8.2 --2016Trig(Polar) Form of Complex Numbers

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8.2 Trigonometric (Polar) Form of Complex Numbers

The Complex Plane and Vector Representation ▪ Trigonometric (Polar) Form ▪ Converting Between Rectangular and Trigonometric (Polar) Forms ▪ An Application of Complex Numbers to Fractals

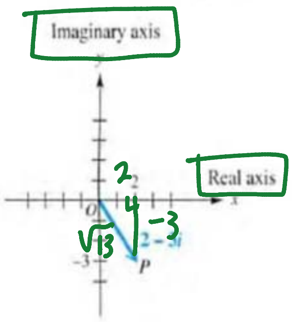


Figure 3

The Complex Plane and Vector Representation Unlike real numbers, complex numbers cannot be ordered. One way to organize and illustrate them is by using a graph. To graph a complex number such as $2 - 3i$, we modify the familiar coordinate system by calling the horizontal axis the **real axis** and the vertical axis the **imaginary axis.** Then complex numbers can be graphed in this **complex plane,** as shown in Figure 3. Each complex number $a + bi$ determines a unique position vector with initial point $(0, 0)$ and terminal point (a, b). This relationship shows the close connection between vectors and complex numbers.

▶ **Note** This geometric representation is the reason that $\boxed{a + bi}$ is called the **rectangular form** of a complex number. (*Rectangular form* is also called *standard form.*)

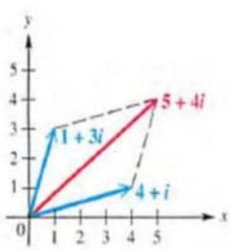


Figure 4

Recall that the sum of the two complex numbers $4 + i$ and $1 + 3i$ is

$$(4 + i) + (1 + 3i) = 5 + 4i. \quad \text{(Section 8.1)}$$

Graphically, the sum of two complex numbers is represented by the vector that is the resultant of the vectors corresponding to the two numbers, as shown in Figure 4.

a+bi

Trigonometric (Polar) Form Figure 6 shows the complex number $x + yi$ that corresponds to a vector **OP** with direction angle θ and magnitude r. The following relationships among x, y, r, and θ can be verified from Figure 6.

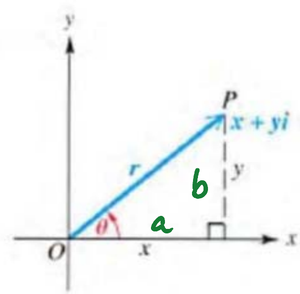


Figure 6

RELATIONSHIPS AMONG x, y, r, AND θ

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}, \quad \text{if } x \neq 0$$

Substituting $x = r\cos\theta$ and $y = r\sin\theta$ into $x + yi$ gives

$$\begin{aligned} x + yi &= r\cos\theta + (r\sin\theta)i \\ &= r(\cos\theta + i\sin\theta). \quad \text{Factor out } r. \end{aligned}$$

TRIGONOMETRIC (POLAR) FORM OF A COMPLEX NUMBER

The expression

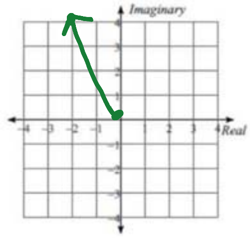
$$r(\cos\theta + i\sin\theta)$$

is called the **trigonometric form** (or **polar form**) of the complex number $x + yi$. The expression $\cos\theta + i\sin\theta$ is sometimes abbreviated **cis θ.** Using this notation, $r(\cos\theta + i\sin\theta)$ is written r **cis θ.**

Graph each complex number. See Example 1.

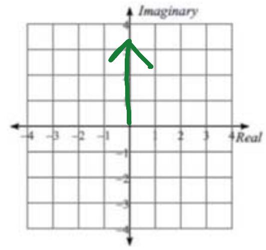
4. $-2 + 4i$

$r = 2\sqrt{5}$



8. $3i$

$r = 3$



Write each complex number in rectangular form. See Example 2.

$a+bi$

26. $4(\cos 60° + i \sin 60°)$

$4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\boxed{2 + 2\sqrt{3}i}$

36. $\sqrt{3}$ cis $315°$

$\sqrt{3}(\cos 315 + i \sin 315)$

$\sqrt{3}\left(\frac{\sqrt{2}}{2} + \frac{-\sqrt{2}}{2}i\right)$

$\boxed{\frac{\sqrt{6}}{2} + \frac{-\sqrt{6}}{2}i}$

Converting Between Rectangular and Trigonometric (Polar) Forms To convert from rectangular form to trigonometric form, we use the following procedure.

CONVERTING FROM RECTANGULAR FORM TO TRIGONOMETRIC FORM

Step 1 Sketch a graph of the number $x + yi$ in the complex plane.

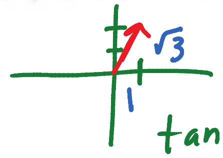
Step 2 Find r by using the equation $r = \sqrt{x^2 + y^2}$.

Step 3 Find θ by using the equation $\tan \theta = \frac{y}{x}$, $x \neq 0$, choosing the quadrant indicated in Step 1.

▶ **Caution** Errors often occur in Step 3. ***Be sure to choose the correct quadrant for θ by referring to the graph sketched in Step 1.***

Write each complex number in trigonometric form $r(\cos\theta + i\sin\theta)$, *with* θ *in the interval* $[0°, 360°)$. *See Example 3.*

40. $1 + i\sqrt{3}$



$r = \sqrt{1^2 + (\sqrt{3})^2}$

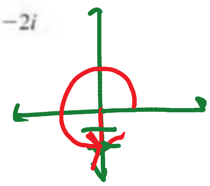
$r = 2$

$\tan\theta = \frac{\sqrt{3}}{1}$

$2(\cos 60° + i\sin 60°)$

$2(\text{cis } 60°)$

48. $-2i$



$r = 2$

$2(\cos 270° + i\sin 270°)$

$2 \text{ cis } 270°$

Perform each conversion, using a calculator to approximate answers as necessary. See Example 4, express degrees to the nearest hundredth, real and imaginary parts to four decimal places.

	Rectangular Form	Trigonometric Form
54.	$\boxed{-4 + i}$	$\sqrt{17}(\cos 165.96 + i\sin 165.96)$
58.	$-.3502 + .9367i$	$1 \text{ cis } 110.5°$

$r = \sqrt{17}$

$\theta = \tan^{-1} = \frac{1}{-4}$ -14.04

$1(\cos 110.5 + i\sin 110.5)$

$1(-.3502 + .9367i)$