Section 4.5 (Day 2) Deriving Quadratic Functions

Warm Up

Solve each system using elimination.

1. ① $3x + 3y + z = 1$
   ② $2x - 3y + z = 2$
   ③ $x + y + z = 3$

① $3x+3y+\cancel{z}=1$
② $-2x+3y-\cancel{z}=-2$

④ $x+6y=-1$

① $3x+3y+\cancel{z}=1$
③ $-x-y-\cancel{z}=-3$

⑤ $2x+2y=-2$

-2

④ $(x+6y)=(-1)\cdot -2$

⑤ $\cancel{2x}+2y=-2$
$-\cancel{2x}-12y=2$

$-10y=0$

$y=0$ ✓

$2x=-2$

$x=-1$ ✓

③ $x+y+z=3$

$-1+0+z=3$

$-1+z=3$

$z=4$

$(-1, 0, 4)$

2. $2x + y = -5$
   $2\cdot(-x + 3y) = (6)\cdot 2$

$\cancel{2x}+y=-5$
$-\cancel{2x}+6y=12$

$7y=7$

$y=1$ ✓

$2x+1=-5$

$2x=-6$

$x=-3$

$(-3,1)$

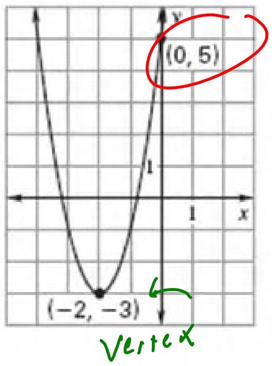
Today we will use our knowledge of quadratic functions and solving systems to help us write quadratic models given certain information. I hope you understood how to solve 3x3 systems from the previous lesson….If not, you better have a TI-84 with you!!!

Write a Quadratic Function in Vertex Form.

Example 1:

Vertex Form: $y = a(x-h)^2 + k$

Hint: Use vertex form because the vertex is given!!



Step 1: Substitute the vertex (h, k) into vertex form.

$y = a(x - -2)^2 - 3 \rightarrow y = a(x+2)^2 - 3$

Step 2: Substitute the other given point (x, y) into vertex form.

$5 = a(0+2)^2 - 3 \rightarrow 5 = a(2)^2 - 3$

Step 3: Solve for *a*.

$5 = 4a - 3$

$8 = 4a$

$a = 2$

Step 4: Rewrite the quadratic function in vertex form.

$f(x) = 2(x+2)^2 - 3$

Example 2: Write a quadratic function in vertex form.

Given: Vertex: (2, 1) Point of graph: (0, 4)

$y = a(x-2)^2 + 1$

$4 = a(0-2)^2 + 1$

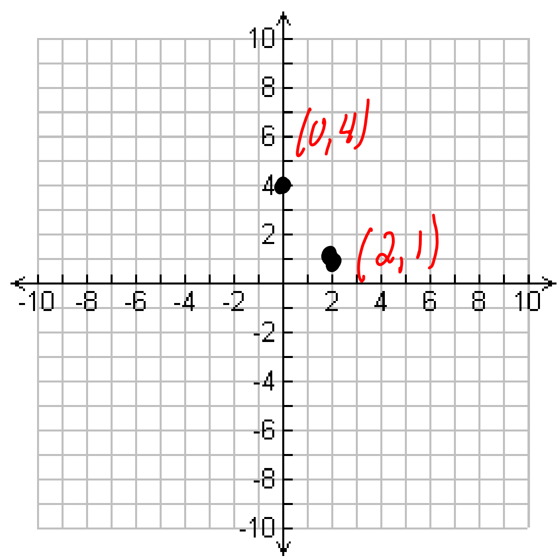
$4 = a(-2)^2 + 1$

$4 = 4a + 1$

$3 = 4a$

$a = \frac{3}{4}$

$f(x) = \frac{3}{4}(x-2)^2 + 1$



Write a Quadratic Function in Standard Form

Example 5:

Write a quadratic function in standard form for the parabola that passes through the points (-2, -6), (0, 6) and (2, 2).

