrt{8}}{2}$

10. $\sin \theta$, given that $\csc \theta = \frac{\sqrt{24}}{3}$

11. $\tan \theta$, given that $\cot \theta = -2.5$

12. $\tan \theta$, given that $\cot \theta = -.01$

13. $\sin \theta$, given that $\csc \theta = 1.42716321$

14. $\cos \theta$, given that $\sec \theta = 9.80425133$

15. Can a given angle θ satisfy both $\sin \theta > 0$ and $\csc \theta < 0$? Explain.

16. Explain what is wrong with the following item that appears on a trigonometry test:

$$\text{“}\textit{Find } \sec \theta, \textit{ given that } \cos \theta = \frac{3}{2}.\text{”}$$

17. *Concept Check* What is wrong with the following statement? $\tan 90° = \frac{1}{\cot 90°}$.

18. *Concept Check* One form of a particular reciprocal identity is $\tan \theta = \frac{1}{\cot \theta}$. Give two other equivalent forms of this identity.

Determine the signs of the trigonometric functions of an angle in standard position with the given measure. See Example 2.

19. 74° **20.** 84° **21.** 218° **22.** 195°

23. 178° **24.** 125° **25.** −80° **26.** −15°

27. 845° **28.** 1005° **29.** −345° **30.** −705°

Identify the quadrant (or possible quadrants) of an angle θ that satisfies the given conditions. See Example 3.

31. $\sin \theta > 0, \csc \theta > 0$

32. $\cos \theta > 0, \sec \theta > 0$

33. $\cos \theta > 0, \sin \theta > 0$

34. $\sin \theta > 0, \tan \theta > 0$

35. $\tan \theta < 0, \cos \theta < 0$

36. $\cos \theta < 0, \sin \theta < 0$

37. $\sec \theta > 0, \csc \theta > 0$

38. $\csc \theta > 0, \cot \theta > 0$

39. $\sec \theta < 0, \csc \theta < 0$

40. $\cot \theta < 0, \sec \theta < 0$

41. $\sin \theta < 0, \csc \theta < 0$

42. $\tan \theta < 0, \cot \theta < 0$

43. Explain why the answers to Exercises 33 and 37 are the same.

44. Explain why there is no angle θ that satisfies $\tan \theta > 0$, $\cot \theta < 0$.

Decide whether each statement is possible *or* impossible *for an angle θ. See Example 4.*

45. $\sin \theta = 2$
46. $\sin \theta = 3$
47. $\cos \theta = -.96$
48. $\cos \theta = -.56$
49. $\tan \theta = .93$
50. $\cot \theta = .93$
51. $\sec \theta = -.3$
52. $\sec \theta = -.9$
53. $\csc \theta = 100$
54. $\csc \theta = -100$
55. $\cot \theta = -4$
56. $\cot \theta = -6$
57. $\sin \theta = \frac{1}{2}$ and $\csc \theta = 2$
58. $\tan \theta = 2$ and $\cot \theta = -2$
59. $\cos \theta = -2$ and $\sec \theta = \frac{1}{2}$

60. Explain why there is no angle θ that satisfies $\cos \theta = \frac{1}{2}$ and $\sec \theta = -2$.

Use identities to solve each of the following. See Examples 5–7.

61. Find $\cos \theta$, given that $\sin \theta = \frac{3}{5}$ and θ is in quadrant II.
62. Find $\sin \theta$, given that $\cos \theta = \frac{4}{5}$ and θ is in quadrant IV.
63. Find $\csc \theta$, given that $\cot \theta = -\frac{1}{2}$ and θ is in quadrant IV.
64. Find $\sec \theta$, given that $\tan \theta = \frac{\sqrt{7}}{3}$ and θ is in quadrant III.
65. Find $\tan \theta$, given that $\sin \theta = \frac{1}{2}$ and θ is in quadrant II.
66. Find $\cot \theta$, given that $\csc \theta = -2$ and θ is in quadrant III.
67. Find $\cot \theta$, given that $\csc \theta = -3.5891420$ and θ is in quadrant III.
68. Find $\tan \theta$, given that $\sin \theta = .49268329$ and θ is in quadrant II.

