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Shape and Structure
Forms of Quadratic Functions

Vocabulary

Write an example for each form of quadratic function and tell whether the form helps determine the x-intercepts, the y-intercept, or the vertex of the graph. Then describe how to determine the concavity of a parabola.

1. Standard form: **Example: $f(x) = 4x^2 + 8x + 3$; Standard form helps determine the y-intercept.**

2. Factored form: **Example: $f(x) = (2x + 1)(2x + 3)$; Factored forms helps determine the x-intercepts.**

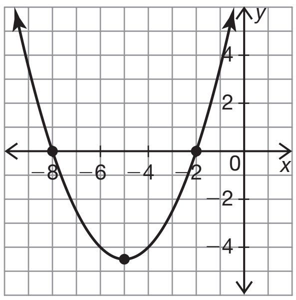
3. Vertex form: **Example: $f(x) = 4(x - 1)^2 + 2$; Vertex form helps determine the vertex.**

4. Concavity of a parabola: **A parabola is concave down, or opens downward, when the a-value of a quadratic function is negative. It is concave up, or opens upward, when the a-value is positive.**

Problem Set

Circle the function that matches each graph. Explain your reasoning.

1.



$f(x) = 6(x - 2)(x - 8)$

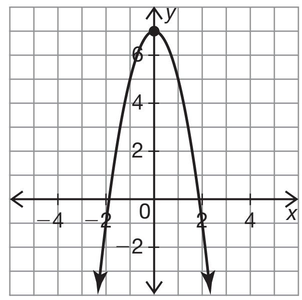
$f(x) = -\frac{1}{2}(x + 2)(x + 8)$

$f(x) = \frac{1}{2}(x + 2)(x + 8)$

$f(x) = \frac{1}{2}(x - 2)(x - 8)$

The *a*–value is positive so the parabola opens up. Also, the roots are at −2 and −8.

2.



$f(x) = 2x^2 - x + 7$

$f(x) = -2x^2 - x + 7$

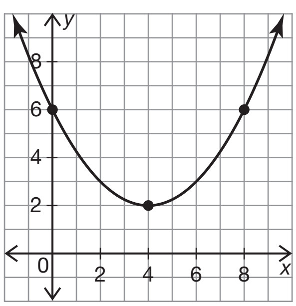
$f(x) = -x^2 - 2x + 7$

$f(x) = -2x^2 - x - 2$

The *a*–value is negative so the parabola opens down. Also, the *y*-intercept is (0, 7).

4

3.



$f(x) = 0.25(x - 4)^2 + 2$

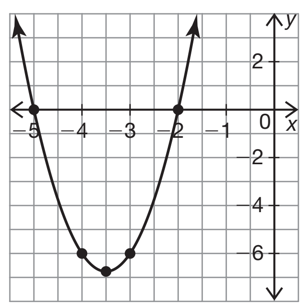
$f(x) = 4(x - 2)^2 - 2$

$f(x) = -0.25(x + 4)^2 + 2$

$f(x) = 0.25(x - 2)^2 + 4$

The *a*–value is positive so the parabola opens up. The vertex is at (4, 2).

4.



$f(x) = -3(x + 2)(x - 5)$

$f(x) = 3(x + 2)(x + 5)$

$f(x) = 3(x - 2)(x - 5)$

$f(x) = -3(x - 2)(x - 5)$

The *a*–value is positive so the parabola opens up. The *x*-intercepts are −5 and −2.

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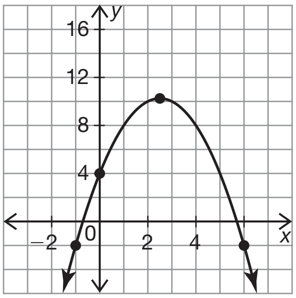
5.



$f(x) = x^2 + 5x - 4$

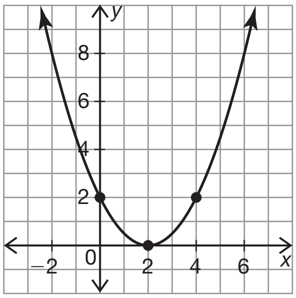
$f(x) = -x^2 + 5x + 10$

$f(x) = x^2 + 5x + 4$

$f(x) = -x^2 + 5x + 4$

The *a*–value is negative so the parabola opens down. The *y*-intercept is (0, 4).

6.



$f(x) = -\frac{1}{2}(x - 2)^2$

$f(x) = \frac{1}{2}(x - 2)^2 + 2$

$f(x) = \frac{1}{2}(x - 2)^2$

$f(x) = \frac{1}{2}(x + 2)^2$

The *a*–value is positive so the parabola opens up. The minimum point is (2, 0).

Use the given information to determine the most efficient form you could use to write the quadratic function. Write standard form, factored form, or vertex form.

7. vertex (3, 7) and point (1, 10)
vertex form

8. points (1, 0), (4, –3), and (7, 0)
factored form

9. *y*-intercept (0, 3) and axis of symmetry $-\frac{3}{8}$
standard form

10. points (–1, 12), (5, 12), and (–2, –2)
standard form

11. roots (–5, 0), (13, 0) and point (–7, 40)
factored form

12. maximum point (–4, –8) and point (–3, –15)
vertex form

Convert each quadratic function in factored form to standard form.

13. $f(x) = (x + 5)(x - 7)$

$f(x) = x^2 - 7x + 5x - 35$

$= x^2 - 2x - 35$

14. $f(x) = (x + 2)(x + 9)$

$f(x) = x^2 + 9x + 2x + 18$

$= x^2 + 11x + 18$

15. $f(x) = 2(x - 4)(x + 1)$

$f(x) = 2(x^2 + x - 4x - 4)$

$= 2x^2 - 6x - 8$

16. $f(x) = -3(x - 1)(x - 3)$

$f(x) = -3(x^2 - 3x - x + 3)$

$= -3x^2 + 12x - 9$

17. $f(x) = \frac{1}{3}(x + 6)(x + 3)$

$f(x) = \frac{1}{3}(x^2 + 3x + 6x + 18)$

$= \frac{1}{3}x^2 + 3x + 6$

18. $f(x) = -\frac{5}{8}(x - 6)(x + 2)$

$f(x) = -\frac{5}{8}(x^2 + 2x - 6x - 4)$

$= -\frac{5}{8}x^2 + \frac{15}{4}x + \frac{15}{2}$

4

Convert each quadratic function in vertex form to standard form.

19. $f(x) = 3(x - 4)^2 + 7$

$f(x) = 3(x^2 - 8x + 16) + 7$

$= 3x^2 - 24x + 55$

20. $f(x) = -2(x + 1)^2 - 5$

$f(x) = -2(x^2 + 2x + 1) - 5$

$= -2x^2 - 4x - 7$

21. $f(x) = 2\left(x + \frac{7}{2}\right)^2 - \frac{3}{2}$

$f(x) = 2\left(x^2 + \frac{14}{2}x + \frac{49}{4}\right) - \frac{3}{2}$

$= 2x^2 + 14x + 23$

22. $f(x) = -(x - 6)^2 + 4$

$f(x) = -(x^2 - 12x + 36) + 4$

$= -x^2 + 12x - 32$

23. $f(x) = -\frac{1}{2}(x - 10)^2 - 12$

$f(x) = -\frac{1}{2}(x^2 - 20x + 100) - 12$

$= -\frac{1}{2}x^2 + 10x - 62$

24. $f(x) = \frac{1}{20}(x + 100)^2 + 60$

$f(x) = \frac{1}{20}(x^2 + 200x + 10{,}000) + 60$

$= \frac{1}{20}x^2 + 10x + 560$

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Write a quadratic function to represent each situation using the given information.

25. Cory is training his dog, Cocoa, for an agility competition. Cocoa must jump through a hoop in the middle of a course. The center of the hoop is 8 feet from the starting pole. The dog runs from the starting pole for 5 feet, jumps through the hoop, and lands 4 feet from the hoop. When Cocoa is 1 foot from landing, Cory measures that she is 3 feet off the ground. Write a function to represent Cocoa's height in terms of her distance from the starting pole.

$h(d) = a(d - r_1)(d - r_2)$

$3 = a(11 - 5)(11 - 12)$

$3 = a(6)(-1)$

$3 = -6a$

$\frac{3}{-6} = a$

$-0.5 = a$

$h(d) = -0.5(d - 5)(d - 12)$

26. Sasha is training her dog, Bingo, to run across an arched ramp, which is in the shape of a parabola. To help Bingo get across the ramp, Sasha places a treat on the ground where the arched ramp begins and one at the top of the ramp. The treat at the top of the ramp is a horizontal distance of 2 feet from the first treat, and Bingo is 6 feet above the ground when he reaches the top of the ramp. Write a function to represent Bingo's height above the ground as he walks across the ramp in terms of his distance from the beginning of the ramp.

$h(d) = a(d - 2)^2 + 6$

$0 = a(0 - 2)^2 + 6$

$0 = 4a + 6$

$-6 = 4a$

$-\frac{6}{4} = a$

$-\frac{3}{2} = a$

$h(d) = -1.5(d - 2)^2 + 6$

27. Ella's dog, Doug, is performing in a special tricks show. Doug can fling a ball off his nose into a bucket 20 feet away. Ella places the ball on Doug's nose, which is 4 feet off the ground. Doug flings the ball through the air into a bucket sitting on a 4-foot platform. Halfway to the bucket, the ball is 10 feet in the air. Write a function to represent the height of the ball in terms of its distance from Doug.

$$h(d) = a(d - 10)^2 + 10$$
$$4 = a(20 - 10)^2 + 10$$
$$4 = 100a + 10$$
$$\frac{-6}{100} = \frac{100a}{100}$$
$$-0.06 = a$$
$$h(d) = -0.06(d - 10)^2 + 10$$

28. A spectator in the crowd throws a treat to one of the dogs in a competition. The spectator throws the treat from the bleachers 19 feet above ground. The treat amazingly flies 30 feet and just barely crosses over a hoop which is 7.5 feet tall. The dog catches the treat 6 feet beyond the hoop when his mouth is 1 foot from the ground. Write a function to represent the height of the treat in terms of its distance.

$$h(d) = ad^2 + bd + c$$
$$(0, 19); (30, 7.5); (36, 1)$$
$$19 = a(0)^2 + b(0) + c$$
$$19 = c$$
$$7.5 = a(30)^2 + b(30) + 19$$
$$7.5 = 900a + 30b + 19$$
$$\frac{-900a}{30} - \frac{11.5}{30} = \frac{30b}{30}$$
$$-30a - 0.38 \approx b$$
$$1 = a(36)^2 + b(36) + 19$$
$$1 = 1{,}296a + 36b + 19$$
$$\frac{-1{,}296a}{36} - \frac{18}{36} = \frac{36b}{36}$$
$$-36a - 0.5 = b$$
$$-30a - 0.38 = -36a - 0.5$$
$$\frac{6a}{6} = \frac{-0.12}{6}$$
$$a = -0.02$$
$$-30(-0.02) - 0.38 = b$$
$$0.6 - 0.38 = b$$
$$0.22 = b$$
$$h(d) = -0.02d^2 + 0.02d + 19$$

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29. Hector's dog, Ginger, competes in a waterfowl jump. She jumps from the edge of the water, catches a toy duck at a horizontal distance of 10 feet from the edge of the water and a height of 2 feet above the water, and lands in the water at a horizontal distance of 15 feet from the edge of the water. Write a function to represent the height of Ginger's jump in terms of her horizontal distance.

$$h(d) = a(d - r_1)(d - r_2)$$
$$2 = a(10 - 0)(10 - 15)$$
$$2 = a(10)(-5)$$
$$\frac{2}{-50} = \frac{-50a}{-50}$$
$$-0.04 = a$$
$$h(d) = -0.04(d - 0)(d - 15)$$

30. Ping is training her dog, TinTin, to jump across a row of logs. He takes off from a platform that is 7 feet high with a speed of 18 feet per second. Write a function to represent TinTin's height in terms of time as he jumps across the logs.

$$h(t) = -16t^2 + v_0t + h_0$$
$$h(t) = -16t^2 + 18t + 7$$

4

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Function Sense
Translating Functions

Vocabulary

Complete each sentence with the correct term from the word bank.

transformation	reference point
translation	argument of a function

1. A(n) reference point is one of a set of key points that help identify the basic function.

2. The mapping, or movement, of all the points of a figure in a plane according to a common operation is called a(n) transformation.