Step 1: Substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain a system of three linear equations.

(-2, -6)
$-6 = a(-2)^2 + b(-2) + c$
$-6 = 4a - 2b + c$
① $4a - 2b + c = -6$

(0, 6)
$6 = a(0)^2 + b(0) + c$
$6 = c$
② $c = 6$

(2, 2)
$2 = a(2)^2 + b(2) + c$
$2 = 4a + 2b + c$
③ $4a + 2b + c = 2$

$4a - 2b + 6 = -6$ $\quad$ $4a + 2b + 6 = 2$

④ $4a - 2b = -12$ $\quad$ ⑤ $4a + 2b = -4$

Step 2: Rewrite the system as a system of two equations using substitution or elimination.

④ $4a - \cancel{2b} = -12$
⑤ $4a + \cancel{2b} = -4$
$8a = -16$

$a = -2$

Step 3: Solve the system for each variable

$4(-2) - 2b = -12$ $\quad$ $b = 2$
$-8 - 2b = -12$
$-2b = -4$

Step 4: Write the quadratic function. $f(x) = -2x^2 + 2x + 6$

Example 6:

Write a quadratic function in standard form for the parabola that passes through the points (-2, 30), (1, 6), and (4, 36).

(-2, 30)
$30 = a(-2)^2 + b(-2) + c$
① $4a - 2b + c = 30$

(1, 6)
$6 = a(1)^2 + b(1) + c$
② $a + b + c = 6$

(4, 36)
$36 = a(4)^2 + b(4) + c$
③ $16a + 4b + c = 36$

Always Eliminate 'c'

① $4a - 2b + c = 30$
② $a + b + c = 6$
③ $16a + 4b + c = 36$

① $4a - 2b + \cancel{c} = 30$
② $-a - b - \cancel{c} = -6$
④ $3a - 3b = 24$

③ $16a + 4b + \cancel{c} = 36$
② $-a - b - \cancel{c} = -6$
⑤ $15a + 3b = 30$

④ $3a - \cancel{3b} = 24$
⑤ $15a + \cancel{3b} = 30$
$18a = 54$
$a = 3$

$3(3) - 3b = 24$ $\quad$ $9 - 3b = 24$

$-3b = 15$
$b = -5$ ✓

$3 - 5 + c = 6$
$-2 + c = 6$
$c = 8$

$f(x) = 3x^2 - 5x + 8$

So you should be saying to yourself: “There has to be an easier way to write a function provided 3 different points.” Well I mean it’s 2015, of course there’s a way to do this using technology! Let’s follow these directions and use our calculator to complete the previous 2 problems and check our answers.

You can use a graphing calculator to determine a quadratic regression equation given three points on the parabola.

Step 1: Diagnostics must be turned on so that all needed data is displayed. Press **2nd CATALOG** to display the catalog. Scroll to **DiagnosticOn** and press **ENTER**. Then press **ENTER** again. The calculator should display the word **Done**.

Step 2: Press **STAT** and then press **ENTER** to select **1:Edit**. In the **L1** column, enter the x-values by typing each value followed by **ENTER**. Use the right arrow key to move to the **L2** column. **ENTER** the y-values.

If there is already data in your L1 list, highlight the heading L1, Press CLEAR, then Press ENTER to delete it.

Step 3: Press **STAT** and use the right arrow key to show the **CALC** menu. Choose **5:QuadReg**. Press Enter. The values for a, b, and c will be displayed.

Step 4: To have the calculator graph the exact equation, press **Y=**, **VARS**, **5:Statistics**, scroll right to **EQ**, press **1:RegEQ**, **GRAPH**.

Use your graphing calculator to determine the quadratic equation for each set of three points that lie on a parabola.

1. (-2,-6), (0,6) and (2,2)

$f(x) = -2x^2 + 2x + 6$

2. (-2,30), (1,6) and (4,36)

$f(x) = 3x^2 - 5x + 8$