Find the five remaining trigonometric function values for each angle θ. See Examples 5–7.

69. $\tan \theta = -\frac{15}{8}$, given that θ is in quadrant II
70. $\cos \theta = -\frac{3}{5}$, given that θ is in quadrant III
71. $\sin \theta = \frac{\sqrt{5}}{7}$, given that θ is in quadrant I
72. $\tan \theta = \sqrt{3}$, given that θ is in quadrant III
73. $\cot \theta = \frac{\sqrt{3}}{8}$, given that θ is in quadrant I
74. $\csc \theta = 2$, given that θ is in quadrant II
75. $\sin \theta = \frac{\sqrt{2}}{6}$, given that $\cos \theta < 0$
76. $\cos \theta = \frac{\sqrt{5}}{8}$, given that $\tan \theta < 0$
77. $\sec \theta = -4$, given that $\sin \theta > 0$
78. $\csc \theta = -3$, given that $\cos \theta > 0$
79. $\sin \theta = .164215$, given that θ is in quadrant II
80. $\cot \theta = -1.49586$, given that θ is in quadrant IV

Work each problem.

81. Derive the identity $1 + \cot^2 \theta = \csc^2 \theta$ by dividing $x^2 + y^2 = r^2$ by y^2.

82. Using a method similar to the one given in this section showing that $\frac{\sin \theta}{\cos \theta} = \tan \theta$, show that $\frac{\cos \theta}{\sin \theta} = \cot \theta$.

83. *Concept Check* *True* or *false*: For all angles θ, $\sin \theta + \cos \theta = 1$. If false, give an example showing why it is false.

84. *Concept Check* *True* or *false*: Since $\cot \theta = \frac{\cos \theta}{\sin \theta}$, if $\cot \theta = \frac{1}{2}$ with θ in quadrant I, then $\cos \theta = 1$ and $\sin \theta = 2$. If false, explain why.

Concept Check *Suppose that* $90° < \theta < 180°$. *Find the sign of each function value.*

85. $\sin 2\theta$

86. $\tan \dfrac{\theta}{2}$

87. $\cot(\theta + 180°)$

88. $\cos(-\theta)$

Concept Check *Suppose that* $-90° < \theta < 90°$. *Find the sign of each function value.*

89. $\cos \dfrac{\theta}{2}$

90. $\cos(\theta + 180°)$

91. $\sec(-\theta)$

92. $\sec(\theta - 180°)$

Find a value of each variable.

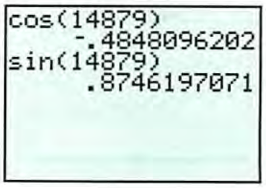
93. $\tan(3\theta - 4°) = \dfrac{1}{\cot(5\theta - 8°)}$

94. $\sec(2\theta + 6°)\cos(5\theta + 3°) = 1$

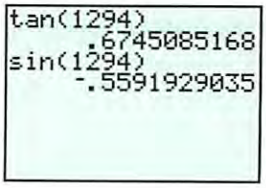
95. $\sin(4\theta + 2°)\csc(3\theta + 5°) = 1$

96. $\cos(6A + 5°) = \dfrac{1}{\sec(4A + 15°)}$

97. *Concept Check* The screen below was obtained with the calculator in degree mode. How can we use it to justify that an angle of 14,879° is a quadrant II angle?



98. *Concept Check* The screen below was obtained with the calculator in degree mode. In which quadrant does a 1294° angle lie?



Chapter 1 Summary

KEY TERMS

1.1 line, line segment (or segment), ray, endpoint of a ray, angle, side of an angle, vertex of an angle, initial side, terminal side, positive angle, negative angle, degree, acute angle, right angle, obtuse angle, straight angle, complementary angles (complements), supplementary angles (supplements), minute, second, angle in standard position, quadrantal angle, coterminal angles

1.2 vertical angles, parallel lines, transversal, similar triangles, congruent triangles

1.3 sine (sin), cosine (cos), tangent (tan), cotangent (cot), secant (sec), cosecant (csc), degree mode

1.4 reciprocal

NEW SYMBOLS

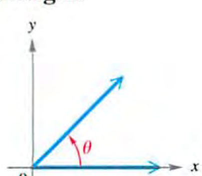
┐ right angle symbol (for a right triangle)
θ Greek letter theta
° degree
′ minute
″ second

