Name ______________________________ Date ____________

Small Investment, Big Reward
Exponential Functions

Vocabulary

Define each term in your own words.

1. exponential function

   A geometric sequence written in function notation. It gets its name because the variable is in the exponent.

2. half-life

   The amount of time it takes a substance to decay to half of its original amount.

Problem Set

Write the explicit formula for each geometric sequence. Then, use the equation to determine the 10th term. Round answers to the nearest thousandth, if necessary.

1.

1	2	3	4	5	6	10
5	15	45	135	405	1,215	98,415

$a_n = 5 \cdot 3^{n-1}$

$a_{10} = 5 \cdot 3^{10-1}$

$= 5 \cdot 3^9$

$= 5 \cdot 19{,}683$

$= 98{,}415$

2.

1	2	3	4	5	6	10
200	100	50	25	12.6	6.25	0.391

$a_n = 200 \cdot (0.5)^{n-1}$

$a_{10} = 200(0.5)^{10-1}$

$= 200(0.5)^9$

$\approx 200(0.001953)$

≈ 0.391

12

3.

1	2	3	4	5	6	10
1	1.25	1.563	1.953	2.441	3.052	7.451

$a_n = 1 \cdot 1.25^{n-1}$

$a_{10} = 1 \cdot 1.25^{10-1}$

$= 1 \cdot 1.25^9$

≈ 7.451

4.

1	2	3	4	5	6	10
1	0.8	0.64	0.512	0.410	0.328	0.134

$a_n = 1 \cdot 0.8^{n-1}$

$a_{10} = 1 \cdot 0.8^{10-1}$

$= 1 \cdot 0.8^9$

≈ 0.134

5.

1	2	3	4	5	6	10
0.4	0.8	1.6	3.2	6.4	12.8	204.8

$a_n = 0.4 \cdot 2^{n-1}$

$a_{10} = 0.4 \cdot 2^{10-1}$

$= 0.4 \cdot 2^9$

$= 0.4 \cdot 512$

$= 204.8$

12

6.

1	2	3	4	5	6	10
27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{729}$

$a_n = 27 \cdot \left(\frac{1}{3}\right)^{n-1}$

$a_{10} = 27\left(\frac{1}{3}\right)^{10-1}$

$= 27\left(\frac{1}{3}\right)^9$

$\approx 27(0.00005)$

≈ 0.0014

$= \frac{1}{729}$

Name ____ Date ____________

Write an exponential function to represent each geometric sequence. Evaluate the function for the given value of n. Round to the nearest thousandth, if necessary.

7. $a_n = 4 \cdot 2.5^{n-1}$

$n = 10$

$f(n) = 4 \cdot 2.5^{n-1}$

$= 4 \cdot 2.5^n \cdot \left(\frac{5}{2}\right)^{-1}$

$= 4 \cdot 2.5^n \cdot \frac{2}{5}$

$= 1.6 \cdot 2.5^n$

$f(10) = 1.6 \cdot 2.5^{10}$

$\approx 1.6 \cdot 9536.743$

$\approx 15{,}258.789$

8. $a_n = 0.3 \cdot 8^{n-1}$

$n = 3$

$f(n) = 0.3 \cdot 8^{n-1}$

$= 0.3 \cdot 8^n \cdot 8^{-1}$

$= 0.3 \cdot 8^n \cdot \frac{1}{8}$

$= 0.0375 \cdot 8^n$

$f(3) = 0.0375 \cdot 8^3$

$= 0.0375 \cdot 512$

$= 19.2$

9. $a_n = 150 \cdot 0.8^{n-1}$

$n = 2$

$f(n) = 150 \cdot 0.8^{n-1}$

$= 150 \cdot 0.8^n \cdot 0.8^{-1}$

$= 150 \cdot 0.8^n \cdot \frac{10}{8}$

$= 187.5 \cdot 0.8^n$

$f(2) = 187.5 \cdot 0.8^2$

$= 187.5 \cdot 0.64$

$= 120$

10. $a_n = 0.05 \cdot 1.25^{n-1}$

$n = 24$

$f(n) = 0.5 \cdot 1.25^{n-1}$

$= 0.5 \cdot 1.25^n \cdot 1.25^{-1}$

$= 0.5 \cdot 1.25^n \cdot \frac{100}{125}$

$= 0.4 \cdot 1.25^n$

$f(24) = 0.4 \cdot 1.25^{24}$

$\approx 0.4 \cdot 211.758$

≈ 84.703

11. $a_n = 10 \cdot 4^{n-1}$

$n = 7$

$f(n) = 10 \cdot 4^{n-1}$

$= 10 \cdot 4^n \cdot 4^{-1}$

$= 10 \cdot 4^n \cdot \frac{1}{4}$

$= 2.5 \cdot 4^n$

$f(7) = 2.5 \cdot 4^7$

$= 2.5 \cdot 16{,}384$

$= 40{,}960$

12. $a_n = 1{,}000 \cdot 0.5^{n-1}$

$n = 5$

$f(n) = 100 \cdot 0.5^{n-1}$

$= 100 \cdot 0.5^n \cdot 0.5^{-1}$

$= 100 \cdot 0.5^n \cdot 2$

$= 200 \cdot 0.5^n$

$f(5) = 200 \cdot 0.5^5$

$= 200 \cdot 0.03125$

$= 6.25$

Write an exponential function $A(t)$, where t represents elapsed time, to represent each half-life situation. Then, use the function to complete each table. Round as necessary.

13.

Elapsed Time (hours)	0	2	4	6	8	20
Drug in Bloodstream (mg)	120	60	30	15	7.5	0.1172
Number of Half-Life Cycles	0	1	2	3	4	10

$A(t) = 120\left(\frac{1}{2}\right)^{\frac{t}{2}}$

$A(20) = 120\left(\frac{1}{2}\right)^{\frac{20}{2}}$

$= 120\left(\frac{1}{2}\right)^{10}$

$\approx 120(0.00098)$

≈ 0.1172

14.

Elapsed Time (minutes)	0	5	10	15	20	100
Bacteria Subject to Growth Inhibitor	800	400	200	100	50	0.000763
Number of Half-Life Cycles	0	1	2	3	4	20

$A(t) = 800\left(\frac{1}{2}\right)^{\frac{t}{5}}$

$A(100) = 800\left(\frac{1}{2}\right)^{\frac{100}{5}}$

$= 800\left(\frac{1}{2}\right)^{20}$

$\approx 800(0.00000095)$

≈ 0.000763

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15.

Elapsed Time (years)	0	14	21	28	42	56
Strontium in Rock Sample (grams)	16	8	5.657	4	2	1
Number of Half-Life Cycles	0	1	1.5	2	3	4

$A(t) = 16\left(\frac{1}{2}\right)^{\frac{t}{14}}$

$A(21) = 16\left(\frac{1}{2}\right)^{\frac{21}{14}}$

$= 16\left(\frac{1}{2}\right)^{\frac{3}{2}}$

$\approx 16(0.354)$

≈ 5.657

16.

Elapsed Time (years)	0	5,700	11,400	15,675	17,100	22,800
C-14 in Rock Sample (grams)	1	0.5	0.25	0.1487	0.125	0.0625
Number of Half-Life Cycles	0	1	2	2.75	3	4

$A(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{5,700}}$

$A(15,675) = 1\left(\frac{1}{2}\right)^{\frac{15,675}{5,700}}$

$= 1\left(\frac{1}{2}\right)^{\frac{11}{4}}$

$\approx 1(0.1487)$

≈ 0.1487

12

17.

Elapsed Time (Days)	0	6	12	18	24	42
Rat Population Exposed to Virus	5000	2500	1250	625	313	39
Number of Half-Life Cycles	0	1	2	3	4	7

$A(t) = 5{,}000\left(\frac{1}{2}\right)^{\frac{t}{6}}$

$A(42) = 5{,}000\left(\frac{1}{2}\right)^{\frac{42}{6}}$

$= 5{,}000\left(\frac{1}{2}\right)^{7}$

$\approx 5{,}000(0.0078)$

≈ 39.063

18.

Elapsed Time (Hours)	0	2	4	6	8	16
Participants in Tennis Tournament	256	128	64	32	16	1
Number of Half-Life Cycles	0	1	2	3	4	8

12

$A(t) = 256\left(\frac{1}{2}\right)^{\frac{t}{2}}$

$A(16) = 256\left(\frac{1}{2}\right)^{\frac{16}{2}}$

$= 256\left(\frac{1}{2}\right)^{8}$

$\approx 256(0.0039)$

$= 1$

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We Have Liftoff!
Properties of Exponential Graphs

Vocabulary

Explain how the natural base e is similar to and different from π.