3. The argument of a function is the variable, term, or expression on which the function operates.

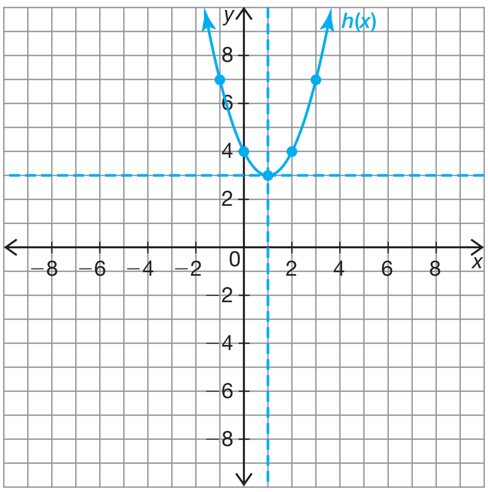
4. A(n) translation is a type of transformation that shifts an entire figure or graph the same distance and direction.

Problem Set

Given $f(x) = x^2$, complete the table and graph $h(x)$.

1. $h(x) = (x - 1)^2 + 3$

Reference Points on $f(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(1, 3)
(1, 1)	→	(2, 4)
(2, 4)	→	(3, 7)



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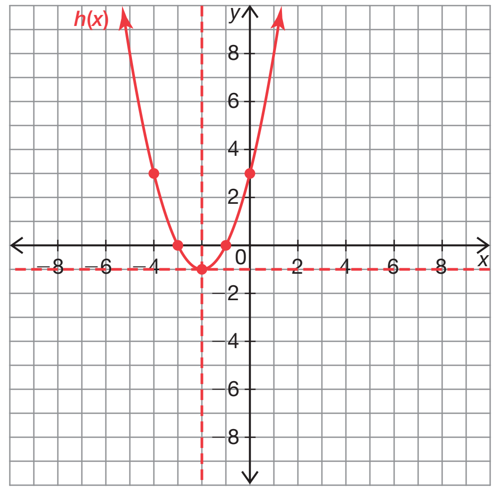
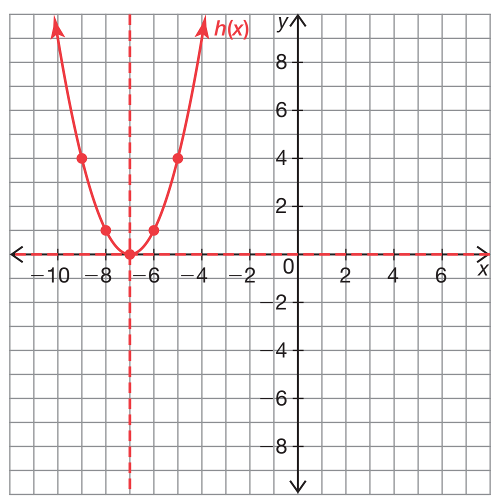
2. $h(x) = (x + 2)^2 - 1$

Reference Points of $f(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(−2, −1)
(1, 1)	→	(−1, 0)
(2, 4)	→	(0, 3)



3. $h(x) = (x + 7)^2$

Reference Points of $f(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(−7, 0)
(1, 1)	→	(−6, 1)
(2, 4)	→	(−5, 4)



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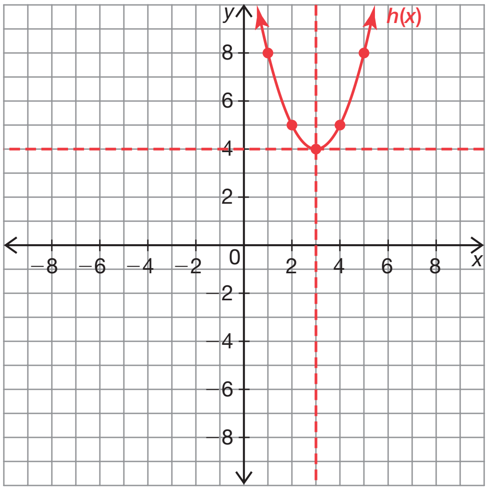
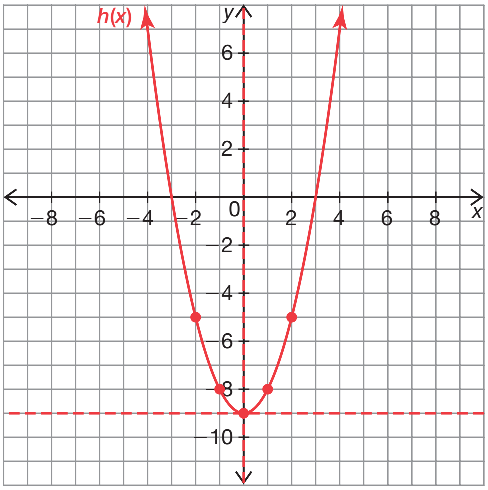
4. $h(x) = (x - 3)^2 + 4$

Reference Points of $f(x)$	$\rightarrow$	Corresponding Points on $h(x)$
(0, 0)	$\rightarrow$	(3, 4)
(1, 1)	$\rightarrow$	(4, 5)
(2, 4)	$\rightarrow$	(5, 8)



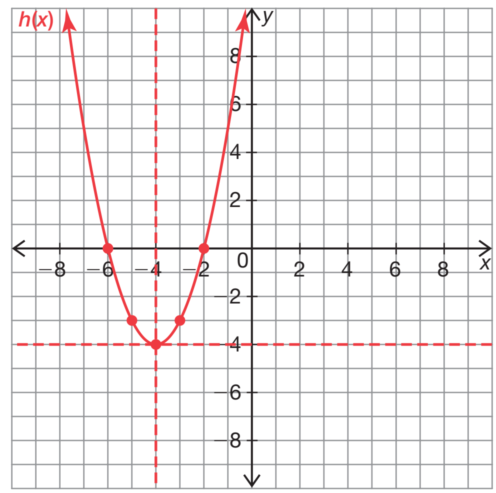
5. $h(x) = x^2 - 9$

Reference Points of $f(x)$	$\rightarrow$	Corresponding Points on $h(x)$
(0, 0)	$\rightarrow$	(0, −9)
(1, 1)	$\rightarrow$	(1, −8)
(2, 4)	$\rightarrow$	(2, −5)



6. $h(x) = (x + 4)^2 - 4$

Reference Points of $f(x)$	$\rightarrow$	Corresponding Points on $h(x)$
(0, 0)	$\rightarrow$	(−4, −4)
(1, 1)	$\rightarrow$	(−3, −3)
(2, 4)	$\rightarrow$	(−2, 0)



Each given function is in transformational function form $g(x) = Af(B(x - C)) + D$, where $f(x) = x^2$. Identify the values of C and D for the given function. Then, describe how the vertex of the given function compares to the vertex of $f(x)$.

7. $g(x) = f(x - 4) + 12$

The C–value is 4 and the D–value is 12, so the vertex will be shifted 4 units to the right and 12 units up to (4, 12).

8. $g(x) = f(x + 8) - 9$

The C–value is −8 and the D–value is −9, so the vertex will be shifted 8 units to the left and 9 units down to (−8, −9)

9. $g(x) = f(x - 5) - 11$

The C–value is 5 and the D–value is −11, so the vertex will be shifted 5 units to the right and 11 units down to (5, −11).

10. $g(x) = f(x - 6) + 10$

The C–value is 6 and the D–value is 10, so the vertex will be shifted 6 units to the right and 10 units up to (6, 10).

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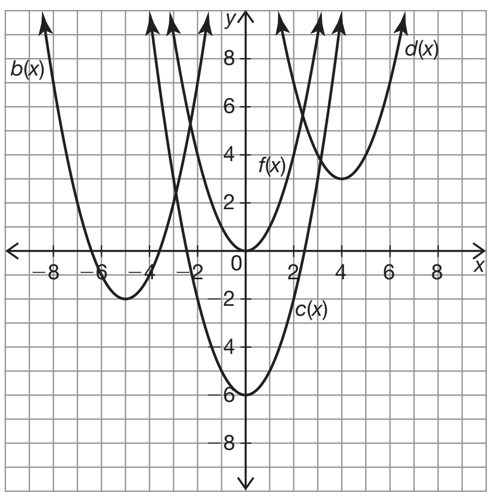
11. $g(x) = f(x + 2) + 3$

The C–value is -2 and the D–value is 3 so the vertex will be shifted 2 units to the left and 3 units up to $(-2, 3)$.

12. $g(x) = f(x + 4) - 2$

The C–value is -4 and the D–value is -2 so the vertex will be shifted 4 units to the left and 2 units down to $(-4, -2)$.

Analyze the graphs of $b(x)$, $c(x)$, $d(x)$, and $f(x)$. Write each function in terms of the indicated function.



13. Write $b(x)$ in terms of $f(x)$.

$b(x) = f(x + 5) - 2$

14. Write $c(x)$ in terms of $f(x)$.

$c(x) = f(x) - 6$

15. Write $d(x)$ in terms of $f(x)$.

$d(x) = f(x - 4) + 3$

16. Write $d(x)$ in terms of $b(x)$.

$d(x) = b(x - 9) + 5$

17. Write $c(x)$ in terms of $b(x)$.

$c(x) = b(x - 5) - 4$

18. Write $b(x)$ in terms of $c(x)$.

$b(x) = c(x + 5) + 4$

4

4

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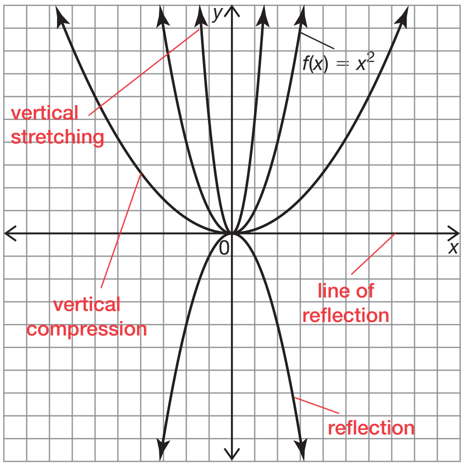
Up and Down
Vertical Dilations of Quadratic Functions

Vocabulary

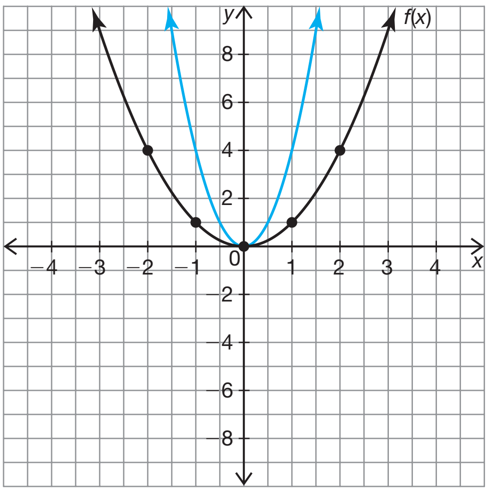
1. Label the graph to identify the vertical dilations (vertical compression and vertical stretching) and the reflection of the function $f(x) = x^2$. Also label the line of reflection.