QUICK REVIEW

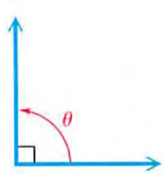
CONCEPTS	EXAMPLES

1.1 Angles

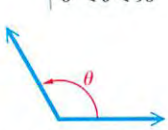
Types of Angles



Acute angle
$0° < \theta < 90°$



Right angle
$\theta = 90°$



Obtuse angle
$90° < \theta < 180°$

Straight angle
$\theta = 180°$

If $\theta = 46°$, then angle θ is an acute angle.

If $\theta = 90°$, then angle θ is a right angle.

If $\theta = 148°$, then angle θ is an obtuse angle.

If $\theta = 180°$, then angle θ is a straight angle.

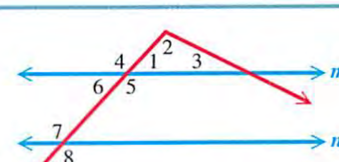
The acute angle θ in the figure at the left is in standard position. If θ measures 46°, find the measure of a negative coterminal angle.

$$46° - 360° = -314°$$

1.2 Angle Relationships and Similar Triangles

Vertical angles have equal measures.

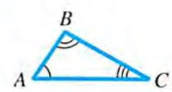
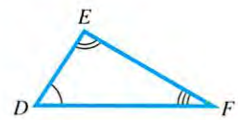
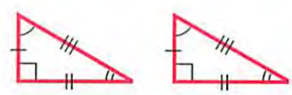
The sum of the measures of the angles of any triangle is 180°.



m and *n* are parallel lines.

Vertical angles 4 and 5 are equal.

The sum of angles 1, 2, and 3 is 180°.

CONCEPTS	EXAMPLES
When a transversal intersects parallel lines, the following angles formed have equal measure: alternate interior, alternate exterior, and corresponding. Interior angles on the same side of the transversal are supplementary.	Refer to the diagram at the bottom of the previous page. Angles 5 and 7 are alternate interior angles, so they are equal. Angles 4 and 8 are alternate exterior angles, so they are equal. Angles 4 and 7 are corresponding angles, so they are equal. Angles 6 and 7 are interior angles on the same side of the transversal, so they are supplementary.
Similar triangles have corresponding angles with the same measures, and corresponding sides proportional.	 Pairs of corresponding angles as marked in triangles *ABC* and *DEF* are equal.  Also, $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$.
Congruent triangles are the same size and the same shape.	 Corresponding angles are equal, and corresponding sides are equal.

1.3 Trigonometric Functions

Definitions of the Trigonometric Functions Let (x, y) be a point other than the origin on the terminal side of an angle θ in standard position. Let $r = \sqrt{x^2 + y^2}$ represent the distance from the origin to (x, y). Then $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}\ (x \neq 0)$ $\csc\theta = \frac{r}{y}\ (y \neq 0)$ $\sec\theta = \frac{r}{x}\ (x \neq 0)$ $\cot\theta = \frac{x}{y}\ (y \neq 0)$. See the summary table of trigonometric function values for quadrantal angles on page 27.	If the point $(-2, 3)$ is on the terminal side of angle θ in standard position, then $x = -2$, $y = 3$, and $r = \sqrt{(-2)^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$. Then $\sin\theta = \frac{3\sqrt{13}}{13}$, $\cos\theta = -\frac{2\sqrt{13}}{13}$, $\tan\theta = -\frac{3}{2}$, $\csc\theta = \frac{\sqrt{13}}{3}$, $\sec\theta = -\frac{\sqrt{13}}{2}$, $\cot\theta = -\frac{2}{3}$.