Both are symbols that represent irrational numbers and are constants that are used to simplify calculations. The natural base e represents continuous growth and is used to model population changes, radioactive decay of a substance, and other physics and calculus applications. It is approximately equal to 2.71828. The number π represents the ratio of a circle's circumference to its diameter and is used in many geometry formulas for area and volume of geometric shapes. It is approximately equal to 3.14159.

Problem Set

Identify each function as exponential "growth" or "decay." Explain your reasoning.

1. $f(x) = 8^x$

The function represents exponential growth because the base is greater than 1.

2. $f(x) = 0.2^x$

The function represents exponential decay because the base is between 0 and 1.

3. $f(x) = \left(\frac{5}{2}\right)^x$

The function represents exponential growth because the base is greater than 1.

4. $f(x) = 25^x$

The function represents exponential growth because the base is greater than 1.

5. $f(x) = \left(\frac{1}{6}\right)^x$

The function represents exponential decay because the base is between 0 and 1.

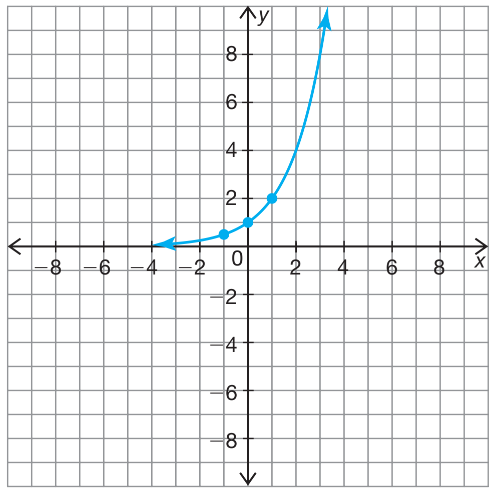
6. $f(x) = 7.5^x$

The function represents exponential growth because the base is greater than 1.

Complete each table and graph the exponential function.

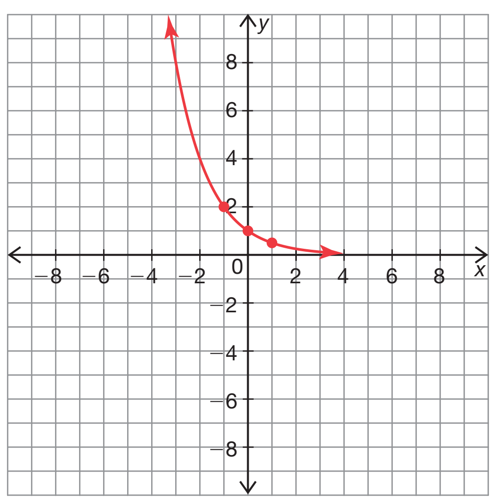
7. $f(x) = 2^x$

x	$f(x)$
−1	$\frac{1}{2}$
0	1
1	2



8. $f(x) = \left(\frac{1}{2}\right)^x$

x	$f(x)$
−1	2
0	1
1	$\frac{1}{2}$



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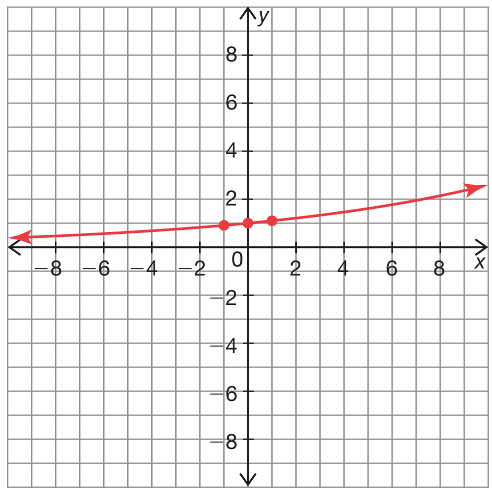
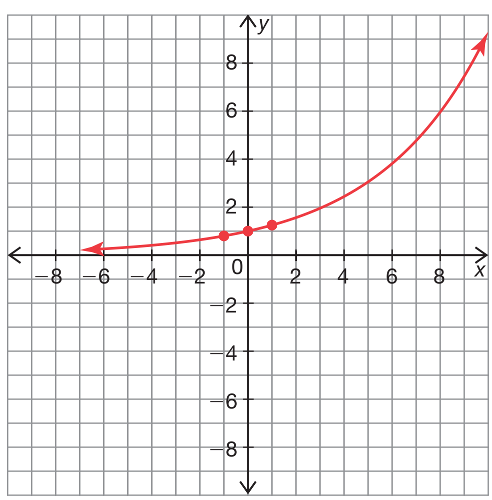
9. $f(x) = 1.1^x$

x	$f(x)$
-1	$\frac{10}{11}$
0	1
1	$\frac{11}{10}$



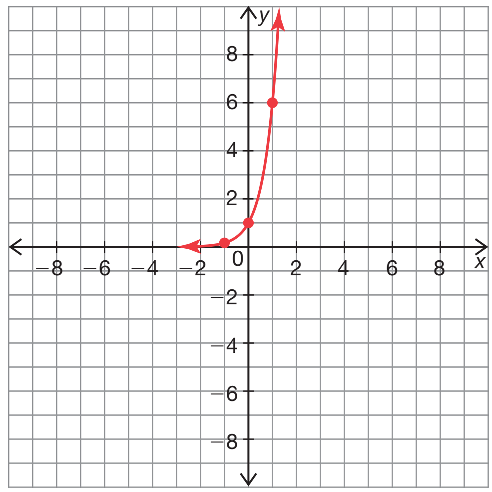
10. $f(x) = \left(\frac{5}{4}\right)^x$

x	$f(x)$
-1	$\frac{4}{5}$
0	1
1	$\frac{5}{4}$



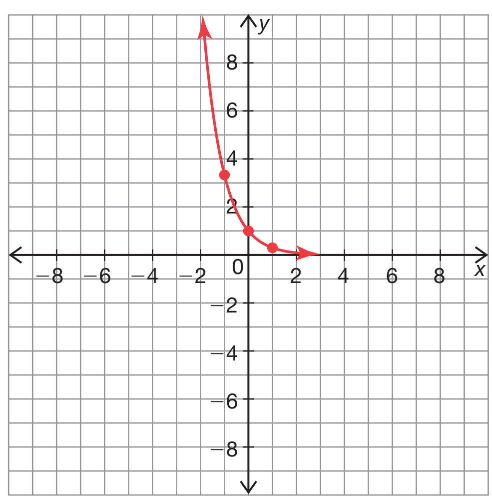
11. $f(x) = 6^x$

x	$f(x)$
−1	$\frac{1}{6}$
0	1
1	6



12. $f(x) = 0.3^x$

x	$f(x)$
−1	$\frac{10}{3}$
0	1
1	$\frac{3}{10}$



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Write an exponential function with the given characteristics.

13. increasing over $(-\infty, \infty)$

reference point $\left(-1, \frac{1}{9}\right)$

Answers will vary.

$f(x) = 9^x$

14. decreasing over $(-\infty, \infty)$

reference point $\left(1, \frac{2}{3}\right)$

Answers will vary.

$f(x) = \left(\frac{2}{3}\right)^x$

15. end behavior: as $x \to -\infty$, $f(x) \to 0$; as $x \to \infty$, $f(x) \to \infty$

reference point (2, 2.25)

Answers will vary.

$f(x) = 1.5^x$

16. decreasing over $(-\infty, \infty)$

reference point $(-2, 16)$

Answers will vary.

$f(x) = \left(\frac{1}{4}\right)^x$

17. increasing over $(-\infty, \infty)$

reference point $\left(-3, \frac{1}{8}\right)$

Answers will vary.

$f(x) = 2^x$

18. end behavior: as $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to 0$

reference point $\left(-4, \frac{81}{16}\right)$

Answers will vary.

$f(x) = \left(\frac{2}{3}\right)^x$

12

Use the formula for compound interest to determine the amount of money in each account after interest is accrued.

19. An investor deposits $1,000 in an account that promises 5% interest calculated at the end of each year. How much will be in the account after seven years?

There will be $1,407.10 in the account after seven years.

$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

$A(7) = 1{,}000\left(1 + \frac{0.05}{1}\right)^{1 \cdot 7}$

$= 1{,}000(1.05)^7$

$\approx 1{,}407.10$

20. At the start of the school year, Fairview High School deposits PTA dues in an account that offers 3.5% compound interest at the end of a year. If $2500 is collected in PTA dues, how much money will the school have at the start of the next school year?

The school will have $2,587.50 at the start of the next school year.

