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All the Pieces of the Puzzle

Exponential and Logarithmic Forms

Vocabulary

1. Give an example of a logarithmic equation.

   Answers will vary. Sample answer: $\log_2 256 = 8$

Problem Set

Arrange the given terms to create a true exponential equation and a true logarithmic equation.

1. 25, 5, 2

   Exponential equation: $5^2 = 25$

   Logarithmic equation: $\log_5 25 = 2$

2. 3, 11, 1331

   Exponential equation: $11^3 = 1331$

   Logarithmic equation: $\log_{11} 1331 = 3$

3. 2, 256, 8

   Exponential equation: $2^8 = 256$

   Logarithmic equation: $\log_2 256 = 8$

4. 1296, 4, 6

   Exponential equation: $6^4 = 1296$

   Logarithmic equation: $\log_6 1296 = 4$

5. $-3, \frac{1}{512}, 8$

   Exponential equation: $8^{-3} = \frac{1}{512}$

   Logarithmic equation: $\log_8 \left(\frac{1}{512}\right) = -3$

6. $\frac{1}{2}, 4, 16$

   Exponential equation: $16^{\frac{1}{2}} = 4$

   Logarithmic equation: $\log_{16} 4 = \frac{1}{2}$

7. $17, \frac{1}{289}, -2$

   Exponential equation: $17^{-2} = \frac{1}{289}$

   Logarithmic equation: $\log_{17} \left(\frac{1}{289}\right) = -2$

8. $\frac{1}{3}, 9, 729$

   Exponential equation: $729^{\frac{1}{3}} = 9$

   Logarithmic equation: $\log_{729} 9 = \frac{1}{3}$

Solve for the unknown.

9. $\log_7 343 = n$

$\log_7 343 = n$

$7^n = 343$

$7^n = 7^3$

$n = 3$

10. $\log_{\frac{1}{4}} 64 = n$

$\log_{\frac{1}{4}} 64 = n$

$\left(\frac{1}{4}\right)^n = 64$

$\left(\frac{1}{4}\right)^n = 4^3$

$\left(\frac{1}{4}\right)^n = \left(\frac{1}{4}\right)^{-3}$

$n = -3$

11 $\log_n 1024 = 5$

$\log_n 1024 = 5$

$n^5 = 1024$

$n^5 = 4^5$

$n = 4$

12. $\log_n \left(\frac{1}{625}\right) = -4$

$\log_n \left(\frac{1}{625}\right) = -4$

$n^{-4} = \frac{1}{625}$

$n^{-4} = \left(\frac{1}{5}\right)^4$

$n^{-4} = 5^{-4}$

$n = 5$

13. $\log_n (\sqrt[4]{8}) = \frac{3}{4}$

$\log_n (\sqrt[4]{8}) = \frac{3}{4}$

$n^{\frac{3}{4}} = \sqrt[4]{8}$

$n^{\frac{3}{4}} = 8^{\frac{1}{4}}$

$n^{\frac{3}{4}} = (2^3)^{\frac{1}{4}}$

$n^{\frac{3}{4}} = 2^{\frac{3}{4}}$

$n = 2$

14. $\log n = 6$

$\log n = 6$

$10^6 = n$

$1{,}000{,}000 = n$

15. $\log_4 16 = n$

$\log_4 16 = n$

$4^n = 16$

$4^n = 4^2$

$n = 2$

16. $\log_{81} \left(\frac{1}{9}\right) = n$

$\log_{81} \left(\frac{1}{9}\right) = n$

$81^n = \frac{1}{9}$

$(9^2)^n = 9^{-1}$

$9^{2n} = 9^{-1}$

$2n = -1$

$n = -\frac{1}{2}$

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Estimate the logarithm to the tenths place. Explain how you determined your answer.

17. $\log_2 48$

The logarithm $\log_2 48$ is approximately equal to 5.5.

$\log_2 32 < \log_2 48 < \log_2 64$

$5 < n < 6$

Because 48 is halfway between 32 and 64, the approximation should be halfway between 5 and 6.

