Section 4.7 (Day 1) Quadratics and Complex Numbers

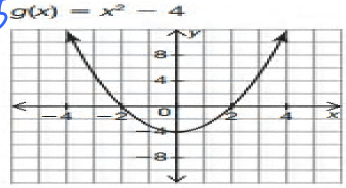
Warm-Up

PROBLEM 1 **X-Axis Intersection Inspection**

1. Analyze each graph. Identify the x-intercepts, if possible.

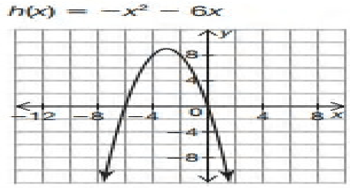
Group A

$g(x) = x^2 - 4$



2 x-intercepts
(2,0) & (-2,0)

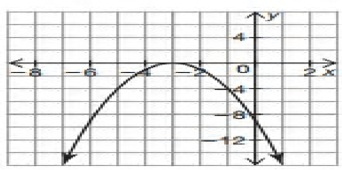
$h(x) = -x^2 - 6x$



2 x-intercepts
(-6,0) & (0,0)

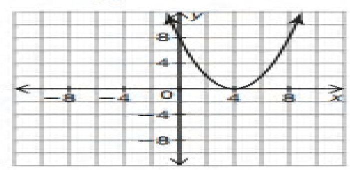
Group B

$m(x) = -x^2 - 6x - 9$



1 x-intercept
(-3,0)

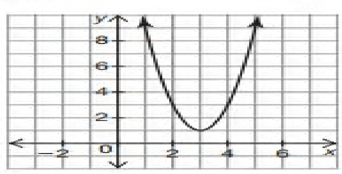
$n(x) = \frac{1}{2}x^2 - 4x + 8$



1 x-intercept
(4,0)

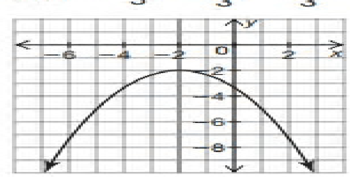
Group C

$p(x) = 2x^2 - 12x + 19$



No x-intercepts

$q(x) = -\frac{1}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$



No x-intercepts

The x-intercepts of a quadratic function $f(x)$ are the solutions of the equation $f(x) = 0$.

2. If $f(x)$ is a quadratic function, explain how to determine the number of solutions of $f(x) = 0$ given the graph of $f(x)$.

If the graph of $f(x)$ intersects the x-axis 2 times, then the quadratic equation $f(x) = 0$ has 2 real solutions.

If the graph of $f(x)$ intersects the x-axis 1 time, then the quadratic equation $f(x) = 0$ has 1 real solution.

If the graph of $f(x)$ does not intersect the x-axis, then the quadratic equation $f(x) = 0$ has no real solution.

Review the Quadratic Formula

Quadratic Formula	Example Equation
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\underset{\uparrow \atop a}{x^2} + \underset{\uparrow \atop b}{2x} + \underset{\uparrow \atop c}{1} = 0$

Example Equation Worked Out → $\dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)} \rightarrow \dfrac{-2 \pm \sqrt{4-4}}{2} \rightarrow \dfrac{-2 \pm 0}{2} \rightarrow -1$

When should we use the Quadratic Formula? Which of the groups from the Warm-Up would we have no choice but to use the Quadratic Formula to find the zeros?

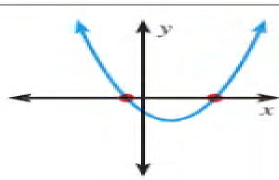
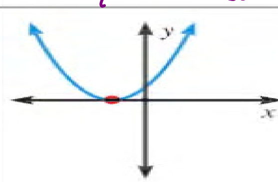
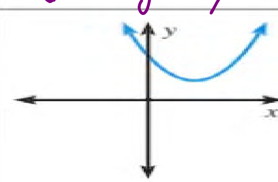
We should use the Quadratic Formula to find zeros if and only if we can't factor or take a square root. Group C would need the Quadratic Formula because they have imaginary zeros.

