CAT- Practice Problems
Trig sections 5.5-5.6 **Name:** KEY

1. Find the exact value of sin 22.5° using the **half-angle** identity. $\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

2. Find the exact value of tan 75° using ***each*** half-angle identity listed for tangent.

a. $\boxed{2+\sqrt{3} \quad \text{OR} \quad \sqrt{7+4\sqrt{3}}}$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\sqrt{2+\sqrt{3}}\,\sqrt{2+\sqrt{3}} = \sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3}$$

$$\tan\frac{150}{2} = \pm\sqrt{\frac{1-\cos 150}{1+\cos 150}} = \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{1+\frac{-\sqrt{3}}{2}}\cdot\frac{2}{2}}\ \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\cdot\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \frac{2+\sqrt{3}\ \pm\ \boxed{2+\sqrt{3}}}{\sqrt{4-\sqrt{9}}} = 1$$

b. $\boxed{2+\sqrt{3}}$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

$$\tan\frac{150}{2} = \frac{\frac{1}{2}}{1+-\frac{\sqrt{3}}{2}}\cdot\frac{2}{2} = \frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = \boxed{2+\sqrt{3}}$$

c. $\boxed{2+\sqrt{3}}$

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$\frac{\pi}{2} < \frac{n}{2} < \frac{3\pi}{\frac{2}{2}}$

$90 < n < 135$

II

3. Given $\boxed{\cos n = \frac{-3}{7}}$ with $\pi < n < \frac{3\pi}{2}$ (III), find $\sin\frac{n}{2}$, $\cos\frac{n}{2}$, and $\tan\frac{n}{2}$. (In which quadrant does n/2 lie?)

a. $\sin\frac{n}{2} = +\frac{\sqrt{35}}{7}$

$\sin\frac{n}{2} = \pm\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1 - \frac{-3}{7}}{\frac{2}{1}}\cdot\frac{7}{7}} = \sqrt{\frac{7+3}{14}} = \sqrt{\frac{5}{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$

b. $\cos\frac{n}{2} = \frac{-\sqrt{14}}{7}$

$\cos\frac{n}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+\frac{-3}{7}}{2}\cdot\frac{7}{7}} = \frac{\sqrt{7-3}}{\sqrt{14}} = \sqrt{\frac{2}{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$

$\sqrt{\frac{4}{14}}$ $\quad \frac{\sqrt{2}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}$

c. $\tan\frac{n}{2} = \frac{-\sqrt{10}}{2}$

$\tan\frac{n}{2} = \frac{1-\cos n}{\sin n} = \frac{1 - \frac{-3}{7}}{\frac{-2\sqrt{10}}{7}}\cdot\frac{7}{7} = \frac{7+3}{-2\sqrt{10}} = \frac{10}{-2\sqrt{10}} = \frac{5}{-\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}$

$-\frac{5\sqrt{10}}{10} = \frac{-\sqrt{10}}{2}$

$\cos^2 n + \sin^2 n = 1$

$\frac{9}{49} + \sin^2 n = 1 \quad \sqrt{\sin^2 n} = \sqrt{\frac{40}{49}} = -\frac{2\sqrt{10}}{7}$

4. Simplify each expression using the **half-angle** identities.

a. $\pm\sqrt{\frac{1-\cos 8x}{2}}$

$\sin 4x$

b. $\pm\sqrt{\frac{1-\cos 9B}{1+\cos 9B}}$

$\tan\frac{9B}{2}$

5. Verify that the equation is an identity: $\tan^2\left(\frac{x}{2}\right) = \frac{\sec x + \cos x - 2}{\sec x - \cos x}$.

1. $\dfrac{\frac{1}{\cos x} + \cos x - 2}{\frac{1}{\cos x} - \cos x}\cdot\dfrac{\cos x}{\cos x}$	1. Reciprical Id
2. $\dfrac{1 + \cos^2 x - 2\cos x}{1 - \cos^2 x}$	2. multiply by 1 $\left(\frac{\cos x}{\cos x}\right)$
3. $\dfrac{(\cos x - 1)(\cos x - 1)}{(1 - \cos x)(1 + \cos x)}$	3. factor
4. $\dfrac{1 - \cos x}{1 + \cos x}$	4. Simplify
5. $\boxed{\tan^2\frac{x}{2} = \tan^2\frac{x}{2}}$	5. Half Angle Identity

$x^2 - 2x + 1$

$(x-1)(x-1)$

Dots

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6. Write cos (3x) in terms of cos x.

$\cos(2x + x) = \boxed{\cos 2x}\cos x - \boxed{\sin 2x}\sin x$

$(2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$

$2\cos^3 x - \cos x - 2\sin^2 x\cos x$

$2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$\boxed{2\cos^3 x}\ \boxed{-\cos x}\ \boxed{-2\cos x} + \boxed{2\cos^3 x}$

$\boxed{4\cos^3 x - 3\cos x}$

7. Given $\sin\theta = \frac{8}{17}$, and $\cos\theta < 0$ (*is negative!*), find sin **2θ**, cos **2θ**, and tan **2θ.**

sin 2θ = -240/289

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{8}{17}\right)\left(\frac{-15}{17}\right)$

$\left(\frac{8}{17}\right)^2 + \cos^2 = 1$
$\cos\theta = \frac{-15}{17}$

cos 2θ = 161/289

$\cos 2\theta = 1 - 2\sin^2\theta$
$1 - 2\left(\frac{8}{17}\right)^2$

$\frac{\sin\theta}{\cos\theta} = \frac{-8}{15}$

$\tan 2\theta = \frac{2\tan x}{1 - \tan^2 x}$ $\quad -\frac{8}{15}$ $\quad -\frac{16}{15}$ $\quad \frac{161}{225}$

$\left(-\frac{8}{15}\right)^2 \quad \frac{64}{225} \quad 225 - 64 = 161$

$\frac{-16}{15} \cdot \frac{15}{225}$

tan 2θ = -240/161

8. Find the value of the 6 trig functions of **θ** if cos **2θ** = $\frac{-12}{13}$ and 180° < θ < 270°.

Hint: use one of the cos 2x identities first!

$\cos\theta = \frac{-\sqrt{26}}{26}$ $\quad \sec\theta = -\sqrt{26}$

$\sin\theta = \frac{-5\sqrt{26}}{26}$ $\quad \csc\theta = \frac{-\sqrt{26}}{5}$

$\tan\theta = 5$ $\quad \cot\theta = \frac{1}{5}$

$2\cos^2\theta - 1 = \frac{-12}{13}$

$1 - 2\sin^2\theta = \frac{-12}{13} - \frac{13}{13}$
-1
$\frac{1}{-2} \cdot -2\sin^2\theta = -\frac{25}{13} \cdot \frac{1}{-2}$

$\sqrt{\sin^2\theta} = \sqrt{\frac{25}{26}} = -\frac{5\sqrt{26}}{26}$

$\tan\theta \quad \frac{\frac{-5\sqrt{26}}{26}}{\frac{-\sqrt{26}}{26}} = 5$