18. $\log_3 2$

Answers will vary.

The logarithm $\log_3 2$ is approximately equal to 0.5.

$\log_3 1 < \log_3 2 < \log_3 3$

$0 < n < 1$

Because 2 is halfway between 1 and 3, the approximation should be halfway between 0 and 1.

19. $\log_8 \left(\frac{1}{495}\right)$

Answers will vary.

The logarithm $\log_8 \left(\frac{1}{495}\right)$ is approximately equal to −2.9.

$\log_8 \left(\frac{1}{512}\right) < \log_8 \left(\frac{1}{495}\right) < \log_8 \left(\frac{1}{64}\right)$

$-3 < n < -2$

Because $\frac{1}{495}$ is closer to $\frac{1}{512}$ than $\frac{1}{64}$, the approximation should be closer to −3 than −2.

20. $\log_6 53$

Answers will vary.

The logarithm $\log_6 53$ is approximately equal to 2.1.

$\log_6 36 < \log_6 53 < \log_6 216$

$2 < n < 3$

Because 53 is closer to 36 than 216, the approximation should be closer to 2 than 3.

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21. $\log_9 6000$

Answers will vary.

The logarithm $\log_9 6000$ is approximately equal to 3.9.

$\log_9 729 < \log_9 6000 < \log_9 6561$

$3 < n < 4$

Because 6000 is closer to 6561 than 729, the approximation should be closer to 4 than 3.

22. $\log_7 \left(\frac{1}{10}\right)$

Answers will vary.

The logarithm $\log_7 \left(\frac{1}{10}\right)$ is approximately equal to −1.1.

$\log_7 \left(\frac{1}{49}\right) < \log_7 \left(\frac{1}{10}\right) < \log_7 \left(\frac{1}{7}\right)$

$-2 < n < -1$

Because $\frac{1}{10}$ is closer to $\frac{1}{7}$ than $\frac{1}{49}$, the approximation should be closer to −1 than −2.

23. $\log_{\frac{1}{2}} \left(\frac{1}{24}\right)$

Answers will vary.

The logarithm $\log_{\frac{1}{2}} \left(\frac{1}{24}\right)$ is approximately equal to 4.5.

$\log_{\frac{1}{2}} \left(\frac{1}{16}\right) < \log_{\frac{1}{2}} \left(\frac{1}{24}\right) < \log_{\frac{1}{2}} \left(\frac{1}{32}\right)$

$4 < n < 5$

Because $\frac{1}{24}$ is halfway between $\frac{1}{16}$ and $\frac{1}{32}$, the approximation should be halfway between 4 and 5.

24. $\log_{\frac{4}{3}} 0.85$

Answers will vary.

The logarithm $\log_{\frac{4}{3}} 0.85$ is approximately equal to −0.6.

$\log_{\frac{4}{3}} 0.75 < \log_{\frac{4}{3}} 0.85 < \log_{\frac{4}{3}} 1$

$-1 < n < 0$

Because 0.85 is closer to 0.75 than 1, the approximation should be closer to −1 than 0.

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Determine the appropriate base of each logarithm. Explain your reasoning.

25. $\log_b 29 = 2.1$

$\log_5 25 < \log_b 29 < \log_5 125$

$2 < 2.1 < 3$

Because the value of the exponent is 2.1, that means that the argument 29 should be close to its lower limit. When the base is 5, 29 is very close to the lower limit of 25, whereas when the base is 4, 29 is not very close to the lower limit of 16.

26. $\log_b 35 = 1.9$

$\log_6 6 < \log_b 35 < \log_6 36$

$1 < 1.9 < 2$

Because the value of the exponent is 1.9, that means that the argument 35 should be close to its upper limit. When the base is 6, 35 is very close to the upper limit of 36, whereas when the base is 7, 35 is not very close to the upper limit of 49.