1.4 Using the Definitions of the Trigonometric Functions

Reciprocal Identities $\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$	If $\cot\theta = -\frac{2}{3}$, find $\tan\theta$. $\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-\frac{2}{3}} = -\frac{3}{2}$

(continued)

CONCEPTS	EXAMPLES

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Use the function values for the example from **Section 1.3** to illustrate the Pythagorean identities.

$$\sin^2\theta + \cos^2\theta = \left(\frac{3\sqrt{13}}{13}\right)^2 + \left(-\frac{2\sqrt{13}}{13}\right)^2$$
$$= \frac{9}{13} + \frac{4}{13} = 1,$$
$$\tan^2\theta + 1 = \left(-\frac{3}{2}\right)^2 + 1 = \frac{13}{4} = \left(-\frac{\sqrt{13}}{2}\right)^2 = \sec^2\theta,$$
$$1 + \cot^2\theta = 1 + \left(-\frac{2}{3}\right)^2 = \frac{13}{9} = \left(\frac{\sqrt{13}}{3}\right)^2 = \csc^2\theta.$$

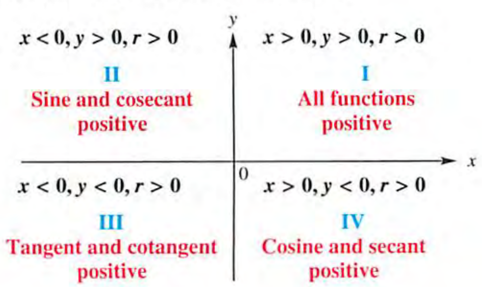
Quotient Identities

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Use the function values for the example from **Section 1.3** to illustrate $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{3\sqrt{13}}{13}}{-\frac{2\sqrt{13}}{13}} = \frac{3\sqrt{13}}{13}\left(-\frac{13}{2\sqrt{13}}\right) = -\frac{3}{2} = \tan\theta$$

Signs of the Trigonometric Functions



Identify the quadrant(s) of any angle θ that satisfies $\sin\theta < 0$, $\tan\theta > 0$.

Since $\sin\theta < 0$ in quadrants III and IV, while $\tan\theta > 0$ in quadrants I and III, both conditions are met only in quadrant III.

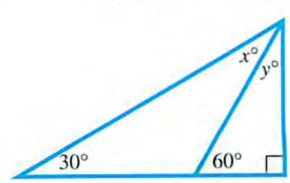
CHAPTER 1 ▶ Review Exercises

1. Give the measures of the complement and the supplement of an angle measuring 35°.

Find the angle of least possible positive measure coterminal with each angle.

2. −51° **3.** −174° **4.** 792°

5. Find the measure of each marked angle.



Work each problem.

6. ***Rotating Pulley*** A pulley is rotating 320 times per min. Through how many degrees does a point on the edge of the pulley move in $\frac{2}{3}$ sec?

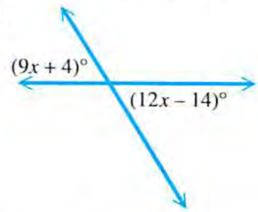
7. ***Rotating Propeller*** The propeller of a speedboat rotates 650 times per min. Through how many degrees will a point on the edge of the propeller rotate in 2.4 sec?

Convert decimal degrees to degrees, minutes, seconds, and convert degrees, minutes, seconds to decimal degrees. Round to the nearest second or the nearest thousandth of a degree, as appropriate. Use a calculator as necessary.

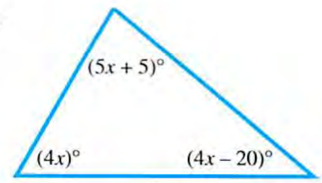
8. $47° \; 25' \; 11''$ **9.** $119° \; 8' \; 3''$ **10.** $-61.5034°$ **11.** $275.1005°$

Find the measure of each marked angle.

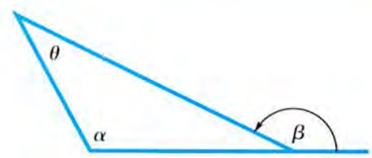
12.



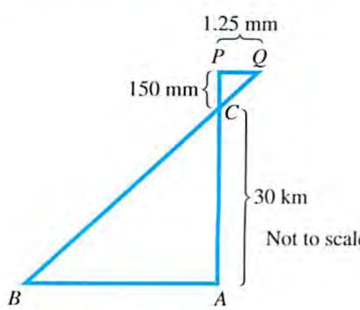
13.