Problem Set

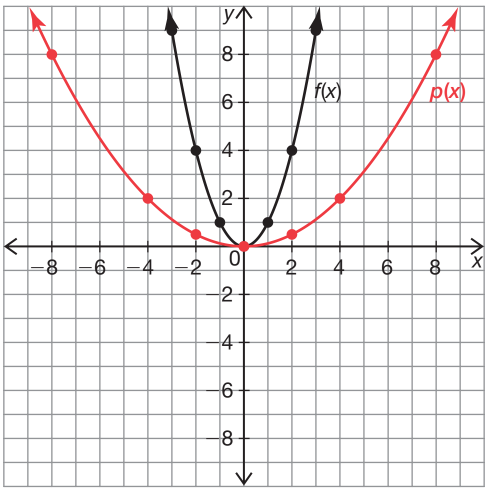
Graph each vertical dilation of $f(x) = x^2$ and tell whether the transformation is a vertical stretch or a vertical compression and if the graph includes a reflection.

1. $g(x) = 4x^2$



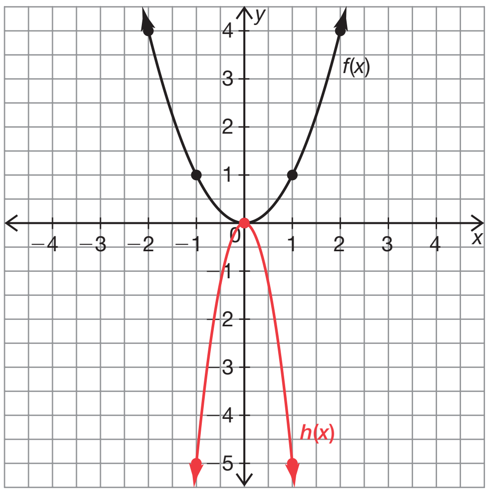
vertical stretch

2. $p(x) = \frac{1}{8}x^2$



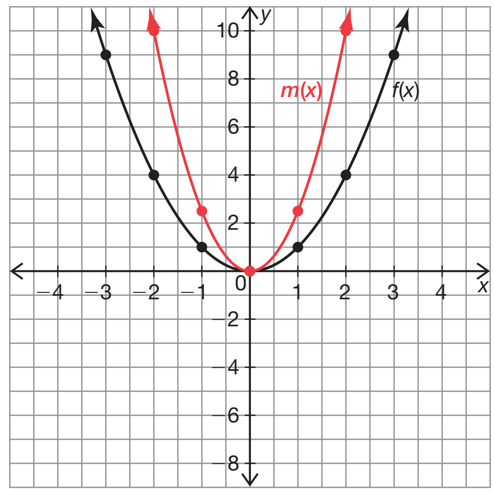
vertical compression

3. $h(x) = -5x^2$



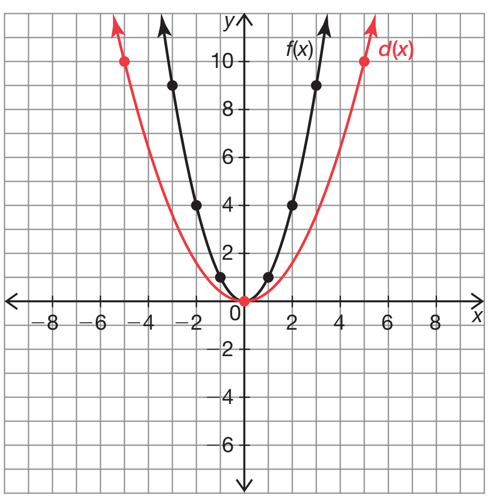
vertical stretch and reflection over $y = 0$

4. $m(x) = 2.5x^2$



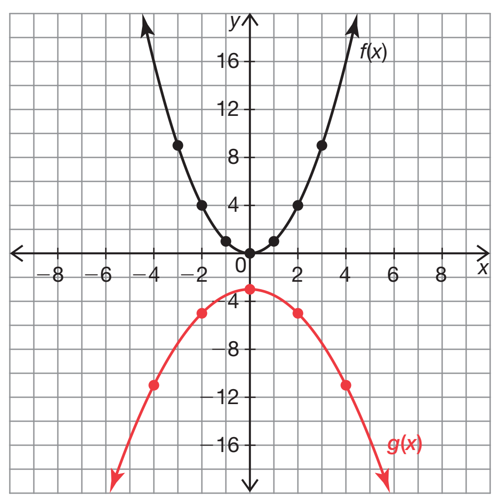
vertical stretch

5. $d(x) = \frac{2}{5}x^2$



vertical compression

6. $g(x) = -\frac{1}{2}x^2 - 3$



vertical compression and reflection over $y = 1.5$

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Each given function is in transformational function form $g(x) = Af(B(x - C)) + D$, where $f(x) = x^2$. Describe how $g(x)$ compares to $f(x)$. Then, use coordinate notation to represent how the A-, C-, and D-values transform $f(x)$ to generate $g(x)$.

7. $g(x) = -3(f(x)) - 1$

The A–value is -3, so the graph will have a vertical stretch by a factor of 3 and will be reflected about the line $y = -1$. The C–value is 0 and the D–value is -1 so the vertex will be shifted 1 unit down to $(0, -1)$.

$(x, y) \rightarrow (x, -3y - 1)$

8. $g(x) = \frac{1}{4}(f(x)) + 8$

The A–value is $\frac{1}{4}$, so the graph will have a vertical compression by a factor of $\frac{1}{4}$ and will be reflected about the line $y = 8$. The C–value is 0 and the D–value is 8, so the vertex will be shifted 8 units up to $(0, 8)$.

$(x, y) \rightarrow (x, \frac{1}{4}y + 8)$

9. $g(x) = -4(f(x + 3))$

The A–value is -4, so the graph will have a vertical stretch by a factor of 4 and will be reflected about the line $y = 0$. The C–value is -3 and the D–value is 0 so the vertex will be shifted 3 units to the left to $(-3, 0)$.

$(x, y) \rightarrow (x - 3, -4y)$

10. $g(x) = \frac{1}{3}f(x - 6) - 3$

The A–value is $\frac{1}{3}$, so the graph will have a vertical compression by a factor of $\frac{1}{3}$. The C–value is 6 and the D–value is -3 so the vertex will be shifted 6 units to the right and 3 units down to $(6, -3)$.

$(x, y) \rightarrow (x + 6, \frac{1}{3}y - 3)$

11. $g(x) = -0.75f(x + 4) - 2$

The A–value is -0.75, so the graph will have a vertical compression by a factor of 0.75 and will be reflected about the line $y = -2$. The C–value is -4 and the D–value is -2 so the vertex will be shifted 4 units to the left and 2 units down to $(-4, -2)$.

$(x, y) \rightarrow (x - 4, -0.75y - 2)$

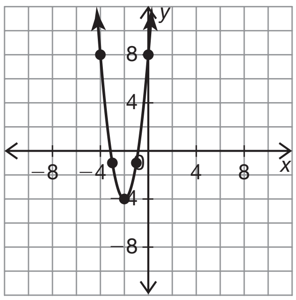
12. $g(x) = \frac{4}{3}f\left(x - \frac{1}{3}\right) + \frac{2}{3}$

The A–value is $\frac{4}{3}$, so the graph will have a vertical stretch by a factor of $\frac{4}{3}$. The C–value is $\frac{1}{3}$ and the D–value $\frac{2}{3}$ is so the vertex will be shifted $\frac{1}{3}$ units to the right and $\frac{2}{3}$ units up to $\left(\frac{1}{3}, \frac{2}{3}\right)$.

$(x, y) \rightarrow \left(x + \frac{1}{3}, \frac{4}{3}y + \frac{2}{3}\right)$

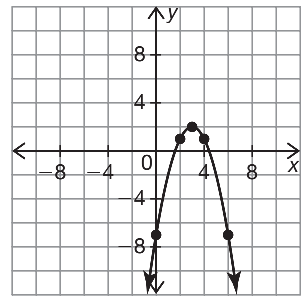
Write the function that represents each graph.

13.



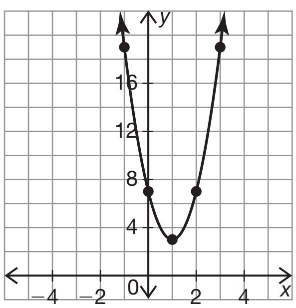
$f(x) = 3(x + 2)^2 - 4$

14.



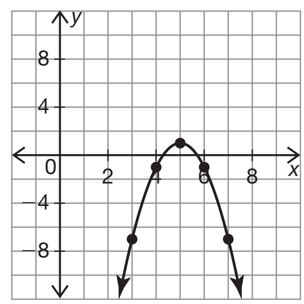
$f(x) = -(x - 3)^2 + 2$

15.



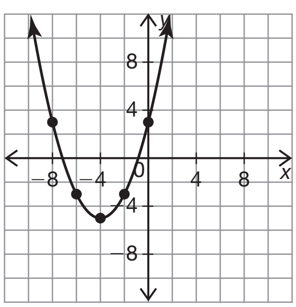
$f(x) = 4(x - 1)^2 + 3$

16.



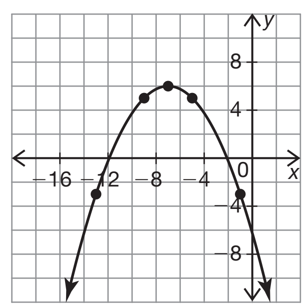
$f(x) = -2(x - 5)^2 + 1$

17.



$f(x) = \frac{1}{2}(x + 4)^2 - 5$

18.



$f(x) = -\frac{1}{4}(x + 7)^2 + 6$

Name ____ Date ____________

Side to Side
Horizontal Dilations of Quadratic Functions

Vocabulary

1. Explain the differences and similarities between horizontal dilation, horizontal stretching, and horizontal compression of a quadratic function.