$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

$A(1) = 2{,}500\left(1 + \frac{0.035}{1}\right)^{1 \cdot 1}$

$= 2{,}500(1.035)^1$

$= 2{,}587.50$

12

21. Kyle put $300 of his birthday money in the bank. The bank compounds interest twice a year at 4%. How much money will Kyle have after three years?

Kyle will have $337.85 after three years.

$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

$A(3) = 300\left(1 + \frac{0.04}{2}\right)^{2 \cdot 3}$

$= 300(1.02)^6$

$\approx 300(1.126)$

≈ 337.85

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22. An investing group has \$50,000 to invest. They put the money in an account that compounds interest monthly at a rate 6%. How much money will the group have at the end of 10 years?

The group will have \$90,969.84 at the end of 10 years.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$\begin{aligned} A(10) &= 50{,}000\left(1 + \frac{0.06}{12}\right)^{12 \cdot 10} \\ &= 50{,}000(1.005)^{120} \\ &\approx 50{,}000(1.82) \\ &\approx 90{,}969.84 \end{aligned}$$

23. Interest is compounded quarterly at Money Bank at a rate of 5.5%. A new client opens an account with \$7200. How much money will be in the account at the end of six years?

There will be \$9,992.48 in the account after six years.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$\begin{aligned} A(6) &= 7{,}200\left(1 + \frac{0.055}{4}\right)^{4 \cdot 6} \\ &= 7{,}200(1.01375)^{24} \\ &\approx 7{,}200(1.3878) \\ &\approx 9{,}992.48 \end{aligned}$$

24. Sasha wants to earn the maximum interest on her money. She decides to deposit \$50 in two different banks for 90 days (3 months) to compare them before she deposits all of her money. She finds a bank that compounds interest daily at 2.2% and another bank that compounds interest monthly at 4.8%. Which bank will earn her more money?

She earns \$50.28 from the first bank and \$50.60 from the second bank. The bank that compounds interest monthly with a 4.8% interest rate will earn Sasha more money.

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$\begin{aligned} A(0.25) &= 50\left(1 + \frac{0.022}{365}\right)^{0.25 \cdot 365} \\ &\approx 50(1.00006)^{90} \\ &\approx 50(1.0055) \\ &\approx 50.28 \end{aligned}$$

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

$$\begin{aligned} A(0.25) &= 50\left(1 + \frac{0.048}{12}\right)^{0.25 \cdot 12} \\ &= 50(1.004)^{3} \\ &\approx 50(1.012) \\ &\approx 50.60 \end{aligned}$$

Use the formula for population growth to predict the population of each city.

25. The population of Austin, Texas is growing 3.9% per year. If the population in 2010 was approximately 790,000, what is the predicted population for 2015?

The population of Austin, Texas will be about 960,096 in 2015.

$N(t) = N_o e^{rt}$

$N(5) = 790{,}000e^{(0.039 \cdot 5)}$

$= 790{,}000e^{0.195}$

$\approx 960{,}096$

26. The population of Boston, Massachusetts is growing at a rate of 1.8%. The population in 2013 was approximately 636,500. What is the predicted population for 2025?

The predicted population of Boston, Massachusetts for 2025 is 789,962.

$N(t) = N_o e^{rt}$

$N(12) = 636{,}500e^{(0.018 \cdot 12)}$

$= 636{,}500e^{0.216}$

$\approx 789{,}962$

12

27. The population of Charlotte, North Carolina in 2013 was approximately 775,000. If the rate of growth is about 3.2%, what is an approximation of Charlotte's population in 2000?

In 2000, the population of Charlotte, North Carolina was approximately 511,252 people.

$N(t) = N_o e^{rt}$

$N(-13) = 775{,}000e^{(0.032 \cdot -13)}$

$= 775{,}000e^{-0.416}$

$\approx 511{,}252$

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28. The population of Beijing, China in 2012 was approximately 20,690,000 and is growing at a rate of about 5.5%. What is an approximation of Beijing's population in 1980?

In 1980, the population of Beijing, China was approximately 3,559,608.

$$N(t) = N_o e^{rt}$$

$$N(-32) = 20{,}690{,}000e^{(0.055 \cdot -32)}$$

$$= 20{,}690{,}000e^{-1.76}$$

$$\approx 3{,}559{,}608$$

29. The population of Detroit, Michigan is decreasing at a rate of about 0.75%. Detroit's population in 2013 was approximately 700,000. What is the predicted population for 2015?

The predicted population for Detroit, Michigan in 2015 is 689,578.

$$N(t) = N_o e^{rt}$$

$$N(2) = 700{,}000e^{(-0.0075 \cdot 2)}$$

$$= 700{,}000e^{-0.015}$$

$$\approx 689{,}578$$

12

30. The population of Berlin, Germany was about 3,290,000 in 2011. Its population is declining at a rate of about 0.2%. What is the predicted population for 2050?

In 2050, the population for Berlin, Germany will be about 3,043,133 people.

$$N(t) = N_o e^{rt}$$

$$N(39) = 3{,}290{,}000e^{(-0.002 \cdot 39)}$$

$$= 3{,}290{,}000e^{-0.078}$$

$$\approx 3{,}043{,}133$$

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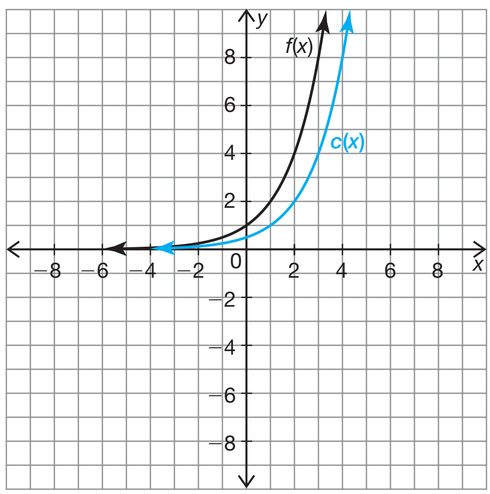
I Like to Move It
Transformations of Exponential Functions

Problem Set

Complete the table to determine the corresponding points on $c(x)$, given reference points on $f(x)$. Then, graph $c(x)$ on the same coordinate plane as $f(x)$ and state the domain, range, and asymptotes of $c(x)$.

1. $f(x) = 2^x$
$c(x) = f(x - 1)$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	$\left(0, \frac{1}{2}\right)$
(0, 1)	(1, 1)
(1, 2)	(2, 2)



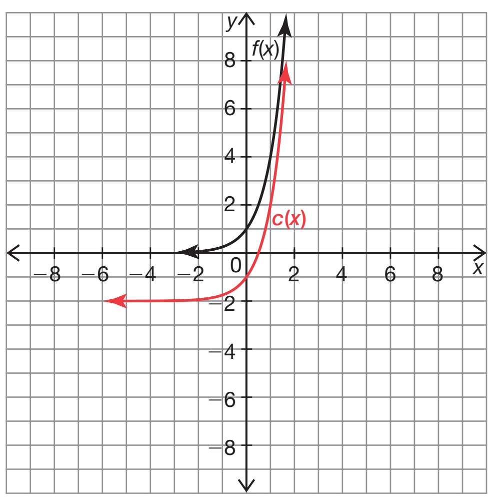
Domain: All real numbers

Range: $y > 0$

Horizontal asymptote: $y = 0$

2. $f(x) = 4^x$
$c(x) = f(x) - 2$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{4}\right)$	$\left(-1, -\frac{7}{4}\right)$
(0, 1)	(0, −1)
(1, 4)	(1, 2)



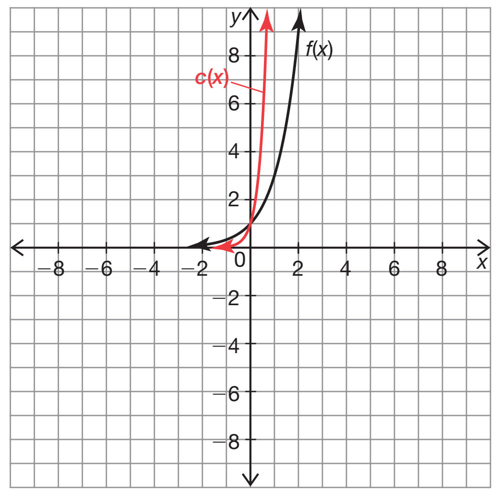
Domain: All real numbers

Range: $y > -2$

Horizontal asymptote: $y = -2$

3. $f(x) = 3^x$
$c(x) = f(3x)$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{3}\right)$	$\left(-\frac{1}{3}, \frac{1}{3}\right)$
(0, 1)	(0, 1)
(1, 3)	$\left(\frac{1}{3}, 3\right)$