14. Express θ in terms of α and β.

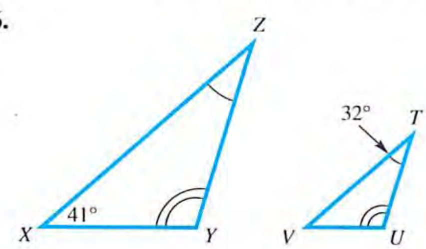


15. ***Length of a Road*** The flight path CP of a satellite carrying a camera with its lens at C is shown in the figure. Length PC represents the distance from the lens to the film PQ, and BA represents a straight road on the ground. Use the measurements given in the figure to find the length of the road. (*Source:* Kastner, B., *Space Mathematics,* NASA, 1985.)

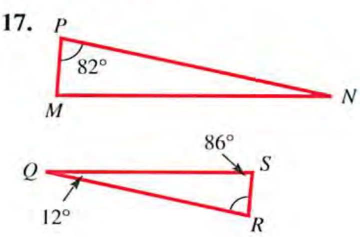


Find all unknown angle measures in each pair of similar triangles.

16.

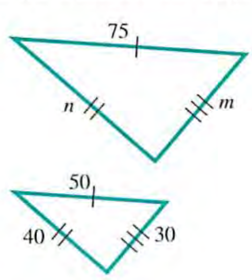


17.

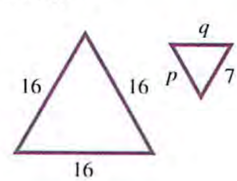


Find the unknown side lengths in each pair of similar triangles.

18.

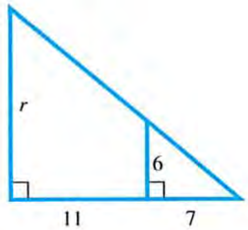


19.

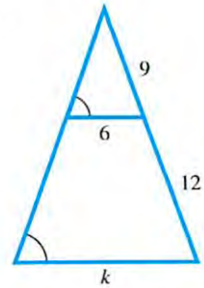


Find the unknown measurement. There are two similar triangles in each figure.

20.



21.

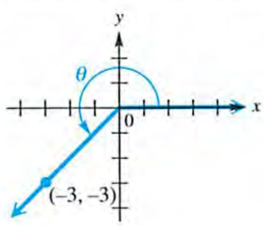


22. ***Concept Check*** Complete the following statement: If two triangles are similar, their corresponding sides are __________ and the measures of their corresponding angles are __________.

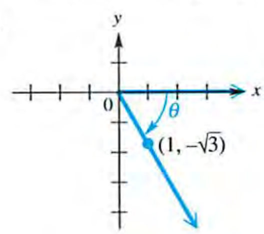
23. ***Length of a Shadow*** If a tree 20 ft tall casts a shadow 8 ft long, how long would the shadow of a 30-ft tree be at the same time and place?

Find the six trigonometric function values for each angle. If a value is undefined, say so.

24.



25.



26. 180°

Find the values of the six trigonometric functions for an angle in standard position having each point on its terminal side.

27. $(3, -4)$

28. $(9, -2)$

29. $(-8, 15)$

30. $(1, -5)$

31. $\left(6\sqrt{3}, -6\right)$

32. $\left(-2\sqrt{2}, 2\sqrt{2}\right)$

33. ***Concept Check*** If the terminal side of a quadrantal angle lies along the y-axis, which of its trigonometric functions are undefined?

34. Find the values of all six trigonometric functions for an angle in standard position having its terminal side defined by the equation $5x - 3y = 0$, $x \geq 0$.

In Exercises 35 and 36, consider an angle θ in standard position whose terminal side has the equation $y = -5x$, with $x \leq 0$.

35. Sketch θ and use an arrow to show the rotation if $0° \leq \theta < 360°$.

36. Find the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.

Complete the table with the appropriate function values of the given quadrantal angles. If the value is undefined, say so.

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
37.	180°						
38.	−90°						

39. Decide whether each statement is *possible* or *impossible*.

(a) $\sec \theta = -\frac{2}{3}$ **(b)** $\tan \theta = 1.4$ **(c)** $\csc \theta = 5$

Find all six trigonometric function values for each angle. Rationalize denominators when applicable.