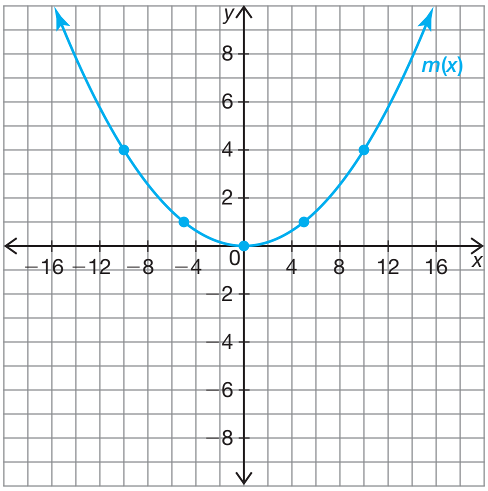
   A horizontal dilation is a type of transformation that stretches or compresses the entire graph. In a horizontal dilation, the x-coordinate of every point on the graph of a function is transformed by a common factor, $\frac{1}{B}$. Horizontal stretching is a type of horizontal dilation that is the stretching of the graph away from the y-axis and where $0 < |B| < 1$. Horizontal compression is a horizontal dilation that is the squeezing of a graph towards the y-axis and where $|B| > 1$.

Problem Set

Complete the table and graph $m(x)$. Then, describe how the graph of $m(x)$ compares to the graph of $f(x)$.

1. $f(x) = x^2$; $m(x) = f\left(\frac{1}{5}x\right)$

Reference Points on $f(x)$	→	Corresponding Points on $m(x)$
(0, 0)	→	(0, 0)
(5, 25)	→	(5, 1)
(10, 100)	→	(10, 4)
(15, 225)	→	(15, 9)

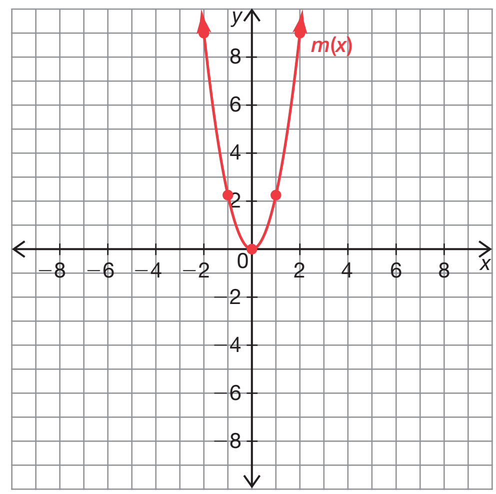


The function $m(x)$ is a horizontal stretch of $f(x)$ by a factor of 5.

2. $f(x) = x^2$; $m(x) = f(1.5x)$

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $m(x)$
(0, 0)	$\rightarrow$	(0, 0)
(1, 1)	$\rightarrow$	(1, 2.25)
(2, 4)	$\rightarrow$	(2, 9)
(4, 16)	$\rightarrow$	(4, 36)

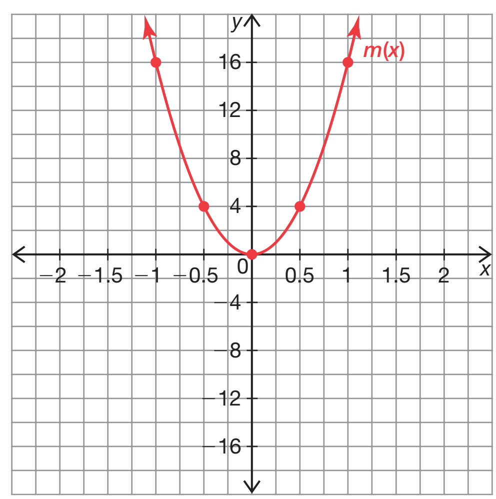
The function $m(x)$ is a horizontal compression of $f(x)$ by a factor of $\frac{2}{3}$.



3. $f(x) = x^2$; $m(x) = f(4x)$

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $m(x)$
(0, 0)	$\rightarrow$	(0, 0)
(0.5, 0.25)	$\rightarrow$	(0.5, 4)
(1, 1)	$\rightarrow$	(1, 16)
(2, 4)	$\rightarrow$	(2, 64)

The function $m(x)$ is a horizontal compression of $f(x)$ by a factor of $\frac{1}{4}$.

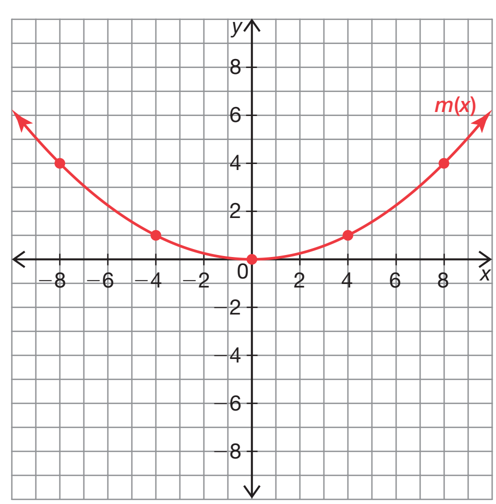


4

4. $f(x) = x^2$; $m(x) = f(0.25x)$

Reference Points on $f(x)$	$\rightarrow$	Corresponding Points on $m(x)$
(0, 0)	$\rightarrow$	(0, 0)
(4, 16)	$\rightarrow$	(4, 1)
(8, 64)	$\rightarrow$	(8, 4)
(12, 144)	$\rightarrow$	(12, 9)

The function $m(x)$ is a horizontal stretch of $f(x)$ by a factor of 4.



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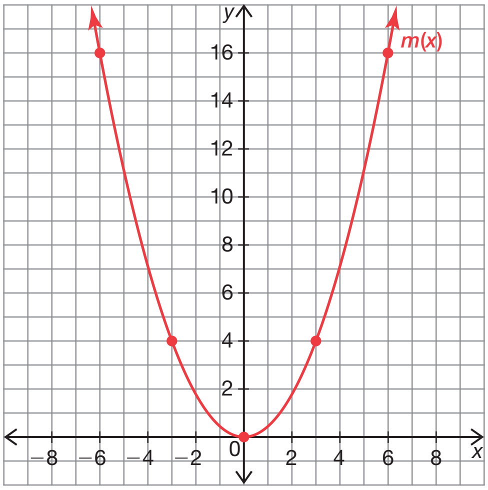
5. $f(x) = x^2$; $m(x) = f\left(\frac{2}{3}x\right)$

Reference Points on $f(x)$	→	Corresponding Points on $m(x)$
(0, 0)	→	(0, 0)
(3, 9)	→	(3, 4)
(6, 36)	→	(6, 16)
(9, 81)	→	(9, 36)

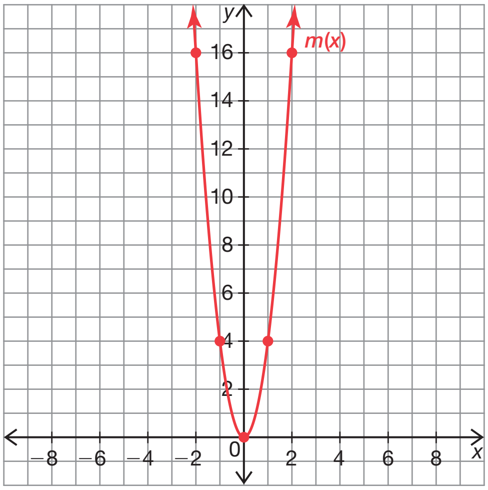
The function $m(x)$ is a horizontal stretch of $f(x)$ by a factor of $\frac{3}{2}$.



6. $f(x) = x^2$; $m(x) = f(2x)$

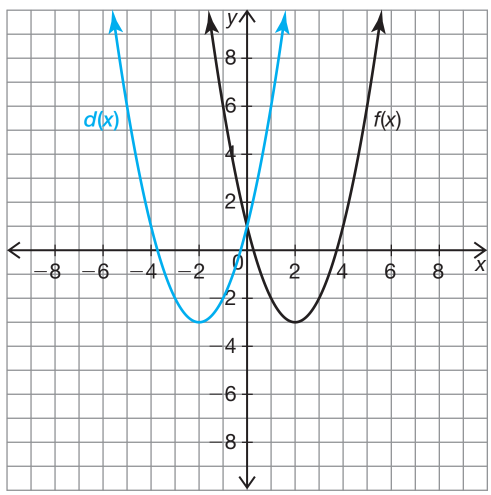
Reference Points on $f(x)$	→	Corresponding Points on $m(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, 4)
(2, 4)	→	(2, 16)
(3, 9)	→	(3, 36)

The function $m(x)$ is a horizontal compression of $f(x)$ by a factor of $\frac{1}{2}$.

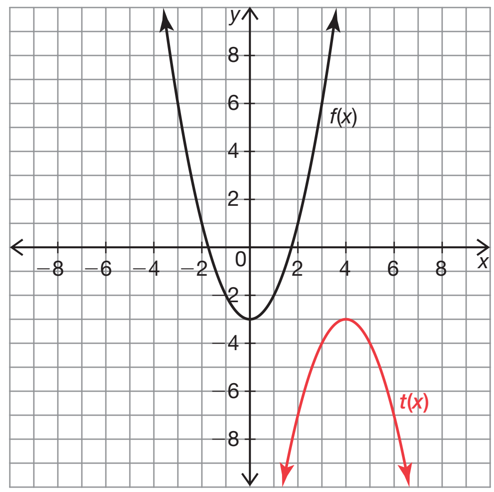


The graph of $f(x)$ is shown. Sketch the graph of the given transformed function.

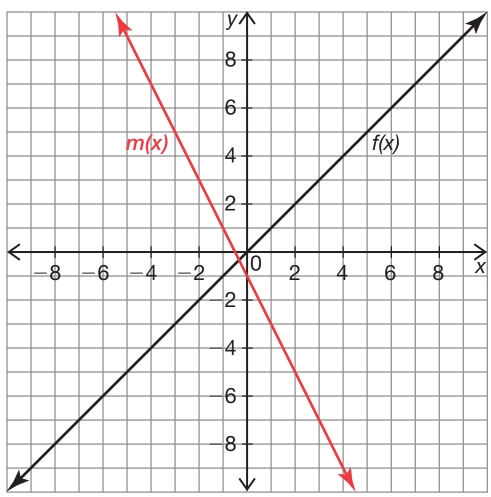
7. $d(x) = f(-x)$



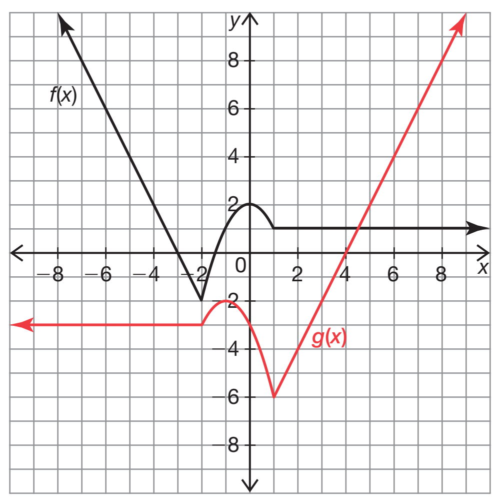
8. $t(x) = -f(x - 4)$



9. $m(x) = -2f(x + 3) + 5$



10. $g(x) = (-x + 1) - 4$



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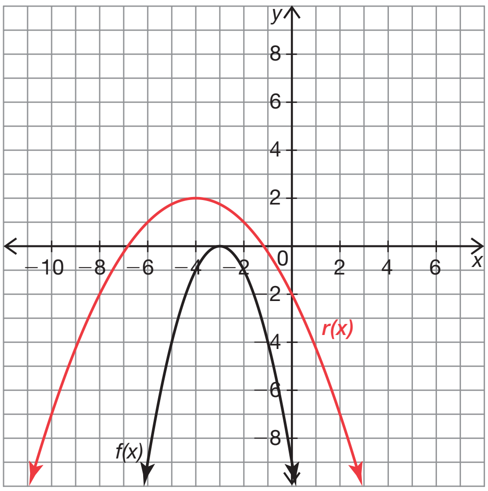
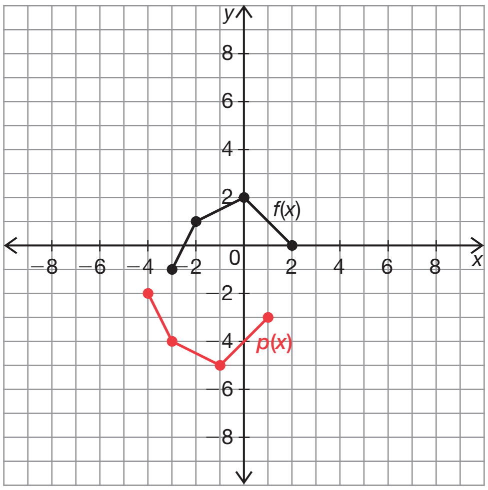
11. $r(x) = f\left(\frac{1}{2}x + 1\right) + 2$