What do we call the highlighted part of the Quadratic Function shown below? Discriminant.

$$x = \frac{-b \pm \boxed{\sqrt{b^2 - 4ac}}}{2a}$$

What can this part of the Quadratic Function quickly tell you? The number of types of zeros.

Using the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	2 Real Zeros	1 "Repeated" Zero	2 Imaginary Zeros
Graph of $y = ax^2 + bx + c$			

USING THE DISCRIMINANT OF $ax^2 + bx + c = 0$

When $b^2 - 4ac > 0$, the equation has 2 Real Zeros. The graph has 2 x-intercepts.

When $b^2 - 4ac = 0$, the equation has 1 Repeated Zero. The graph has 1 x-intercept.

When $b^2 - 4ac < 0$, the equation has 2 Imaginary Zeros. The graph has 0 x-intercepts.

You better try to factor before you listen to the directions below!!! Make your life easier!

Seriously…you better factor Groups A and B. Group C you have to use the Quadratic Formula.

3. Choose one of the two functions from each group in Question 1. Use the Quadratic Formula and what you know about imaginary numbers to solve an equation of the form $f(x) = 0$ for each function you choose.

Group A

$g(x) = x^2 - 4$

$= (x+2)(x-2)$

$x = -2$ & 2

$h(x) = -x^2 - 6x$

$-x(x+6)$

$x = 0$ & -6

Group B

$m(x) = -x^2 - 6x - 9$

$= -1(x^2+6x+9)$

M: 9 3·3

A: 6 3+3

$= -1(x+3)(x+3)$

$x = -3$ & -3 Hence "Repeated" Zero!!

Group C

$p(x) = 2x^2 - 12x + 19$

a b c

$$\frac{12 \pm \sqrt{(-12)^2 - 4(2)(19)}}{2(2)}$$

$$\frac{12 \pm \sqrt{144-152}}{4}$$

$$\frac{12 \pm \sqrt{-8}}{4}$$

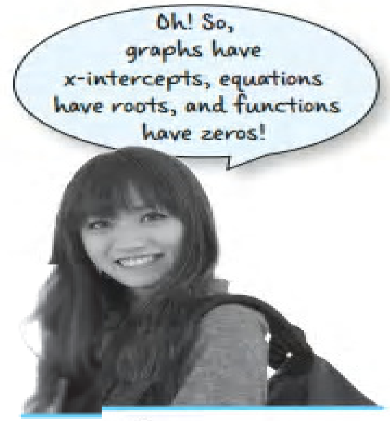
$$\frac{12 \pm \sqrt{8}\,i}{4}$$

$$\frac{12 \pm \sqrt{4}\sqrt{2}\,i}{4}$$

$$\frac{12^{\div 2} \pm 2^{\div 2}\sqrt{2}\,i}{4^{\div 2}}$$

$$x = \frac{6 \pm \sqrt{2}\,i}{2}$$

Just as equations may have imaginary roots, functions may have imaginary zeros. Imaginary zeros are zeros of quadratic functions that do not cross the x-axis. Remember that zeros of a function $f(x)$ are the values of x for which $f(x) = 0$. Zeros, roots, and x-intercepts are all related.



7. Use any method to determine whether each function has real or imaginary zeros. You do not need to calculate the zeros.

a. $f(x) = -3x^2 + 2x - 1$

$$\begin{aligned} b^2 - 4ac &= 2^2 - 4(-3)(-1) \\ &= 4 - 12 \\ &= -8 \end{aligned}$$

The discriminant is negative, so the equation has imaginary zeros.

b. $f(x) = -\frac{1}{2}x^2 + x - \frac{1}{2}$

$$\begin{aligned} b^2 - 4ac &= 1^2 - 4\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

The discriminant is zero, so the equation has real zeros.

c. $f(x) = 2x^2 - 5x - 6$

$$\begin{aligned} b^2 - 4ac &= (-5)^2 - 4(2)(-6) \\ &= 25 + 48 \\ &= 73 \end{aligned}$$

The discriminant is positive, so the equation has real zeros.