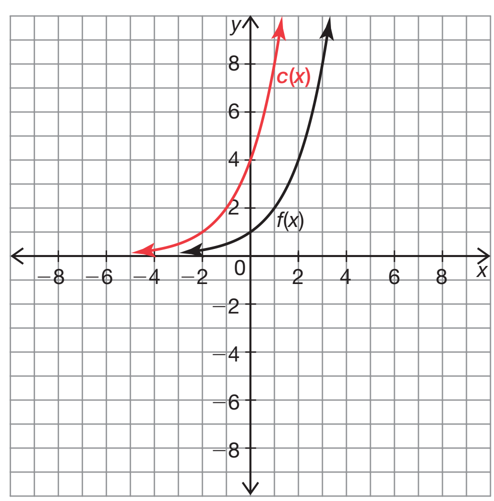
Domain: All real numbers

Range: $y > 0$

Horizontal asymptote: $y = 0$

4. $f(x) = 2^x$
$c(x) = 4f(x)$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{2}\right)$	(−1, 2)
(0, 1)	(0, 4)
(1, 2)	(1, 8)



Domain: All real numbers

Range: $y > 0$

Horizontal asymptote: $y = 0$

12

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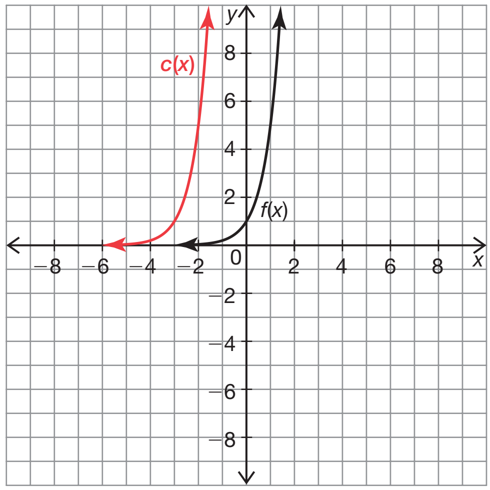
5. $f(x) = 5^x$
$c(x) = f(x + 3)$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{5}\right)$	$\left(-4, \frac{1}{5}\right)$
(0, 1)	(−3, 1)
(1, 5)	(−2, 5)



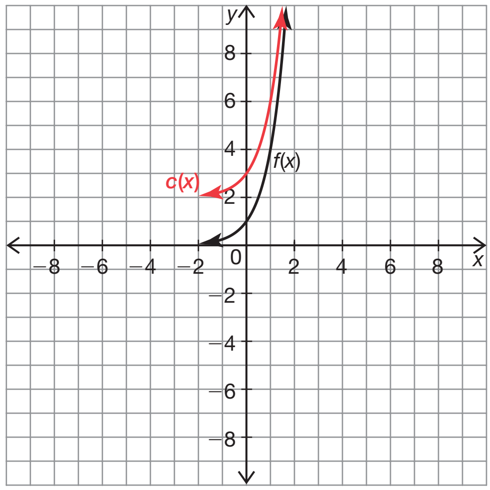
Domain: All real numbers

Range: $y > 0$

Horizontal asymptote: $y = 0$

6. $f(x) = 4^x$
$c(x) = f(x) + 2$

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{4}\right)$	$\left(-1, 2\frac{1}{4}\right)$
(0, 1)	(0, 3)
(1, 4)	(1, 6)



Domain: All real numbers

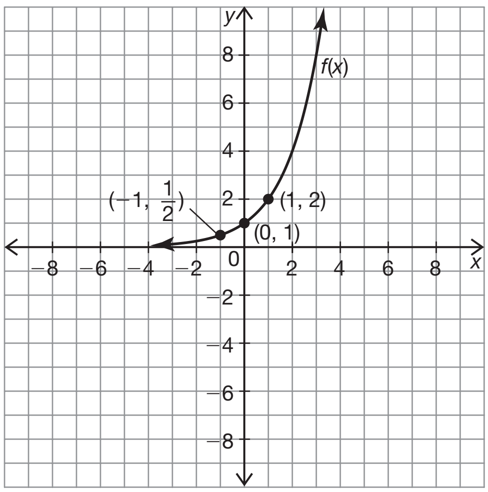
Range: $y > 2$

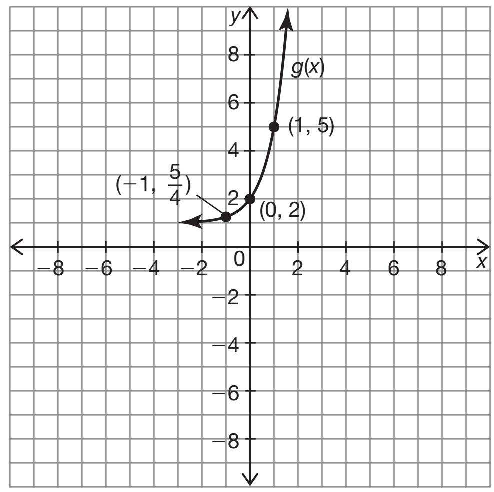
Horizontal asymptote: $y = 2$

12

Describe the transformations performed on $f(x)$ to create $g(x)$. Then, write an equation for $g(x)$ in terms of $f(x)$.

7.

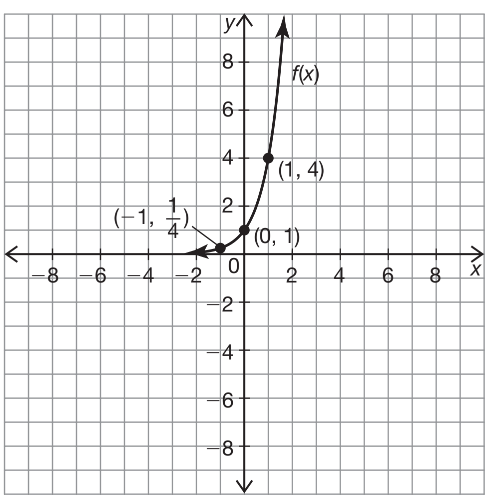


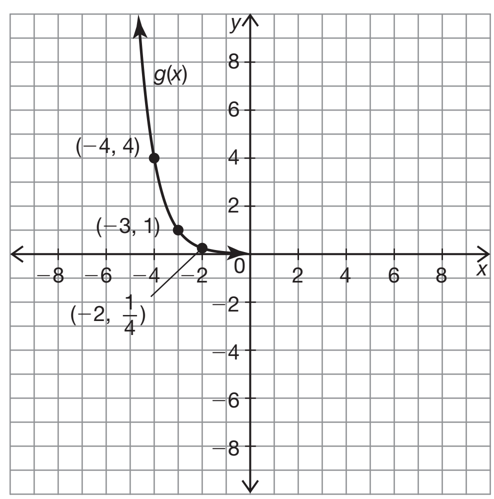


To create $g(x)$, the graph of $f(x)$ is compressed horizontally by a factor of $\frac{1}{2}$ and vertically translated up 1 unit.

$g(x) = f(2x) + 1$

8.





To create $g(x)$, the graph of $f(x)$ is reflected over the y-axis and horizontally translated right 3 units.

$g(x) = f(3 - x)$

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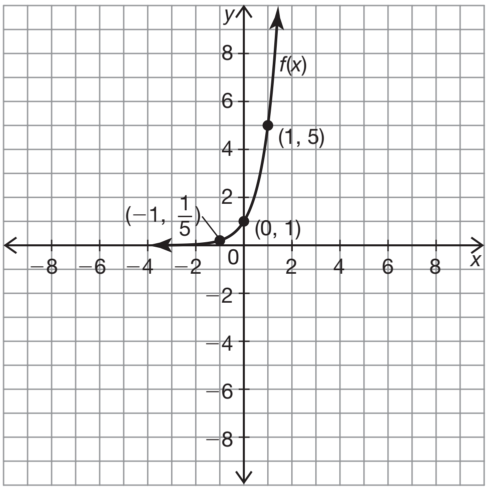
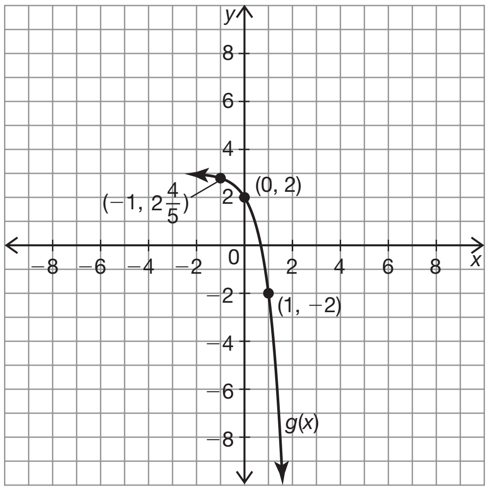
9.