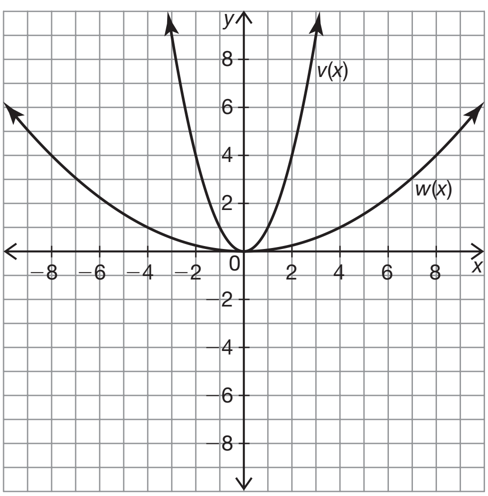


12. $p(x) = -f(x + 1) - 3$



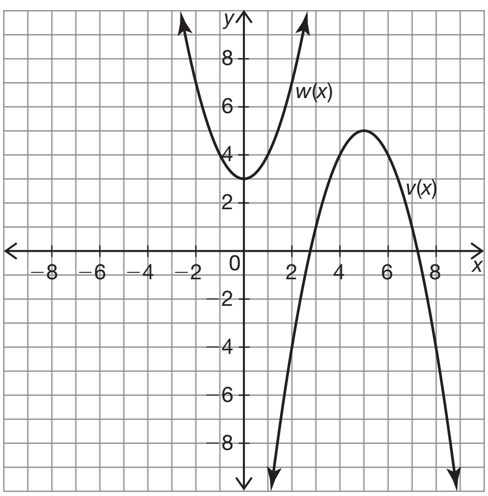
Write an equation for $w(x)$ in terms of $v(x)$.

13.



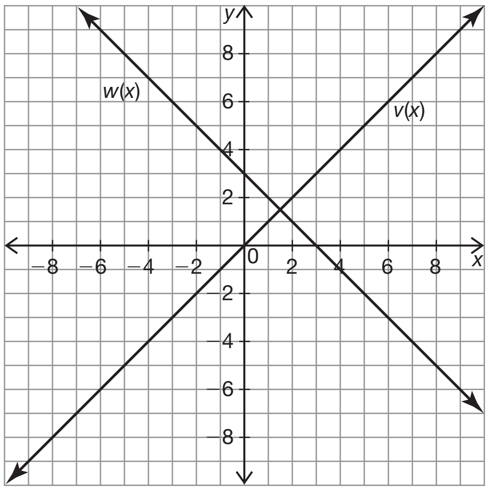
$w(x) = v\left(\frac{1}{4}x\right)$

14.



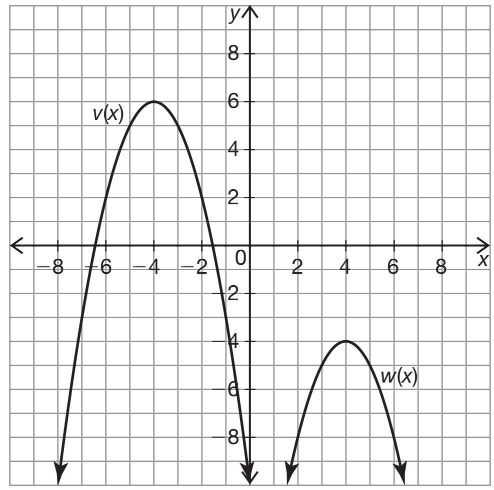
$w(x) = -v(x + 5) - 2$

15.



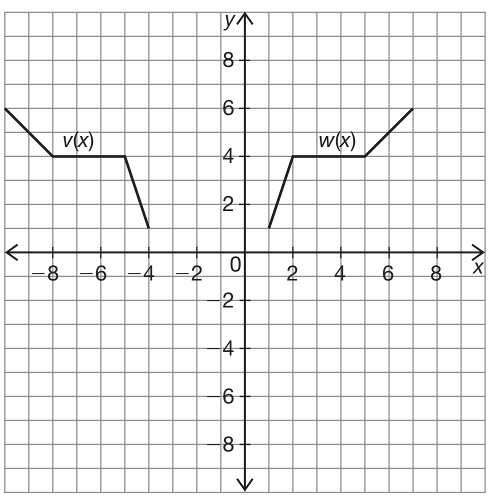
w(*x*) = −*v*(*x*) + 3

16.



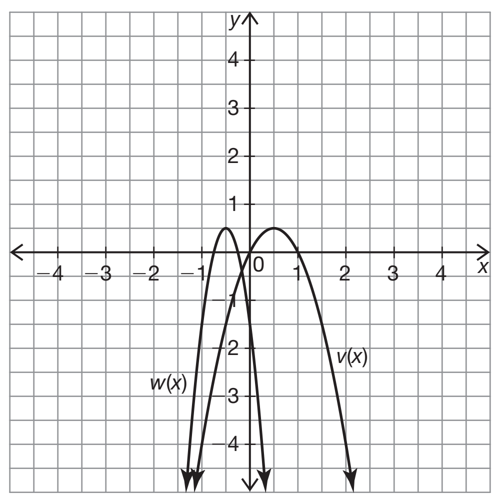
w(*x*) = *v*(*x* − 8) − 10

17.



w(*x*) = *v*(−*x* − 3)

18.



w(*x*) = *v*(−2*x*)

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What's the Point?

Deriving Quadratic Functions

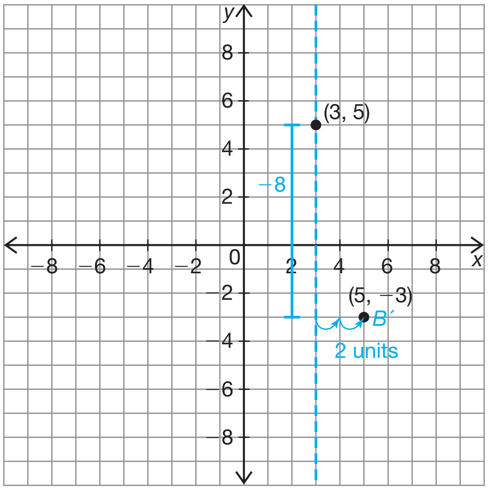
Problem Set

Use your knowledge of reference points to write an equation for the quadratic function that satisfies the given information. Use the graph to help solve each problem.

1. Given: vertex (3, 5) and point (5, −3)

$f(x) = -2(x - 3)^2 + 5$

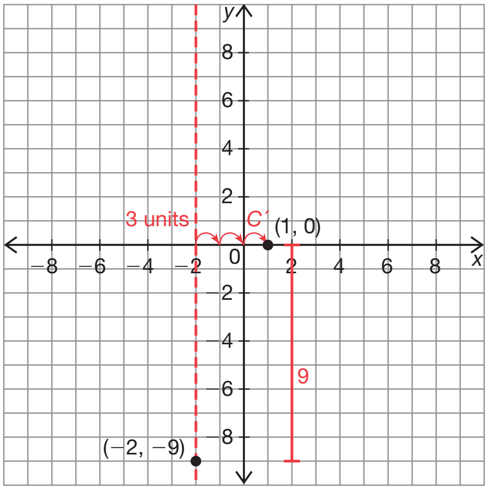
Point (5, −3) is point B' because it is 2 units from the axis of symmetry. The range between the vertex and point B on the basic function is 4. The range between the vertex and point B' is $4 \times (-2)$, therefore the a-value must be −2.



2. Given: vertex (−2, −9) and one of two x-intercepts (1, 0)

$f(x) = (x + 2)^2 - 9$

Point (1, 0) is point C' because it is 3 units from the axis of symmetry. The range between the vertex and point C on the basic function is 9. The range between the vertex and point C' is 9×1, therefore the a-value must be 1.

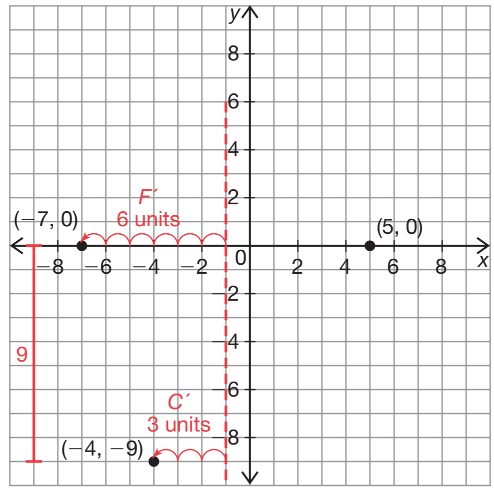


3. Given: two x-intercepts $(-7, 0)$ and $(5, 0)$ and one point $(-4, -9)$

$f(x) = \frac{1}{3}(x - 5)(x + 7)$

$\frac{r_1 + r_2}{2} = \frac{-7 + 5}{2} = \frac{-2}{2} = -1$

The axis of symmetry is $x = -1$. Point $(-7, 0)$ is point F' because it is 6 units from the axis of symmetry. Point $(-4, -9)$ is point C' because it is 3 units from the axis of symmetry. The range between point C and F on the basic function is 27. The range between point C' and point F' is $27 \times \frac{1}{3}$, therefore the a-value must be $\frac{1}{3}$.

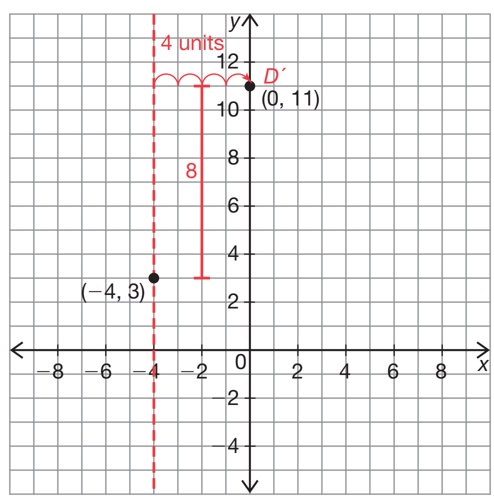


4

4. Given: vertex $(-4, 3)$ and y-intercept $(0, 11)$

$f(x) = \frac{1}{2}(x + 4)^2 + 3$

Point $(0, 11)$ is D' because it is 4 units from the axis of symmetry. The range between the vertex and point D on the basic function is 16. The range between the vertex and point D' is $16 \times \frac{1}{2}$, therefore the a-value must be $\frac{1}{2}$.



