8.3 --2016 Product/Quotient Theorems

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CAT Notes Trig 8.3 The Product & Quotient Theorems

By the end of this lesson, you will be able to:

- Find the product of complex numbers in trigonometric form
- Find the quotient of complex numbers in trigonometric form

Warm up: Convert each standard (rectangular) form number into polar (trig) form or vice versa.

Use θ in the interval [0°, 360°).

❶ 6 + 2i

$r = 2\sqrt{10}$

$2\sqrt{10}\,(\cos 18.4° + i\sin 18.4°)$

❷ -3 + 4i

$5(\cos 126.7 + i\sin 126.7)$

❸ 4 cis 150°

$4(\cos 150 + i\sin 150)$

$4\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$-2\sqrt{3} + 2i$

❹ $\sqrt{2}\ cis\ 315°$

$\sqrt{2}\left(\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}i\right)$

$1 - i$

Part 1: Products

Previously, we learned to multiply binomial expressions using "FOIL" or double distribution.

Example: Find the product of $(1 + i\sqrt{3})(-2\sqrt{3} + 2i)$ $= -2\sqrt{3} + 2i - 6i + 2i^2\sqrt{3}$

$(-2\sqrt{3})(i\sqrt{3})$

$-2i\sqrt{9}$

$-2i \cdot 3 = -6i$

$-2\sqrt{3}$

$-4\sqrt{3} - 4i$

Another way to find this product is to first CONVERT COMPLEX NUMBERS INTO TRIGONOMETRIC (POLAR) FORM.

$(1 + i\sqrt{3}) =$ $2\ cis\ 60°$

$(-2\sqrt{3} + 2i) =$ $4\ cis\ 150°$

$-4\sqrt{3} - 4i = 8\ cis\ 210°$

$(2\sqrt{3})^2$

$2\sqrt{3} \cdot 2\sqrt{3}$

$4 \cdot 3$

Now multiply these trig forms. Use some trig identities to simplify the expression. (Angle sum/difference identities)

$[2(\cos 60° + i\sin 60°)][4(\cos 150° + i\sin 150°)]$

$8[\cos 60°\cos 150° + i^2\sin 60°\sin 150° + i\cos 60°\sin 150° + i\sin 60°\cos 150°$

$8[\cos(60 + 150) + i\sin(60 + 150)$

$8(\cos 210 + i\sin 210°)$

$8(cis\ 210)$

CAT Notes Trig 8.3 The Product & Quotient Theorems

We may generalize this work in the following "PRODUCT THEOREM."

PRODUCT THEOREM

If $r_1(\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, then

$$[r_1(\cos\theta_1 + i\sin\theta_1)]\cdot[r_2(\cos\theta_2 + i\sin\theta_2)]$$
$$= r_1r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)].$$

In compact form, this is written

$$(r_1 \operatorname{cis}\theta_1)(r_2 \operatorname{cis}\theta_2) = r_1r_2 \operatorname{cis}(\theta_1+\theta_2).$$

In other words, to multiply complex numbers in trig form,

multiply their absolute values ($r_1 \cdot r_2$) and add their arguments ($\theta_1 + \theta_2$).

Examples: Using the product theorem, multiply the following & write the result in rectangular form.

❺ $3(\cos 45° + i\sin 45°)$ and $2(\cos 135° + i\sin 135°)$ $= 6(\cos 180 + i\sin 180)$

$= 6(-1 + 0i)$

$= -6$

❻ $[4(\cos 120° + i\sin 120°)][5(\cos 30° + i\sin 30°)]$ $= 20(\cos 150 + i\sin 150°)$

$20\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$-10\sqrt{3} + 10i$

❼ $(\sqrt{2}\, cis 300°)(\sqrt{2}\, cis 270°)$ $2(\operatorname{cis} 570) = 2(\operatorname{cis} 210)$

$570 - 360 = 210°$

$2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$-\sqrt{3} - i$

CAT Notes Trig 8.3 The Product & Quotient Theorems

Part 2: Quotients

Previously, we learned to divide complex number expressions by utilizing a "clever form of 1" made up of the conjugate of the denominator. Then we wrote the answer in standard rectangular form.

__Example:__ Find the quotient of these complex numbers: $\frac{(1+i\sqrt{3})}{(-2\sqrt{3}+2i)} \cdot \frac{-2\sqrt{3}-2i}{-2\sqrt{3}-2i} = \frac{-2\sqrt{3}-2i-6i+2i^2\sqrt{3}}{12+2\sqrt{3}i-2\sqrt{3}i-4i^2}$

$$= \frac{-8i}{16}$$

$$= -\frac{1}{2}i$$

$$= 0 + -\frac{1}{2}i$$

Another way to find the quotient of two complex numbers is by using the "QUOTIENT THEOREM."

QUOTIENT THEOREM

If $r_1(\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, where $r_2(\cos\theta_2 + i\sin\theta_2) \neq 0$, then

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)].$$

In compact form, this is written

$$\frac{r_1 \operatorname{cis}\theta_1}{r_2 \operatorname{cis}\theta_2} = \boxed{\frac{r_1}{r_2}}\operatorname{cis}(\theta_1 - \theta_2).$$

CAT Notes Trig 8.3 The Product & Quotient Theorems

Examples: Using the quotient theorem, find each quotient and write the answer in rectangular form.