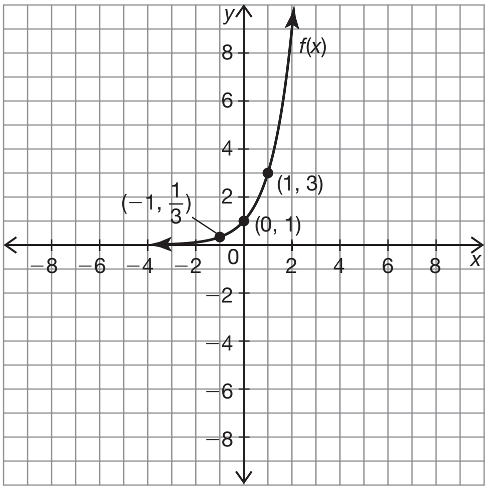


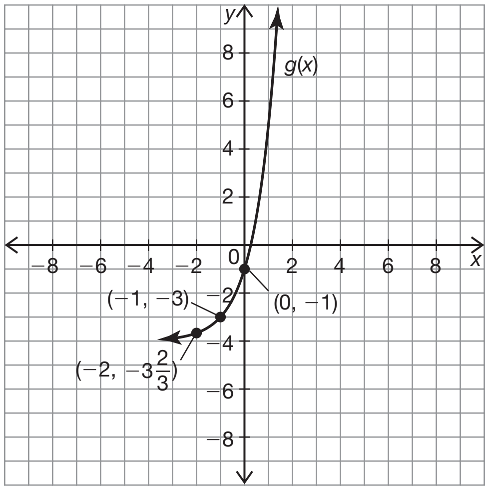


To create $g(x)$, the graph of $f(x)$ was reflected over the x-axis and vertically translated up 3 units.

$g(x) = -f(x) + 3$

10.



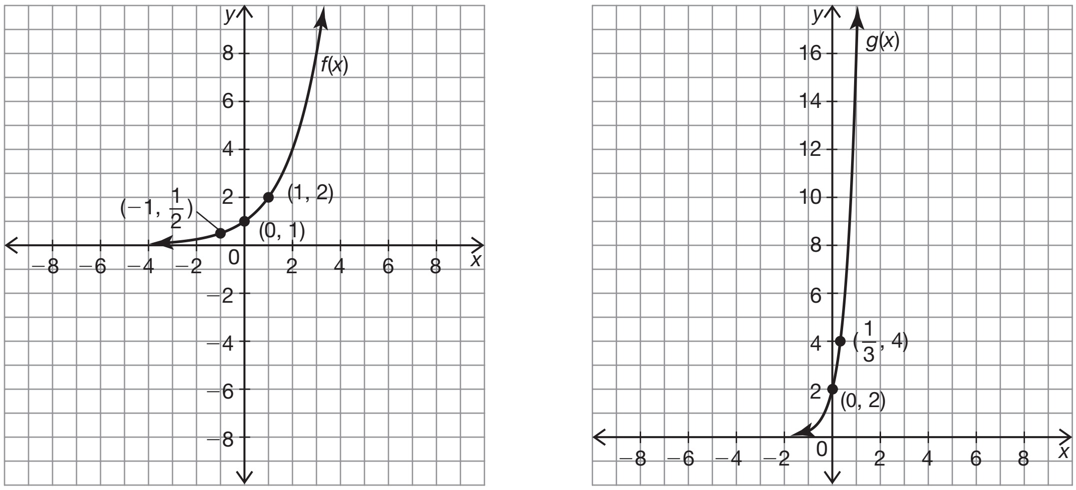


To create $g(x)$, the graph of $f(x)$ was horizontally translated left 1 unit and vertically translated down 4 units.

$g(x) = f(x + 1) - 4$

12

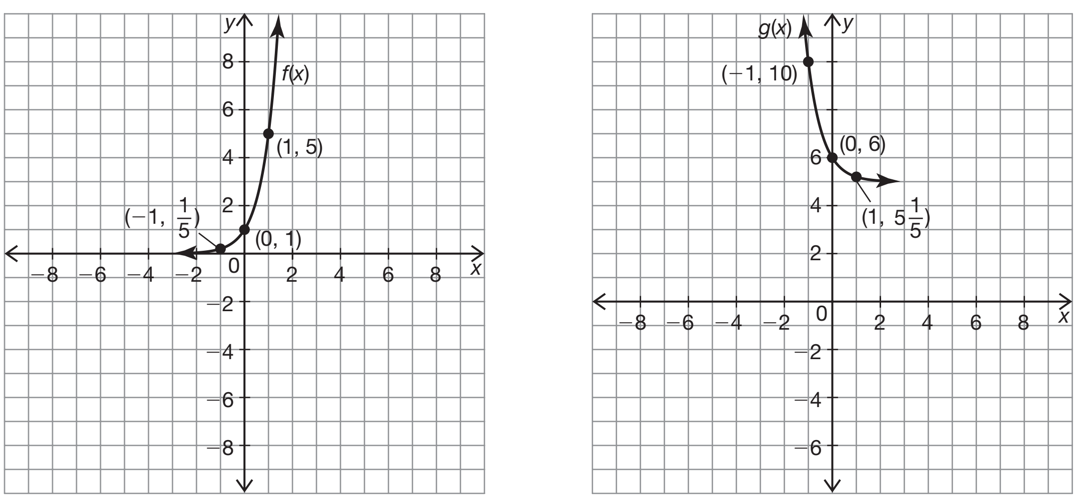
11.



To create $g(x)$, the graph of $f(x)$ was stretched vertically by a factor of 2 and compressed horizontally by a factor of $\frac{1}{3}$.

$g(x) = 2f(3x)$

12.



To create $g(x)$, the graph of $f(x)$ was reflected over the y-axis and vertically translated up 5 units.

$g(x) = f(-x) + 5$

12

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Describe the transformations performed on $m(x)$ that produced $t(x)$. Then, write an exponential equation for $t(x)$.

13. $m(x) = 3^x$

$t(x) = -m(x + 1)$

The graph of the function $m(x)$ is reflected over the x-axis and horizontally translated left 1 unit to produce $t(x)$.

$t(x) = -3^{x+1}$

14. $m(x) = 5^x$

$t(x) = 3m(x) - 2$

The graph of the function $m(x)$ is stretched vertically by a factor of 3 and vertically translated down 2 units to produce $t(x)$.

$t(x) = 3 \cdot 5^x - 2$

15. $m(x) = e^x$

$t(x) = \frac{1}{2}m(x) + 4$

The graph of the function $m(x)$ is compressed vertically by a factor of $\frac{1}{2}$ and vertically translated up 4 units to produce $t(x)$.

$t(x) = \frac{1}{2} \cdot e^x + 4$

16. $m(x) = 4^x$

$t(x) = m(3x - 1)$

The graph of the function $m(x)$ is compressed horizontally by a factor of $\frac{1}{3}$ and horizontally translated right $\frac{1}{3}$ units to produce $t(x)$.

$t(x) = 4^{3x-1}$

12

17. $m(x) = 7^x$
$t(x) = m(0.5x + 2)$

The graph of the function $m(x)$ is stretched horizontally by a factor of 2 and horizontally translated left 4 units to produce $t(x)$.

$t(x) = 7^{0.5x+2}$

18. $m(x) = 6^x$
$t(x) = -2m(-x) + 3$

The graph of the function $m(x)$ is reflected over the x-axis and reflected over the y-axis, stretched vertically by a factor of 2, and vertically translated up 3 units to produce $t(x)$.

$t(x) = -2 \cdot 6^{-x} + 3$

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I Feel the Earth Move

Logarithmic Functions

Vocabulary

Write the term that best completes each sentence.

logarithm	logarithmic function	common logarithm	natural logarithm

1. The **logarithm** of a number for a given base is the exponent to which the base must be raised in order to produce that number.

2. A **natural logarithm** is a logarithm with base *e*, and is usually written as *ln*.

3. A **logarithmic function** is a function involving a logarithm.

4. A **common logarithm** is a logarithm with a base 10 and is usually written without a base specified.

Problem Set

Write each exponential equation as a corresponding logarithmic equation.