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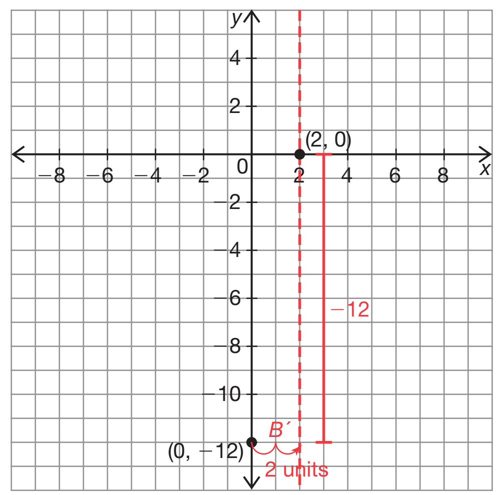
5. Given: exactly one x-intercept $(2, 0)$ and y-intercept $(0, -12)$

$f(x) = -3(x - 2)^2$

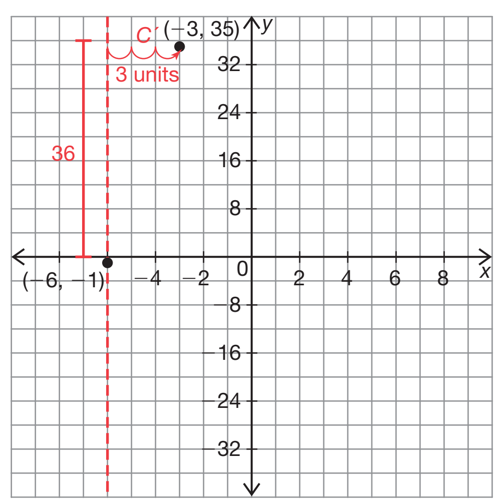
Because there is only 1 x-intercept, it must be the vertex. Point $(0, -12)$ is B' because it is 2 units from the axis of symmetry. The range between the vertex and point B on the basic function is 4. The range between the vertex and B' is $4 \times (-3)$, therefore the a-value must be -3.



6. Given: vertex $(-6, -1)$ and point $(-3, 35)$

$f(x) = 4(x + 6)^2 - 1$

Point $(-3, 35)$ is C' because it is 3 units from the axis of symmetry. The range between the vertex and point C on the basic function is 9. The range between the vertex and C' is 9×4, therefore the a-value must be 4.



Use a graphing calculator to determine the quadratic equation for each set of three points that lie on a parabola.

7. $(-4, 12), (-2, -14), (2, 6)$

$f(x) = 3x^2 + 5x - 16$

8. $(5, -56), (1, -4), (-10, -26)$

$f(x) = -x^2 - 7x + 4$

9. $(-8, 8), (-4, 6), (4, 38)$

$f(x) = 0.375x^2 + 4x + 16$

10. $(-2, 3), (2, -9), (5, -60)$

$f(x) = -2x^2 - 3x + 5$

11. $(0, 3), (-5, -2.4), (15, -7.8)$

$f(x) = -0.09x^2 + 0.63x + 3$

12. $(-2, 13), (1, -17), (7, 31)$

$f(x) = 2x^2 - 8x - 11$

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Create a system of equations and use algebra to write a quadratic equation for each set of three points that lie on a parabola.

13. $(-3, 12), (0, 9), (3, 24)$

Equation 1: $12 = 9a - 3b + c$

Equation 2: $9 = c$

Equation 3: $24 = 9a + 3b + c$

Substitute equation 2 into equation 1 and solve for a.

$$12 = 9a - 3b + 9$$
$$3 = 9a - 3b$$
$$3 + 3b = 9a$$
$$a = \frac{1}{3} + \frac{1}{3}b$$

Substitute the value for a in terms of b and the value for c into equation 3 and solve for b.

$$24 = 9\left(\frac{1}{3} + \frac{1}{3}b\right) + 3b + 9$$
$$24 = 3 + 3b + 3b + 9$$
$$15 = 3 + 6b$$
$$12 = 6b$$
$$b = 2$$

Substitute the values for b and c into equation 1 and solve for a.

$$12 = 9a - 3(2) + 9$$
$$15 = 9a + 3$$
$$9 = 9a$$
$$a = 1$$

Substitute the values for a, b, and c into a quadratic equation in standard form.

$$f(x) = x^2 + 2x + 9$$

4

14. $(-2, -2)$, $(1, -5)$, $(2, -18)$

Equation 1: $-2 = 4a - 2b + c$

Equation 2: $-5 = a + b + c$

Equation 3: $-18 = 4a + 2b + c$

Subtract equation 2 from equation 1 and solve in terms of a:

$$\begin{aligned} -2 &= 4a - 2b + c \\ -(-5 &= a + b + c) \\ \hline 3 &= 3a - 3b \\ 3a &= 3 + 3b \\ a &= 1 + b \end{aligned}$$

Subtract equation 3 from equation 2:

$$\begin{aligned} -5 &= a + b + c \\ -(-18 &= 4a + 2b + c) \\ \hline 13 &= -3a - b \end{aligned}$$

Substitute the value for a into this equation:

$$\begin{aligned} 13 &= -3(1 + b) - b \\ 13 &= -3 - 3b - b \\ 16 &= -4b \\ -4 &= b \end{aligned}$$

Substitute the value of b into the equation for the value of a:

$$\begin{aligned} a &= 1 + (-4) \\ a &= -3 \end{aligned}$$

Substitute the values of a and b into equation 1:

$$\begin{aligned} -2 &= 4(-3) - 2(-4) + c \\ -2 &= -12 + 8 + c \\ -2 &= -4 + c \\ c &= 2 \end{aligned}$$

Substitute the values for a, b, and c into a quadratic equation in standard form.

$f(x) = -3x^2 - 4x + 2$

4

Name ____________________ Date ____________

15. (2, 9), (0, −5), (−10, −15)

Equation 1: $9 = 4a + 2b + c$

Equation 2: $-5 = c$

Equation 3: $-15 = 100a - 10b + c$

Substitute equation 2 into equation 1 and solve for *a*:

$$9 = 4a + 2b - 5$$
$$14 = 4a + 2b$$
$$14 - 2b = 4a$$
$$a = \frac{7}{2} - \frac{1}{2}b$$

Substitute the value for *a* in terms of *b* and the value for *c* into equation 3 and solve for *b*.

$$-15 = 100\left(\frac{7}{2} - \frac{1}{2}b\right) - 10b - 5$$
$$-15 = 350 - 50b - 10b - 5$$
$$-360 = -60b$$
$$b = 6$$

Substitute the values for *b* and *c* into the value for *a*.

$$a = \frac{7}{2} - \frac{1}{2}(6)$$
$$a = \frac{1}{2}$$

Substitute the values for *a*, *b*, and *c* into a quadratic equation in standard form.

$$f(x) = \frac{1}{2}x^2 + 6x - 5$$

4

16. $(-1, 2), (4, 27), (-3, 20)$

Equation 1: $2 = a - b + c$

Equation 2: $27 = 16a + 4b + c$

Equation 3: $20 = 9a - 3b + c$

Subtract equation 2 from equation 1 and solve in terms of a:

$$\begin{array}{rrl} & 2 = & a - b + c \\ - & (27 = & 16a + 4b + c) \\ \hline & -25 = & -15a - 5b \end{array}$$

$$15a = 25 - 5b$$

$$a = \frac{5}{3} - \frac{1}{3}b$$

Subtract equation 3 from equation 2:

$$\begin{array}{rrl} & 27 = & 16a + 4b + c \\ - & (20 = & 9a - 3b + c) \\ \hline & 7 = & 7a + 7b \end{array}$$

Substitute the value for a into this equation:

$$7 = 7\left(\frac{5}{3} - \frac{1}{3}b\right) + 7b$$

$$7 = \frac{35}{3} - \frac{7}{3}b + 7b$$

$$-\frac{14}{3} = \frac{14}{3}b$$

$$-1 = b$$

Substitute the value of b into the equation for the value of a:

$$a = \frac{5}{3} - \frac{1}{3}(-1)$$

$$a = 2$$

Substitute the values of a and b into equation 1:

$$2 = 2 - (-1) + c$$

$$2 = 3 + c$$

$$c = -1$$

Substitute the values for a, b, and c into a quadratic equation in standard form.

$$f(x) = 2x^2 - x - 1$$

4

Name ____________________ Date ____________

17. (5, −6), (−2, 8), (3, 4)

Equation 1: $-6 = 25a + 5b + c$

Equation 2: $8 = 4a - 2b + c$

Equation 3: $4 = 9a + 3b + c$

Subtract equation 2 from equation 1 and solve in terms of a:

$$\begin{array}{rrl} & -6 = & 25a + 5b + c \\ - & (8 = & 4a - 2b + c) \\ \hline & -14 = & 21a + 7b \end{array}$$

$$-21a = 14 + 7b$$

$$a = -\frac{2}{3} - \frac{1}{3}b$$

Subtract equation 3 from equation 2:

$$\begin{array}{rrl} & 8 = & 4a - 2b + c \\ - & (4 = & 9a + 3b + c) \\ \hline & 4 = & -5a - 5b \end{array}$$

Substitute the value for a into this equation:

$$4 = -5\left(-\frac{2}{3} - \frac{1}{3}b\right) - 5b$$

$$4 = \frac{10}{3} + \frac{5}{3}b - 5b$$

$$\frac{2}{3} = -\frac{10}{3}b$$

$$-\frac{1}{5} = b$$

Substitute the value of b into the equation for the value of a:

$$a = -\frac{2}{3} - \frac{1}{3}\left(\frac{-1}{5}\right)$$

$$a = -\frac{2}{3} + \frac{1}{15}$$

$$a = -\frac{9}{15} = -\frac{3}{5}$$

Substitute the values of a and b into equation 1:

$$-6 = 25\left(-\frac{3}{5}\right) + 5\left(-\frac{1}{5}\right) + c$$

$$-6 = -15 - 1 + c$$

$$10 = c$$

Substitute the values for a, b, and c into a quadratic equation in standard form.

$$f(x) = -\frac{3}{5}x^2 - \frac{1}{5}x + 10$$

4

18. $(1, 17), (-1, -9), (2, 105)$

Equation 1: $17 = a + b + c$

Equation 2: $-9 = a - b + c$

Equation 3: $105 = 4a + 2b + c$

Subtract equation 2 from equation 1 and solve for b:

$$\begin{array}{rl} & 17 = a + b + c \\ - & (-9 = a - b + c) \\ \hline & 26 = \quad 2b \\ & b = 13 \end{array}$$

Subtract equation 2 from equation 3 and solve for a in terms of b:

$$\begin{array}{rl} & 105 = 4a + 2b + c \\ - & (-9 = \quad a - \quad b + c) \\ \hline & 114 = 3a + 3b \\ & 3a = 114 - 3b \\ & a = 38 - b \end{array}$$

4

Substitute the value for b into this equation:

$a = 38 - 13$

$a = 25$

Substitute the values of a and b into equation 1:

$17 = 25 + 13 + c$

$17 = 38 + c$

$-21 = c$

Substitute the values for a, b, and c into a quadratic equation in standard form.

