PROBLEM 4 Rewriting Quotients of Complex Numbers

Division of complex numbers requires the use of complex conjugates, thus changing the divisor into a real number. Recall, the complex conjugate of $a + bi$ is $a + bi$.

1. For each complex number, write its conjugate. Then calculate each product.

a. $(7 + i)(7 - i)$

$49 - (+1)$

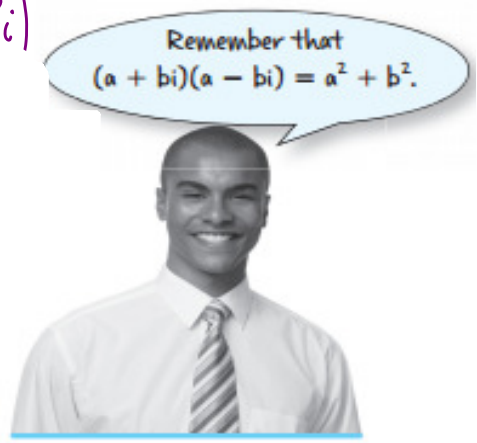
50

b. $(-5 - 3i)(-5 + 3i)$

$25 - 9(-1)$

$25 + 9$

34



c. $(12 + 11i)(12 - 11i)$

$144 - 121i^2$

$144 - 121(-1)$

$144 + 121 = 265$

d. $-4i \cdot i$

$-4i^2$

$-4(-1)$

4

* When you have just 1 term..... only multiply by i *

$i = \sqrt{-1}$

You can rewrite the division of a complex number by multiplying both the divisor and the dividend by the conjugate of the divisor, thus changing the divisor into a real number.

You can rewrite $\frac{3 - 2i}{4 + 3i}$ without a complex number in the denominator.

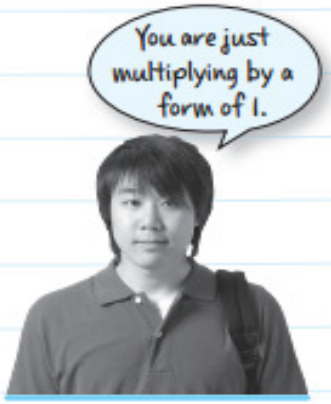
$$\frac{3 - 2i}{4 + 3i} = \frac{3 - 2i}{4 + 3i} \cdot \frac{4 - 3i}{4 - 3i}$$

$$= \frac{12 - 9i - 8i + 6i^2}{4^2 + 3^2}$$

$$= \frac{12 - 17i - 6}{16 + 9}$$

$$= \frac{6 - 17i}{25}$$

$$\frac{3 - 2i}{4 + 3i} = \frac{6}{25} - \frac{17}{25}i$$



2. Rewrite each quotient without a complex number in the denominator.

a. $\frac{2-i}{3+2i} = \frac{2-i}{3+2i} \cdot \frac{3-2i}{3-2i}$

$= \frac{6-4i-3i+2i^2}{9-6i+6i-4i^2}$

$= \frac{4-7i}{13}$

$= \frac{4}{13} - \frac{7}{13}i$

$\frac{(2-i)(3-2i)}{(3+2i)(3-2i)} = \frac{6-4i-3i+2i^2}{9-4i^2} = \frac{6-7i+2(-1)}{9-4(-1)} = \frac{4-7i}{13}$

b. $\frac{3-4i}{2-3i} = \frac{3-4i}{2-3i} \cdot \frac{2+3i}{2+3i}$

$= \frac{6+9i-8i-12i^2}{4+6i-6i-9i^2}$

$= \frac{18+i}{13}$

$=$

$\frac{(3-4i)(2+3i)}{(2-3i)(2+3i)} = \frac{6+9i-8i-12i^2}{4-9i^2} = \frac{6+i-12(-1)}{4-9(-1)} = \frac{18+i}{13}$

c. $\frac{5+2i}{1+i} = \frac{5+2i}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{5-5i+2i-2i^2}{1-i+i-i^2}$

$= \frac{7-3i}{2}$

$=$

d. $\frac{20-5i}{2-4i} = \frac{20-5i}{2-4i} \cdot \frac{2+4i}{2+4i}$

$= \frac{40+80i-10i-20i^2}{4+8i-8i-16i^2}$

$= \frac{60+70i}{20}$

$= 3 + \frac{7}{2}i$

$\frac{6+7i}{2}$

Check for Students' Understanding

Consider the following sets of numbers:

- Natural numbers 1, 2, 3. . . .
- Rational numbers Fractions
- Imaginary numbers i
- Whole numbers 0, 1, 2 . . . .
- Irrational numbers Non-ending / Non-repeating Decimals
- Complex numbers Every Number is a complex number
- Integers Number Line
- Real numbers Not imaginary

List all of the number sets form the above list that describes each given number.

1. 6

Natural number, Whole number, Integer, Rational number, Real number, Complex number

2. $\sqrt{6}$

Irrational number, Real number, Complex number

3. $6i$

Imaginary number, Complex number

4. $\frac{1}{6}$

rational number, Real number, Complex number

5. $6 - i$

Imaginary number, Complex number