1. $3^2 = 9$

   $\log_3 (9) = 2$

2. $5^4 = 625$

   $\log_5 (625) = 4$

3. $4^{-3} = \frac{1}{64}$

   $\log_4 \left(\frac{1}{64}\right) = -3$

4. $10^{-5} = \frac{1}{100{,}000}$

   $\log \left(\frac{1}{100{,}000}\right) = -5$

5. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

   $\log_{\frac{1}{2}} \left(\frac{1}{32}\right) = 5$

6. $\left(\frac{1}{11}\right)^{-2} = 121$

   $\log_{\frac{1}{11}} (121) = -2$

Write each logarithmic equation as a corresponding exponential equation.

7. $\log_7 \left(\frac{1}{49}\right) = -2$

$7^{22} = \frac{1}{49}$

8. $\log_{\frac{1}{3}} \left(\frac{1}{729}\right) = 6$

$\left(\frac{1}{3}\right)^6 = \frac{1}{729}$

9. $\log_2 (128) = 7$

$2^7 = 128$

10. $\log_6 \left(\frac{1}{1296}\right) = -4$

$6^{-4} = \frac{1}{1296}$

11. $\log_{\frac{1}{5}} \left(\frac{1}{125}\right) = 3$

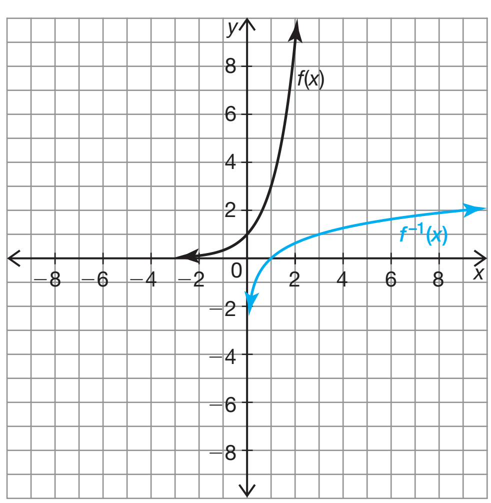
$\left(\frac{1}{5}\right)^3 = \frac{1}{125}$

12. $\log_9 (729) = 3$

$9^3 = 729$

Graph the inverse of each exponential function $f(x)$. Then, describe the domain, range, asymptotes, and end behavior of the inverse.

13. $f(x) = 3^x$



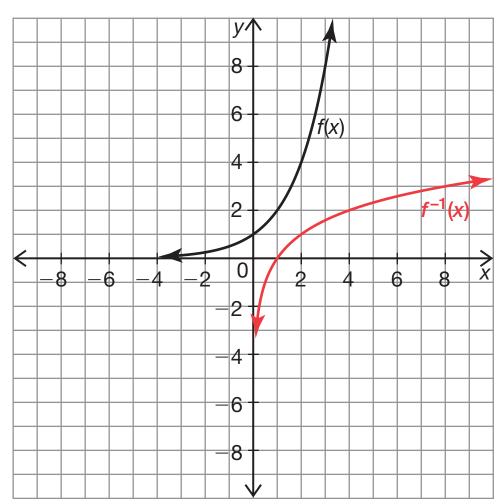
Domain: $x > 0$

Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

14. $f(x) = 2^x$



Domain: $x > 0$

Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

12

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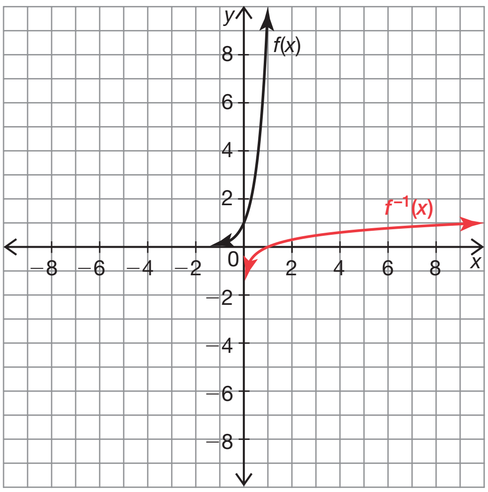
15. $f(x) = 10^x$



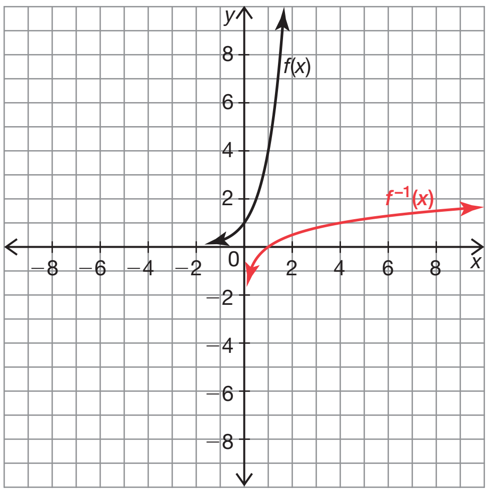
Domain: $x > 0$

Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

16. $f(x) = 4^x$



Domain: $x > 0$

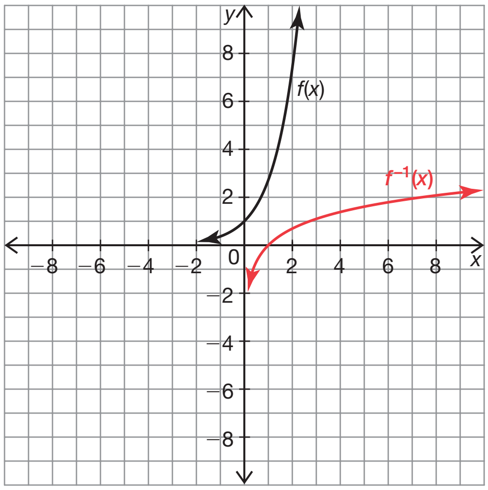
Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

12

17. $f(x) = e^x$



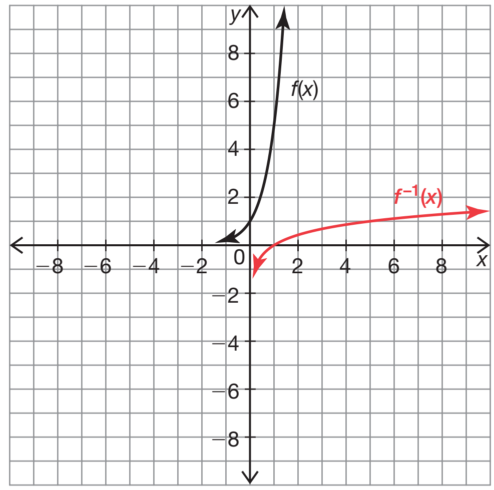
Domain: $x > 0$

Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

18. $f(x) = 5^x$



Domain: $x > 0$

Range: All real numbers

Asymptotes: $x = 0$

End behavior: As $x \to 0$, $y \to -\infty$.
As $x \to +\infty$, $y \to +\infty$.

Solve each logarithmic equation.

19. $-2 = \log_9 \left(\frac{1}{b}\right)$

$\frac{1}{b} = 9^{-2}$

$\frac{1}{b} = \frac{1}{81}$

$b = 81$

20. $-0.903 \approx x \cdot \log(0.5)$

$-0.903 \approx x \cdot -0.301$

$\frac{-0.903}{-0.301} = x$

$3 = x$

21. $\frac{1}{2} = \log_n(3)$

$n^{\left(\frac{1}{2}\right)} = 3$

$n = 3^2$

$n = 9$

22. $2.398 \approx \log b$

$10^{2.398} \approx b$

$250 \approx b$

23. $0.058 \approx \ln z$

$e^{0.058} \approx z$

$1.06 \approx z$

24. $-1.349 = \frac{1}{2} \log\left(\frac{g}{1000}\right)$

$2 \cdot (-1.349) = \log\left(\frac{g}{1000}\right)$

$-2.698 = \log\left(\frac{g}{1000}\right)$

$10^{-2.698} = \frac{g}{1000}$

$0.002 \approx \frac{g}{1000}$

$2 \approx g$

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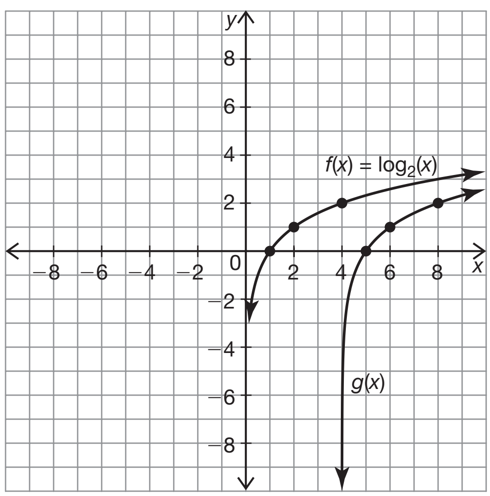
More Than Meets the Eye
Transformations of Logarithmic Functions

Problem Set

Analyze the graphs of $f(x)$ and $g(x)$. Describe the transformations performed on the graph of $f(x)$ to produce the graph of the transformed function $g(x)$. Then, write an equation for $g(x)$.