$f(x) = 25x^2 + 13x - 21$

Name ______________________________ Date ____________

Now It's Getting Complex . . . But It's Really Not Difficult!
Complex Number Operations

Vocabulary

Match each term to its corresponding definition.

1. the number i
   E
2. imaginary number
   A
3. pure imaginary number
   H
4. complex number
   F
5. real part of a complex number
   B
6. imaginary part of a complex number
   I
7. complex conjugates
   D
8. monomial
   J
9. binomial
   C
10. trinomial
    G

A. a number in the form $a + bi$ where a and b are real numbers and b is not equal to 0

B. term a of a number written in the form $a + bi$

C. a polynomial with two terms

D. pairs of numbers of the form $a + bi$ and $a - bi$

E. a number such that its square equals -1

F. a number in the form $a + bi$ where a and b are real numbers

G. a polynomial with three terms

H. a number of the form bi where b is not equal to 0

I. term bi of a number written in the form $a + bi$

J. a polynomial with one term

Problem Set

Calculate each power of i.

1. i^{48}

$i^{48} = (i^4)^{12}$
$= 1^{12}$
$= 1$

2. i^{361}

$i^{361} = (i^4)^{90}(i^1)$
$= 1^{90}(\sqrt{-1})$
$= \sqrt{-1}$
$= i$

3. i^{55}

$i^{55} = (i^4)^{13}(i^2)(i^1)$
$= 1^{13}(-1)(\sqrt{-1})$
$= -\sqrt{-1}$
$= -i$

4. i^{1000}

$i^{1000} = (i^4)^{250}$
$= 1^{250}$
$= 1$

5. i^{-22}

$i^{-22} = \dfrac{1}{(i^4)^5 \cdot (i^2)}$
$= \dfrac{1}{1^5(-1)}$
$= -1$

6. i^{-7}

$i^{-7} = \dfrac{1}{(i^2)^3 \cdot i}$
$= \dfrac{1}{(-1)^3 \cdot i}$
$= \dfrac{1}{-i}$
$= \dfrac{1}{i^{-1}}$
$= i$
$= \sqrt{-1}$

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Rewrite each expression using i.

7. $\sqrt{-72}$

$\sqrt{-72} = \sqrt{36(2)(-1)}$
$= 6\sqrt{2}i$

8. $\sqrt{-49} + \sqrt{-23}$

$\sqrt{-49} + \sqrt{-23} = \sqrt{49(-1)} + \sqrt{23(-1)}$
$= 7i + \sqrt{23}i$

9. $38 - \sqrt{-200} + \sqrt{121}$

$38 - \sqrt{-200} + \sqrt{121} = 38 - \sqrt{100(2)(-1)} + 11$
$= 49 - 10\sqrt{2}i$

10. $\sqrt{-45} + 21$

$\sqrt{-45} + 21 = \sqrt{9(5)(-1)} + 21$
$= 3\sqrt{5}i + 21$

11. $\dfrac{\sqrt{-48} - 12}{4}$

$\dfrac{\sqrt{-48} - 12}{4} = \dfrac{\sqrt{16(3)(-1)}}{4} - \dfrac{12}{4}$

$= \dfrac{4\sqrt{3}i}{4} - 3$

$= \sqrt{3}i - 3$

12. $\dfrac{1 + \sqrt{4} - \sqrt{-15}}{3}$

$\dfrac{1 + \sqrt{4} - \sqrt{-15}}{3} = \dfrac{1}{3} + \dfrac{2}{3} - \dfrac{\sqrt{15(-1)}}{3}$

$= 1 - \dfrac{\sqrt{15}i}{3}$

13. $-\sqrt{-28} + \dfrac{\sqrt{21}}{3} - \dfrac{\sqrt{12}}{6}$

$-\sqrt{-28} + \dfrac{\sqrt{21}}{3} - \dfrac{\sqrt{12}}{6} = -\sqrt{4(7)(-1)} + \dfrac{2\sqrt{21}}{6} - \dfrac{\sqrt{4(3)}}{6}$

$= -2\sqrt{7}i + \dfrac{2\sqrt{21} - 2\sqrt{3}}{6}$

$= -2\sqrt{7}i + \dfrac{2(\sqrt{21} - \sqrt{3})}{6}$

$= -2\sqrt{7}i + \dfrac{(\sqrt{21} - \sqrt{3})}{3}$

14. $\dfrac{\sqrt{-75} + \sqrt{80}}{10}$

$\dfrac{\sqrt{-75} + \sqrt{80}}{10} = \dfrac{\sqrt{25(3)(-1)}}{10} + \dfrac{\sqrt{16(5)}}{10}$

$= \dfrac{5\sqrt{3}i}{10} + \dfrac{4\sqrt{5}}{10}$

$= \dfrac{1}{2}\sqrt{3}i + \dfrac{2}{5}\sqrt{5}$

4

Simplify each expression.

15. $(2 + 5i) - (7 - 9i)$

$(2 + 5i) - (7 - 9i) = 2 + 5i - 7 + 9i$

$= (2 - 7) + (5i + 9i)$

$= -5 + 14i$

16. $-6 + 8i - 1 - 11i + 13$

$-6 + 8i - 1 - 11i + 13$

$= (-6 - 1 + 13) + (8i - 11i)$

$= 6 - 3i$

17. $-(4i - 1 + 3i) + (6i - 10 + 17)$

$-(4i - 1 + 3i) + (6i - 10 + 17)$

$= (-4i - 3i + 6i) + (1 - 10 + 17)$

$= -i + 8$

18. $22i + 13 - (7i + 3 + 12i) + 16i - 25$

$22i + 13 - (7i + 3 + 12i) + 16i - 25$

$= (22i - 7i - 12i + 16i) + (13 - 3 - 25)$

$= 19i - 15$

19. $9 + 3i(7 - 2i)$

$9 + 3i(7 - 2i) = 9 + 21i - 6i^2$

$= 9 + 21i - 6(-1)$

$= (9 + 6) + 21i$

$= 15 + 21i$

20. $(4 - 5i)(8 + i)$

$(4 - 5i)(8 + i) = 32 + 4i - 40i - 5i^2$

$= 32 + 4i - 40i - 5(-1)$

$= (32 + 5) + (4i - 40i)$

$= 37 - 36i$

21. $-0.5(14i - 6) - 4i(0.75 - 3i)$

$-0.5(14i - 6) - 4i(0.75 - 3i)$

$= -7i + 3 - 3i + 12i^2$

$= (-7i - 3i) + 3 + 12(-1)$

$= (-7i - 3i) + (3 - 12)$

$= -10i - 9$

22. $\left(\frac{1}{2}i - \frac{3}{4}\right)\left(\frac{1}{8} - \frac{3}{4}i\right)$

$\left(\frac{1}{2}i - \frac{3}{4}\right)\left(\frac{1}{8} - \frac{3}{4}i\right) = \frac{1}{16}i - \frac{3}{8}i^2 - \frac{3}{32} + \frac{9}{16}i$

$= \left(\frac{1}{16}i + \frac{9}{16}i\right) - \frac{3}{8}(-1) - \frac{3}{32}$

$= \frac{10}{16}i + \frac{3}{8} - \frac{3}{32}$

$= \frac{5}{8}i + \frac{9}{32}$

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Determine each product.

23. $(3 + i)(3 - i)$

$$\begin{aligned}(3 + i)(3 - i) &= 9 - 3i + 3i - i^2\\ &= 9 - (-1)\\ &= 10\end{aligned}$$

24. $(4i - 5)(4i + 5)$

$$\begin{aligned}(4i - 5)(4i + 5) &= 16i^2 + 20i - 20i - 25\\ &= 16(-1) - 25\\ &= -41\end{aligned}$$

25. $(7 - 2i)(7 + 2i)$

$$\begin{aligned}(7 - 2i)(7 + 2i) &= 49 + 14i - 14i - 4i^2\\ &= 49 - 4(-1)\\ &= 54\end{aligned}$$

26. $\left(\frac{1}{3} + 3i\right)\left(\frac{1}{3} - 3i\right)$

$$\begin{aligned}\left(\frac{1}{3} + 3i\right)\left(\frac{1}{3} - 3i\right) &= \frac{1}{9} - i + i - 9i^2\\ &= \frac{1}{9} - 9(-1)\\ &= 9\frac{1}{9}\end{aligned}$$

27. $(0.1 + 0.6i)(0.1 - 0.6i)$

$$\begin{aligned}&(0.1 + 0.6i)(0.1 - 0.6i)\\ &= 0.01 - 0.06i + 0.06i - 0.36i^2\\ &= 0.01 - 0.36(-1)\\ &= 0.01 + 0.36\\ &= 0.37\end{aligned}$$

28. $-2[(-i - 8)(-i + 8)]$

$$\begin{aligned}-2[(-i - 8)(-i + 8)] &= -2(i^2 - 8i + 8i - 64)\\ &= -2(-1 - 64)\\ &= 130\end{aligned}$$

Identify each expression as a monomial, binomial, or trinomial. Explain your reasoning.

29. $4xi + 7x$

The expression is a monomial because it can be rewritten as $(4i + 7)x$, which shows one x term.

30. $-3x + 5 - 8xi + 1$

The expression is a binomial because it can be rewritten as $(-3 - 8i)x + 6$, which shows one x term and one constant term.

31. $6x^2i + 3x^2$

The expression is a monomial because it can be rewritten as $(6i + 3)x^2$, which shows one x^2 term.

32. $8i - x^3 + 7x^2i$

The expression is a trinomial because it shows one x^3 term, one x^2 term, and one constant term.

33. $xi - x + i + 2 - 4i$

The expression is binomial because it can be rewritten as $(i - 1)x + (2 - 3i)$, which shows one x term and one constant term.

34. $-3x^3i - x^2 + 6x^3 + 9i - 1$

The expression is a trinomial because it can be rewritten as $(-3i + 6)x^3 + (-1)x^2 + (9i - 1)$, which shows one x^3 term, one x^2 term, and one constant term.

Simplify each expression, if possible.

35. $(x - 6i)^2$

$$\begin{aligned}(x - 6i)^2 &= x^2 - 6xi - 6xi + 36i^2 \\ &= x^2 - 12xi + 36(-1) \\ &= x^2 - 12xi - 36\end{aligned}$$

4

36. $(2 + 5xi)(7 - xi)$

$$\begin{aligned}(2 + 5xi)(7 - xi) &= 14 - 2xi + 35xi - 5x^2i^2 \\ &= 14 + 33xi - 5x^2(-1) \\ &= 14 + 33xi + 5x^2\end{aligned}$$

37. $3xi - 4yi$

This expression cannot be simplified.