40. $\sin \theta = \frac{\sqrt{3}}{5}$, given that $\cos \theta < 0$

41. $\cos \theta = -\frac{5}{8}$, given that θ is in quadrant III

42. $\tan \theta = 2$, given that θ is in quadrant III

43. $\sec \theta = -\sqrt{5}$, given that θ is in quadrant II

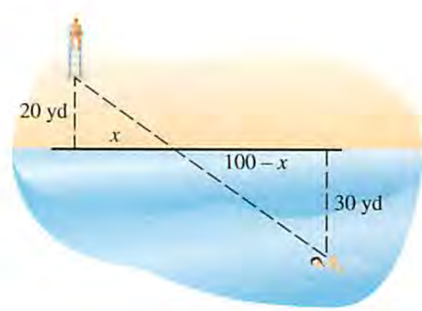
44. $\sin \theta = -\frac{2}{5}$, given that θ is in quadrant III

45. $\sec \theta = \frac{5}{4}$, given that θ is in quadrant IV

46. ***Concept Check*** If, for some particular angle θ, $\sin \theta < 0$ and $\cos \theta > 0$, in what quadrant must θ lie? What is the sign of $\tan \theta$?

Solve each problem.

47. ***Swimmer in Distress*** A lifeguard located 20 yd from the water spots a swimmer in distress. The swimmer is 30 yd from shore and 100 yd east of the lifeguard. Suppose the lifeguard runs, then swims to the swimmer in a direct line, as shown in the figure. How far east from his original position will he enter the water? (*Hint:* Find the value of x in the sketch.)



48. ***Angle the Celestial North Pole Moves*** At present, the north star Polaris is located very near the celestial north pole. However, because Earth is inclined 23.5°, the moon's gravitational pull on Earth is uneven. As a result, Earth slowly precesses (moves in) like a spinning top and the direction of the celestial north pole traces out a circular path once every 26,000 yr. See the figure. For example, in approximately A.D. 14,000 the star Vega will be located at the celestial north pole—and not the star Polaris. As viewed from the center C of this circular path, calculate the angle (to the nearest second) that the celestial north pole moves each year. (*Source:* Zeilik, M., S. Gregory, and E. Smith, *Introductory Astronomy and Astrophysics,* Second Edition, Saunders College Publishers, 1998.)

49. ***Depth of a Crater on the Moon*** The depths of unknown craters on the moon can be approximated by comparing the lengths of their shadows to the shadows of nearby craters with known depths. The crater Aristillus is 11,000 ft deep, and its shadow was measured as 1.5 mm on a photograph. Its companion crater, Autolycus, had a shadow of 1.3 mm on the same photograph. Use similar triangles to determine the depth of the crater Autolycus. (*Source:* Webb, T., *Celestial Objects for Common Telescopes,* Dover Publications, 1962.)

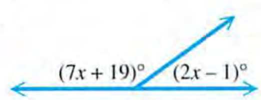
50. ***Height of a Lunar Peak*** The lunar mountain peak Huygens has a height of 21,000 ft. The shadow of Huygens on a photograph was 2.8 mm, while the nearby mountain Bradley had a shadow of 1.8 mm on the same photograph. Calculate the height of Bradley. (*Source:* Webb, T., *Celestial Objects for Common Telescopes,* Dover Publications, 1962.)

CHAPTER 1 ▶ Test

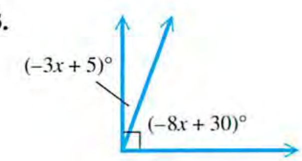
1. For an angle measuring 67°, give the measure of its **(a)** complement and **(b)** supplement.

Find the measure of each unknown angle.

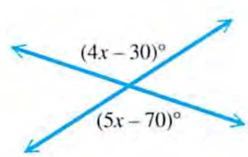
2.



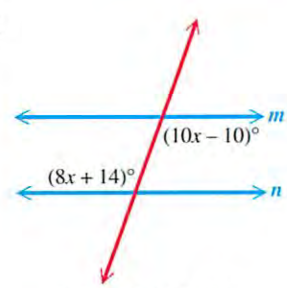
3.



4.

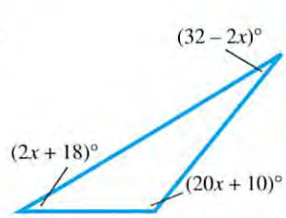


5.



m and *n* are parallel.