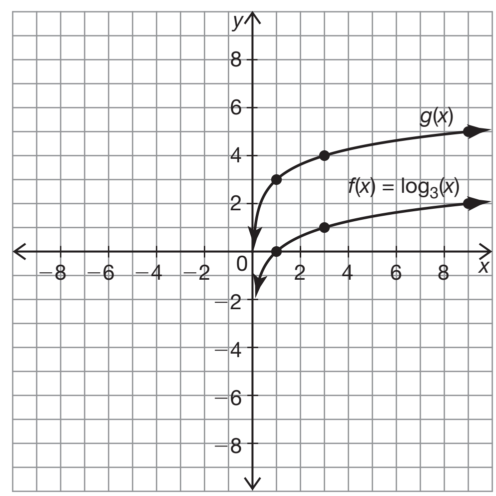
1.



The graph of $g(x)$ was horizontally translated right 4 units to produce the graph of $g(x)$.

$g(x) = \log_2 (x - 4)$

2.

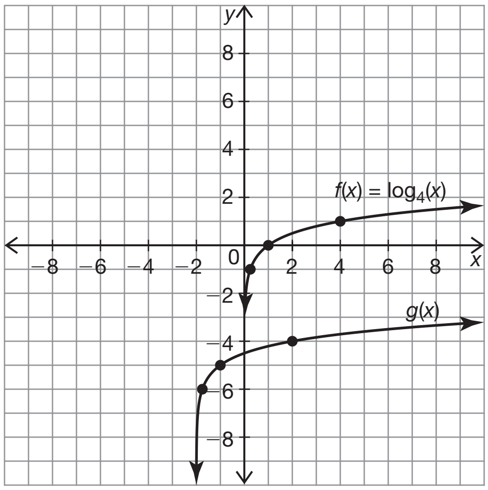


The graph of $g(x)$ was vertically translated up 3 units to produce the graph of $g(x)$.

$g(x) = \log_3 (x) + 3$

12

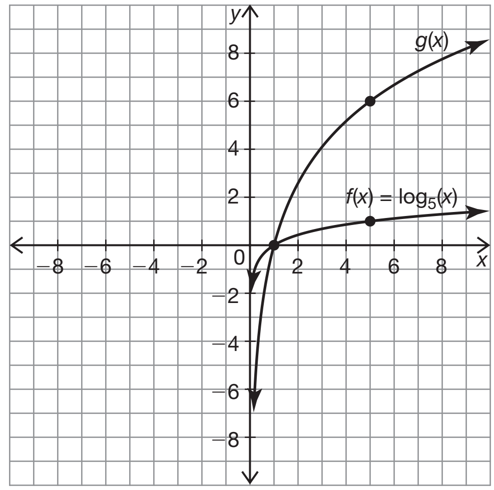
3.



The graph of $g(x)$ was horizontally translated left 2 units and vertically translated down 5 units to produce the graph of $g(x)$.

$g(x) = \log_4(x + 2) - 5$

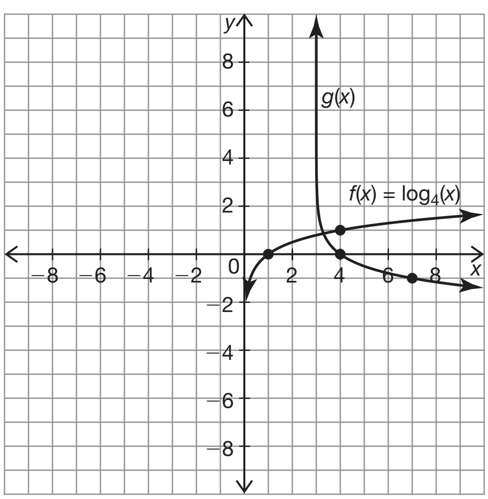
4.



The graph of $g(x)$ was stretched vertically by a factor of 6 to produce the graph of $g(x)$.

$g(x) = 6\log_5(x)$

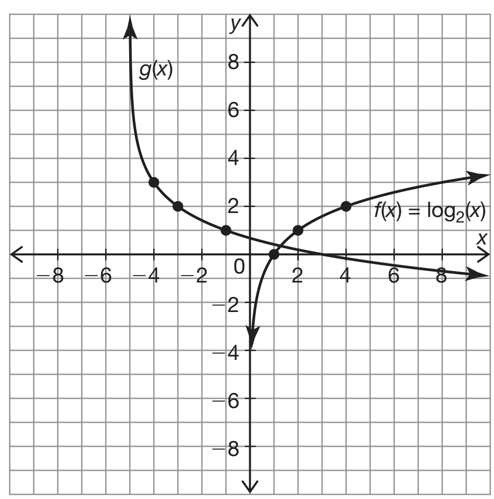
5.



The graph of $g(x)$ was reflected over the x-axis and horizontally translated right 3 units to produce the graph of $g(x)$.

$g(x) = -\log_4(x - 3)$

6.



The graph of $g(x)$ was reflected over the x-axis and horizontally translated up 3 units to produce the graph of $g(x)$.

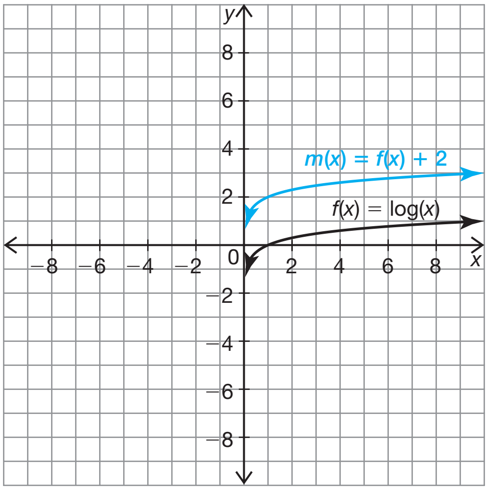
$g(x) = -\log_2(x + 5) + 3$

12

Name ____ Date ______________

The graph of $f(x) = \log(x)$ is shown. Use the graph of $f(x)$ to sketch the transformed function $m(x)$ on the coordinate plane. Then, state the domain, range and asymptotes of $m(x)$.

7. $m(x) = f(x) + 2.$

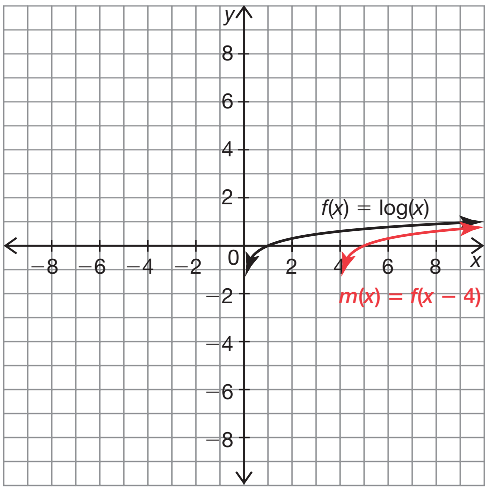


Domain of $m(x)$: $(0, \infty)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = 0$

8. $m(x) = f(x - 4).$



Domain of $m(x)$: $(4, \infty)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = 4$

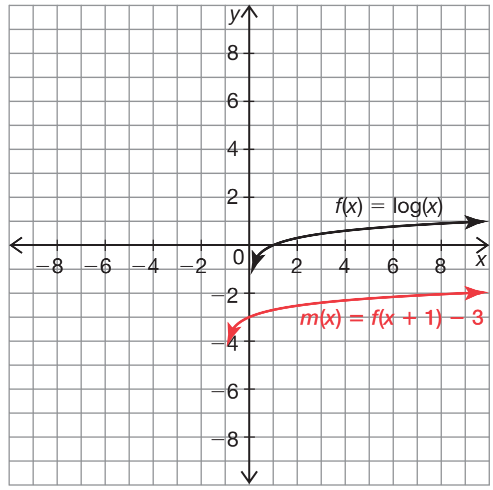
12

9. $m(x) = f(x + 1) - 3$.