38. $(2xi - 9)(3x + 5i)$

$$\begin{aligned}(2xi - 9)(3x + 5i) &= 6x^2i + 10xi^2 - 27x - 45i \\ &= 6x^2i + (-1)10x - 27x - 45i \\ &= 6x^2i - 37x - 45i\end{aligned}$$

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39. $(x + 4i)(x - 4i)(x + 4i)$

$$\begin{aligned}(x + 4i)(x - 4i)(x + 4i) &= (x^2 - 16i^2)(x + 4i)\\ &= (x^2 - 16(-1))(x + 4i)\\ &= (x^2 + 16)(x + 4i)\\ &= (x^3 + 4x^2i + 16x + 64i\end{aligned}$$

40. $(3i - 2xi)(3i - 2xi) + (2i - 3xi)(2 - 3xi)$

$$\begin{aligned}(3i - 2xi)(3i - 2xi) + (2i - 3xi)(2 - 3xi) &= (9i^2 - 6xi^2 - 6xi^2 + 4x^2i^2) + (4i - 6xi^2 - 6xi + 9x^2i^2)\\ &= (9(-1) - 12x(-1) + 4x^2(-1)) + (4i - 6x(-1) - 6xi + 9x^2(-1))\\ &= (-9 + 12x - 4x^2) + (4i + 6x - 6xi - 9x^2)\\ &= (-4x^2 - 9x^2) + (12x + 6x - 6xi) + (-9 + 4i)\\ &= -13x^2 + 18x - 6xi - 9 + 4i\end{aligned}$$

4

For each complex number, write its conjugate.

41. $7 + 2i$

$7 - 2i$

42. $3 + 5i$

$3 - 5i$

43. $8i$

$-8i$

44. $-7i$

$7i$

45. $2 - 11i$

$2 + 11i$

46. $9 - 4i$

$9 + 4i$

47. $-13 - 6i$

$-13 + 6i$

48. $-21 + 4i$

$-21 - 4i$

Calculate each quotient.

49. $\frac{3 + 4i}{5 + 6i}$

$$\frac{3 + 4i}{5 + 6i} = \frac{3 + 4i}{5 + 6i} \cdot \frac{5 - 6i}{5 - 6i} = \frac{15 - 18i + 20i - 24i^2}{25 - 30i + 30i - 36i^2}$$

$$= \frac{15 + 2i + 24}{25 + 36} = \frac{39 + 2i}{61} = \frac{39}{61} + \frac{2}{61}i$$

50. $\frac{8 + 7i}{2 + i}$

$$\frac{8 + 7i}{2 + i} = \frac{8 + 7i}{2 + i} \cdot \frac{2 - i}{2 - i} = \frac{16 - 8i + 14i - 7i^2}{4 + 2i - 2i - i^2}$$

$$= \frac{16 + 6i + 7}{4 + 1} = \frac{23 + 6i}{5} = \frac{23}{5} + \frac{6}{5}i$$

51. $\frac{-6 + 2i}{2 - 3i}$

$$\frac{-6 + 2i}{2 - 3i} = \frac{-6 + 2i}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} = \frac{-12 - 18i + 4i + 6i^2}{4 + 6i - 6i - 9i^2}$$

$$= \frac{-12 - 14i - 6}{4 + 9} = \frac{-18 - 14i}{13} = -\frac{18}{13} - \frac{14}{13}i$$

52. $\frac{-1 + 5i}{1 - 4i}$

$$\frac{-1 + 5i}{1 - 4i} = \frac{-1 + 5i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i} = \frac{-1 - 4i + 5i + 20i^2}{1 + 4i - 4i - 16i^2}$$

$$= \frac{-1 + i - 20}{1 + 16} = \frac{-21 + i}{17} = -\frac{21}{17} + \frac{1}{17}i$$

53. $\frac{6 - 3i}{2 - i}$

$$\frac{6 - 3i}{2 - i} = \frac{6 - 3i}{2 - i} \cdot \frac{2 + i}{2 + i} = \frac{12 + 6i - 6i - 3i^2}{4 + 2i - 2i - i^2}$$

$$= 12 + \frac{3}{4} + 1 = \frac{15}{5} = 3$$

54. $\frac{4 - 2i}{-1 + 2i}$

$$\frac{4 - 2i}{-1 + 2i} = \frac{4 - 2i}{-1 + 2i} \cdot \frac{-1 - 2i}{-1 - 2i} = \frac{-4 - 8i + 2i + 4i^2}{1 + 2i - 2i - 2i^2}$$

$$= \frac{-4 - 6i - 4}{1 + 2} = \frac{-8 - 6i}{3} = -\frac{8}{3} - \frac{6}{3}i = -\frac{8}{3} - 2i$$

4

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You Can't Spell "Fundamental Theorem of Algebra" without F-U-N!

Quadratics and Complex Numbers

Vocabulary

Write a definition for each term in your own words.

1. imaginary roots

Imaginary roots are imaginary solutions to an equation. Imaginary roots are the values of x when $y = 0$ and the graph does not intersect the x-axis.

2. discriminant

The discriminant is the radicand in the Quadratic Formula, $b^2 - 4ac$. The discriminant "discriminates" the number and type of roots of a quadratic equation.

3. imaginary zeros

Imaginary zeros are zeros of quadratic functions that do not cross the x-axis.

4. degree of a polynomial equation

The degree of a polynomial equation is the greatest exponent in the polynomial equation.

5. Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra is a theorem that states that any polynomial equation of degree n must have exactly n complex roots or solutions and any polynomial function of degree n must have exactly n complex zeros.

6. double root

Double roots are two real roots of an equation if the graph has 1 x-intercept.

Problem Set

Use the Quadratic Formula to solve an equation of the form $f(x) = 0$ for each function.

1. $f(x) = x^2 - 2x - 3$

$x^2 - 2x - 3 = 0$

$a = 1, b = -2, c = -3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$

$x = \frac{2 \pm \sqrt{16}}{2}$

$x = \frac{2 \pm 4}{2}$

$x = 3, x = -1$

2. $f(x) = x^2 + 4x + 4$

$x^2 + 4x + 4 = 0$

$a = 1, b = 4, c = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{0}}{2}$

$x = \frac{-4}{2}$

$x = -2$

4

3. $f(x) = 4x^2 - 9$

$4x^2 - 9 = 0$

$a = 4, b = 0, c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{0 \pm \sqrt{(0)^2 - 4(4)(-9)}}{2(4)}$

$x = \frac{0 \pm \sqrt{144}}{8}$

$x = \frac{0 \pm 12}{8}$

$x = \frac{3}{2}, x = -\frac{3}{2}$

4. $f(x) = -x^2 - 5x - 6$

$-x^2 - 5x - 6 = 0$

$a = -1, b = -5, c = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-1)(-6)}}{2(-1)}$

$x = \frac{5 \pm \sqrt{25 - 24}}{-2}$

$x = \frac{5 \pm 1}{-2}$

$x = -3, x = -2$

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5. $f(x) = x^2 + 2x + 10$

$x^2 + 2x + 10 = 0$

$a = 1, b = 2, c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-36}}{2}$$

$$x = \frac{-2 \pm 6i}{2}$$

$x = -1 + 3i, x = -1 - 3i$

6. $f(x) = -3x^2 - 6x - 11$

$-3x^2 - 6x - 11 = 0$

$a = -3, b = -6, c = -11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-3)(-11)}}{2(-3)}$$

$$x = \frac{6 \pm \sqrt{-96}}{-6}$$

$$x = \frac{6 \pm 4\sqrt{-6}}{-6}$$

$$x = \frac{6 \pm 4\sqrt{6}i}{-6}$$

$x = -1 + \frac{2\sqrt{6}}{3}i, x = -1 - \frac{2\sqrt{6}}{3}i$

4

Use the discriminant to determine whether each function has real or imaginary zeros.

7. $f(x) = x^2 + 12x + 35$

$b^2 - 4ac = 12^2 - 4(1)(35)$

$= 144 - 140$

$= 4$

The discriminant is positive, so the function has real zeros.

8. $f(x) = -3x + x - 9$

$b^2 - 4ac = 1^2 - 4(-3)(-9)$

$= 1 - 108$

$= -107$

The discriminant is negative, so the function has imaginary zeros.

9. $f(x) = x^2 - 4x + 7$

$b^2 - 4ac = (-4)^2 - 4(1)(7)$
$= 16 - 28$
$= -12$

The discriminant is negative, so the function has imaginary zeros.

10. $f(x) = 9x^2 - 12x + 4$

$b^2 - 4ac = (-12)^2 - 4(9)(4)$
$= 144 - 144$
$= 0$

The discriminant is zero, so the function has real zeros (double roots).

11. $f(x) = -\frac{1}{4}x^2 + 3x - 8$

$b^2 - 4ac = 3^2 - 4\left(-\frac{1}{4}\right)(-8)$
$= 9 - 8$
$= 1$

The discriminant is positive, so the function has real zeros.

12. $f(x) = x^2 + 6x + 9$

$b^2 - 4ac = 6^2 - 4(1)(9)$
$= 36 - 36$
$= 0$

The discriminant is zero, so the function has real zeros (double roots).

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Use the vertex form of a quadratic equation to determine whether the zeros of each function are real or imaginary. Explain how you know.

13. $f(x) = (x - 4)^2 - 2$

Because the vertex (4, −2) is below the x-axis and the parabola is concave up ($a > 0$), it intersects the x-axis. So, the zeros are real.

14. $f(x) = -2(x - 1)^2 - 5$

Because the vertex (−1, −5) is below the x-axis and the parabola is concave down ($a < 0$), it does not intersect the x-axis. So, the zeros are imaginary.

15. $f(x) = \frac{1}{3}(x - 2)^2 + 7$

Because the vertex (2, 7) is above the x-axis and the parabola is concave up ($a > 0$), it does not intersect the x-axis. So, the zeros are imaginary.

16. $f(x) = -3(x - 1)^2 + 5$

Because the vertex (1, 5) is above the x-axis and the parabola is concave down, it intersects the x-axis. So, the zeros are real.

17. $f(x) = -(x - 6)^2$

Because the vertex (6, 0) is on the x-axis, it intersects the x-axis once. So, the zeros are real.

18. $f(x) = \frac{3}{4}(x + 4)^2 - 6$

Because the vertex (−4, −6) is below the x-axis and the parabola is concave up, it intersects the x-axis. So, the zeros are real.

4

Factor each function over the set of real or imaginary numbers. Then, identify the type of zeros.

19. $k(x) = x^2 - 25$

$k(x) = (x + 5)(x - 5)$

$x = -5, x = 5$

The function $k(x)$ has two real zeros.

20. $n(x) = x^2 - 5x - 14$

$n(x) = (x - 7)(x + 2)$

$x = 7, x = -2$

The function $n(x)$ has two real zeros.

21. $p(x) = -x^2 - 8x - 17$

$p(x) = -[x - (-4 - i)][x - (-4 + i)]$

$x = -4 - i, x = -4 + i$

The function $p(x)$ has two imaginary zeros.

22. $g(x) = x^2 + 6x + 10$

$g(x) = [x - (-3 - i)][x - (-3 + i)]$

$x = -3 - i, x = -3 + i$

The function $g(x)$ has two imaginary zeros.

23. $h(x) = -x^2 + 8x - 7$

$h(x) = -(x - 7)(x - 1)$

$x = 7, x = 1$

The function $h(x)$ has two real zeros.

24. $m(x) = \frac{1}{2}x^2 + 8$

$m(x) = \frac{1}{2}(x + 4i)(x - 4i)$

$x = 4i, = -4i$

The function $m(x)$ has two imaginary zeros.