❽ $\dfrac{24(\cos 150° + i\sin 150°)}{2(\cos 30° + i\sin 30°)}$ $= 12(\text{cis } 120) = 12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

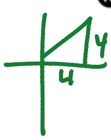
$= -6 + 6\sqrt{3}\,i$

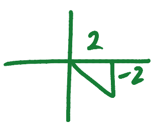
❾ $\dfrac{(16\,\text{cis}310°)}{(8\,\text{cis}70°)}$ $= 2\,\text{cis } 240 = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= -1 - \sqrt{3}\,i$

For numbers 10 & 11, first convert rectangular form complex numbers into trig form!

❿ $\dfrac{(4+4i)}{(2-2i)} = \dfrac{4\sqrt{2}(\text{cis } 45°)}{2\sqrt{2}(\text{cis } 315°)} = 2(\cos -270°)$ or $2(\cos 90)$



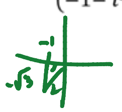


$45 - 315$

$= 2(0 + 1i)$

$= 2i$

⓫ $\dfrac{0+(2i)}{(-1-i\sqrt{3})} = \dfrac{2(\text{cis } 90)}{2(\text{cis } 240)} = 1(\text{cis } -150°) = 1(\text{cis } 210°)$



$= 1\left(-\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right)$

$-\frac{\sqrt{3}}{2} + -\frac{1}{2}i$

For numbers 12 & 13, use a calculator. Express real & imaginary parts to 4 decimal places. Give answers in rectangular form.

⓬ $(4\,\text{cis}19.25°)(7\,\text{cis}41.75°)$

$28(\text{cis } 61)$

$13.5747 + 24.4894i$

⓭ $\dfrac{30(\cos 130° + i\sin 130°)}{10(\cos 21° + i\sin 21°)}$

$3(\text{cis } 109)$

$-.9767 + 2.8366i$

Take 2

CAT Notes Trig 8.3 The Product & Quotient Theorems

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Warm up: Convert each standard (rectangular) form number into polar (trig) form or vice versa.

Use θ in the interval [0°, 360°).

❶ 6 + 2i ❷ -3 + 4i ❸ 4 cis 150° ❹ $\sqrt{2}\ cis\ 315°$

Part 1: Products

Previously, we learned to multiply binomial expressions using "FOIL" or double distribution.

Example: Find the product of $(1 + i\sqrt{3})(-2\sqrt{3} + 2i)$

Another way to find this product is to first CONVERT COMPLEX NUMBERS INTO TRIGONOMETRIC (POLAR) FORM.

$(1 + i\sqrt{3}) =$ ____________________

$(-2\sqrt{3} + 2i) =$ ____________________

Now multiply these trig forms. Use some trig identities to simplify the expression. (Angle sum/difference identities)

CAT Notes Trig 8.3   The Product & Quotient Theorems

We may generalize this work in the following "PRODUCT THEOREM."

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In compact form, this is written

$$(r_1 \text{ cis } \theta_1)(r_2 \text{ cis } \theta_2) = r_1r_2 \text{ cis}(\theta_1+\theta_2).$$

In other words, to multiply complex numbers in trig form,

multiply their absolute values ($r_1\cdot r_2$) and add their arguments ($\theta_1 + \theta_2$).

Examples: Using the product theorem, multiply the following & write the result in rectangular form.

❺ $3(cos45° + i\ sin45°)$ and $2(cos135° + i\ sin135°)$

❻ $[4(cos120° + i\ sin120°)][5(cos30° + i\ sin30°)]$

❼ $(\sqrt{2}\ cis300°)(\sqrt{2}\ cis270°)$

Part 2: Quotients

Previously, we learned to divide complex number expressions by utilizing a "clever form of 1" made up of the conjugate of the denominator. Then we wrote the answer in standard rectangular form.

Example: Find the quotient of these complex numbers: $\dfrac{(1+i\sqrt{3})}{(-2\sqrt{3}+2i)}$

Another way to find the quotient of two complex numbers is by using the "QUOTIENT THEOREM."

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CAT Notes Trig 8.3 The Product & Quotient Theorems

Examples: Using the quotient theorem, find each quotient and write the answer in rectangular form.

❽ $\frac{24(cos150° + i\ sin150°)}{2(cos30° + i\ sin30°)}$

❾ $\frac{(16\ cis310°)}{(8\ cis70°)}$

For numbers 10 & 11, first convert rectangular form complex numbers into trig form!

❿ $\frac{(4 + 4i)}{(2 - 2i)}$

⓫ $\frac{(2i)}{(-1 - i\sqrt{3})}$

For numbers 12 & 13, use a calculator. Express real & imaginary parts to 4 decimal places. Give answers in rectangular form.

⓬ $(4\ cis19.25°)(7\ cis41.75°)$

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Take 3

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