6.



7. Perform the indicated conversion.

(a) 74° 18′ 36″ to decimal degrees **(b)** 45.2025° to degrees, minutes, seconds

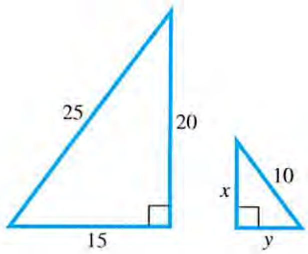
8. Find the least positive measure of an angle coterminal with an angle of the given measure.

(a) $390°$ **(b)** $-80°$ **(c)** $810°$

9. ***Rotating Tire*** A tire rotates 450 times per min. Through how many degrees does a point on the edge of the tire move in 1 sec?

10. ***Length of a Shadow*** If a vertical pole 30 ft tall casts a shadow 8 ft long, how long would the shadow of a 40-ft pole be at the same time and place?

11. Find the unknown side lengths x and y in this pair of similar triangles.



Draw a sketch of an angle in standard position having the given point on its terminal side. Indicate the angle of least positive measure θ, and give the values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, *and* $\csc\theta$. *If any of these are undefined, say so.*

12. $(2, -7)$ **13.** $(0, -2)$

14. Draw a sketch of an angle in standard position having the equation $3x - 4y = 0$, $x \le 0$, as its terminal side. Indicate the angle of least positive measure θ, and give the values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.

15. Complete the table with the appropriate function values of the given quadrantal angles. If the value is undefined, say so.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$90°$						
$-360°$						
$630°$						

16. If the terminal side of a quadrantal angle lies along the negative x-axis, which two of its trigonometric function values are undefined?

17. Identify the possible quadrant or quadrants in which θ must lie under the given conditions.

(a) $\cos\theta > 0$, $\tan\theta > 0$ **(b)** $\sin\theta < 0$, $\csc\theta < 0$ **(c)** $\cot\theta > 0$, $\cos\theta < 0$

18. Decide whether each statement is *possible* or *impossible* for some angle θ.

(a) $\sin\theta = 1.5$ **(b)** $\sec\theta = 4$ **(c)** $\tan\theta = 10{,}000$

19. Find the value of $\sec\theta$ if $\cos\theta = -\frac{7}{12}$.

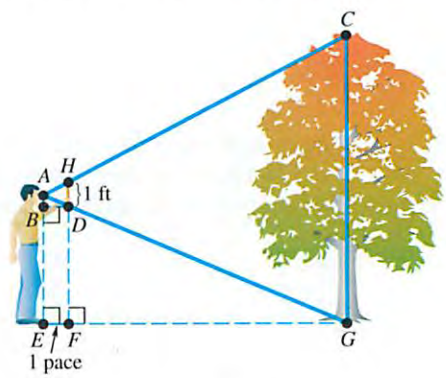
20. Find the five remaining trigonometric function values of θ if $\sin\theta = \frac{3}{7}$ and θ is in quadrant II.

CHAPTER 1 ▶ Quantitative Reasoning

Have you ever gazed up at a redwood tree, a skyscraper, a public monument, or perhaps a dinosaur in a museum and wondered how tall it was?

There is a relatively simple way to make a reasonable estimate. All you need is a 1-ft ruler. Hold the ruler vertically at arm's length as you approach the object to be measured. Stop when one end of the ruler lines up with the top of the object and the other end with its base. Now pace off the distance to the object, taking normal strides. The number of paces will be the approximate height of the object in feet.

The reason this method works depends on a concept presented in this chapter. Furnish the reasons for the following steps, which refer to the figure. (Assume that the length of one pace is EF.) Then answer the question.



Reasons

Step 1 $CG = \dfrac{CG}{1} = \dfrac{AG}{AD}$ ____________________

Step 2 $\dfrac{AG}{AD} = \dfrac{EG}{EF}$ ____________________

Step 3 $\dfrac{EG}{EF} = \dfrac{EG}{BD} = \dfrac{EG}{1}$ ____________________

Step 4 CG ft $= EG$ paces ____________________

What is the height of the tree?