Domain of $m(x)$: $(-1, \infty)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = -1$

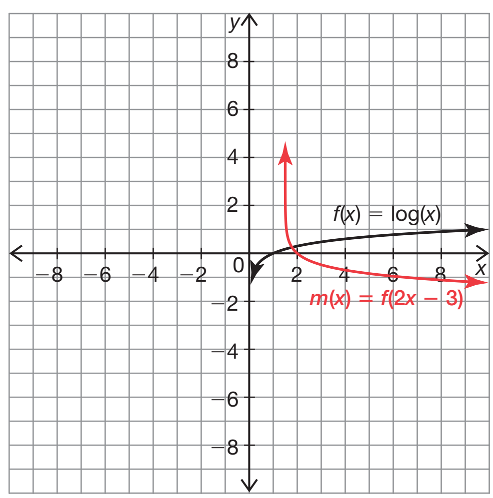


10. $m(x) = -f(2x - 3)$.

Domain of $m(x)$: $\left(\frac{3}{2}, \infty\right)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = \frac{3}{2}$



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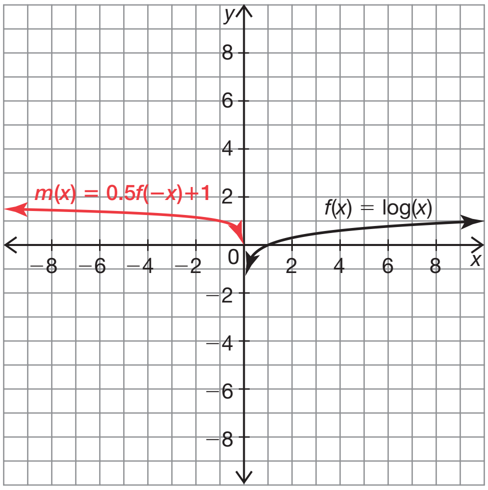
11. $m(x) = 0.5f(-x) + 1.$

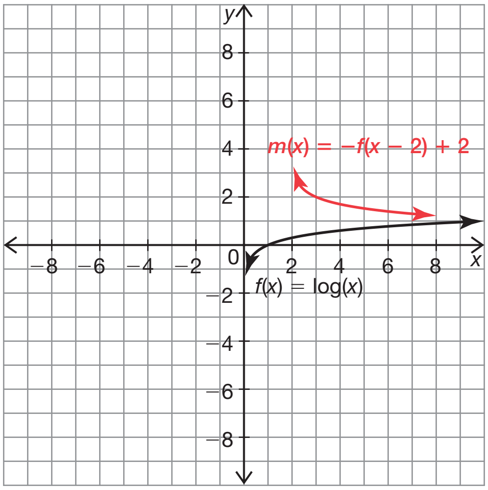


Domain of $m(x)$: $(-\infty, 0)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = 0$

12. $m(x) = -f(x - 2) + 2$



Domain of $m(x)$: $(2, \infty)$

Range of $m(x)$: $(-\infty, \infty)$

Asymptote of $m(x)$: $x = 2$

12

Write a transformed logarithmic function, $c(x)$, in terms of $f(x) = \log_2 (x)$ with the characteristics given.

13. vertical asymptote at $x = 6$

Answers will vary.

$c(x) = f(x - 6)$

14. domain of $(-\infty, 3)$

Answers will vary.

$c(x) = f(-x + 3)$

15.

Reference Points on $f(x)$	→	Corresponding Points on $c(x)$
$\frac{1}{2}, -1$	→	$\left(\frac{1}{2}, -3\right)$
(1, 0)	→	(1, 0)
(2, 1)	→	(2, 3)

$c(x) = 3f(x)$

16. vertical asymptote at $x = -2$

Answers will vary.

$c(x) = f(x + 2)$

12

17. domain: $(4, \infty)$

point: $(5, -4)$

Answers will vary.

$c(x) = f(x - 4) - 4$

18.

Reference Points on $f(x)$	→	Corresponding Points on $g(x)$
$\left(\frac{1}{2}, -1\right)$	→	$\left(\frac{1}{2}, 3\right)$
(1, 0)	→	(1, 2)
(2, 1)	→	(2, 1)

$c(x) = -f(x) + 2$

Name ____ Date ______________

Consider the function $y = f(x)$ and the transformed function $g(x)$. Write an equation for $g^{-1}(x)$ in terms of $f^{-1}(x)$.

19. $g(x) = f(x) + 3$

$g^{-1}(x) = f^{-1}(x - 3)$

20. $g(x) = f(x - 2)$

$g^{-1}(x) = f^{-1}(x) + 2$

21. $g(x) = f(x + 6)$

$g^{-1}(x) = f^{-1}(x) - 6$

22. $g(x) = f(x) - 5$

$g^{-1}(x) = f^{-1}(x + 5)$

23. $g(x) = f(x + 1) - 3$

$g^{-1}(x) = f^{-1}(x + 3) - 1$

24. $g(x) = f(x - 4) + 2$

$g^{-1}(x) = f^{-1}(x - 2) + 4$

Consider the function $f(x) = 4^x$ and its inverse function $f^{-1}(x) = \log_4(x)$. Complete a table for each transformation and write the transformation function in terms of $f^{-1}(x)$. Then, identify the transformation on $f(x)$ and the effect on the inverse.

25.

$h(x) = 2f(x)$	$h^{-1}(x) = f^{-1}\left(\frac{x}{2}\right)$
$\left(-1, \frac{1}{2}\right)$	$\left(\frac{1}{2}, -1\right)$
(0, 2)	(2, 0)
(1, 8)	(8, 1)

Transformation on $f(x)$: vertical dilation of 2

Effect on the inverse: horizontal dilation of 2

12

26.

$m(x) = \frac{1}{4}f(x)$	$m^{-1}(x) = f^{-1}(4x)$
$\left(-1, \frac{1}{16}\right)$	$\left(\frac{1}{16}, -1\right)$
$\left(0, \frac{1}{4}\right)$	$\left(\frac{1}{4}, 0\right)$
(1, 1)	(1, 1)

Transformation on $f(x)$: vertical dilation of $\frac{1}{4}$

Effect on the inverse: horizontal dilation of $\frac{1}{4}$

27.

$h(x) = f(8x)$	$h^{-1}(x) = \frac{1}{8}f^{-1}(x)$
$\left(-\frac{1}{8}, \frac{1}{4}\right)$	$\left(\frac{1}{4}, -\frac{1}{8}\right)$
(0, 1)	(1, 0)
$\left(\frac{1}{8}, 4\right)$	$\left(4, \frac{1}{8}\right)$

Transformation on $f(x)$: horizontal dilation of $\frac{1}{8}$

Effect on the inverse: vertical dilation of $\frac{1}{8}$

28.

$h(x) = f\left(\frac{x}{2}\right)$	$h^{-1}(x) = 2f^{-1}(x)$
$\left(-2, \frac{1}{4}\right)$	$\left(\frac{1}{4}, -2\right)$
(0, 1)	(1, 0)
(2, 4)	(4, 2)

Transformation on $f(x)$: horizontal dilation of 2

Effect on the inverse: vertical dilation of 2

12

Name ______________________________ Date ______________

29.

$h(x) = 12f(x)$	$h^{-1}(x) = f^{-1}\left(\frac{x}{12}\right)$
$(-1, 3)$	$(3, -1)$
$(0, 12)$	$(12, 0)$
$(1, 48)$	$(48, 1)$

Transformation on $f(x)$: vertical dilation of 12

Effect on the inverse: horizontal dilation of 12

30.

$h(x) = f(12x)$	$h^{-1}(x) = \frac{1}{12}f^{-1}(x)$
$\left(-\frac{1}{12}, \frac{1}{4}\right)$	$\left(\frac{1}{4}, -\frac{1}{12}\right)$
$(0, 1)$	$(1, 0)$
$\left(\frac{1}{12}, 4\right)$	$\left(4, \frac{1}{12}\right)$

Transformation on $f(x)$: horizontal dilation of 12

Effect on the inverse: vertical dilation of 12

Given each function $f(x)$, write an equation for the inverse function $f^{-1}(x)$.

31. $f(x) = 2^{3x}$

$f^{-1}(x) = \frac{1}{3}\log_2(x)$

32. $f(x) = 4 \cdot 3^x$

$f^{-1}(x) = \log_3\left(\frac{x}{4}\right)$

33. $f(x) = \log_5\left(\frac{x}{3}\right)$

$f^{-1}(x) = 3 \cdot 5^x$

34. $f(x) = \frac{1}{4}\log_4(x)$

$f^{-1}(x) = 4^{4x}$

35. $f(x) = \log(-x)$

$f^{-1}(x) = -10^x$

36. $f(x) = 2 \cdot 4^{5x}$

$f^{-1}(x) = \frac{1}{5}\log_4\left(\frac{x}{2}\right)$