27. $\log_b \left(\frac{1}{7}\right) = -1.9$

$\log_4 \left(\frac{1}{8}\right) < \log_b \left(\frac{1}{7}\right) < \log_4 \left(\frac{1}{4}\right)$

$-2 < -1.9 < -1$

Because the value of the exponent is -1.9, that means that the argument $\frac{1}{7}$ should be close to its lower limit. When the base is 4, $\frac{1}{7}$ is very close to the lower limit of $\frac{1}{8}$, whereas when the base is 5, $\frac{1}{7}$ is not very close to the lower limit of $\frac{1}{25}$.

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28. $\log_b 80 = 3.9$

$\log_3 27 < \log_b 80 < \log_3 81$

$3 < 3.9 < 4$

Because the value of the exponent is 3.9, that means that the argument 80 should be close to its upper limit. When the base is 3, 80 is very close to the upper limit of 81, whereas when the base is 4, 80 is not very close to the upper limit of 256.

29. $\log_b 6 = 0.9$

$\log_7 1 < \log_b 6 < \log_7 7$

$0 < 0.9 < 1$

Because the value of the exponent is 0.9, that means that the argument 6 should be close to its upper limit. When the base is 7, 6 is very close to the upper limit of 7, whereas when the base is 8, 6 is not very close to the upper limit of 8.

30. $\log_b 66 = 3.1$

$\log_4 64 < \log_b 66 < \log_4 256$

$3 < 3.1 < 4$

Because the value of the exponent is 3.1, that means that the argument 66 should be close to its lower limit. When the base is 4, 66 is very close to the lower limit of 64, whereas when the base is 3, 66 is not very close to the lower limit of 27.

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Mad Props
Properties of Logarithms

Problem Set

Rewrite each logarithmic expression in expanded form using the properties of logarithms.

1. $\log_3 (5x)$

   $\log_3 (5x) = \log_3 5 + \log_3 x$

2. $\log_5 \left(\frac{a}{b}\right)$

   $\log_5 \left(\frac{a}{b}\right) = \log_5 a - \log_5 b$

3. $\log_7 (n^4)$

   $\log_7 (n^4) = 4 \log_7 n$

4. $\log \left(\frac{x}{7}\right)$

   $\log \left(\frac{x}{7}\right) = \log x - \log 7$

5. $\log_2 (mn)$

   $\log_2 (mn) = \log_2 m + \log_2 n$

6. $\log (p^q)$

   $\log (p^q) = q \log p$

7. $\ln (x^2)$

   $\ln (x^2) = 2 \ln x$

8. $\ln \left(\frac{c}{3}\right)$

   $\ln \left(\frac{c}{3}\right) = \ln c - \ln 3$

9. $\log_3 (7x^2)$

   $\log_3 (7x^2) = \log_3 7 + 2 \log_3 x$

10. $\ln (2x^3y^2)$

    $\ln (2x^3 y^2) = \ln 2 + 3 \ln x + 2 \ln y$

11. $\log \left(\frac{xy}{5}\right)$

    $\log \left(\frac{xy}{5}\right) = \log x + \log y - \log 5$

12. $\log_7 \left(\frac{3x^4}{y}\right)$

    $\log_7 \left(\frac{3x^4}{y}\right) = \log_7 3 + 4 \log_7 x - \log_7 y$

13. $\ln \left(\frac{x}{7y}\right)$

    $\ln \left(\frac{x}{7y}\right) = \ln x - \ln 7 - \ln y$

14. $\log_5 \left(\frac{7x^2}{y^3}\right)$

    $\log_5 \left(\frac{7x^2}{y^3}\right) = \log_5 7 + 2 \log_5 x - 3 \log_5 y$

15. $\log (xyz)$

    $\log (xyz) = \log x + \log y + \log z$

16. $\ln \left(\frac{x + 1}{(y + 3)^2}\right)$

    $\ln \left(\frac{x + 1}{(y + 3)^2}\right)$

    $= \ln (x + 1) - 2 \ln (y + 3)$

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Rewrite each logarithmic expression as a single logarithm.

17. $\log x - 2\log y$

$\log x - 2\log y = \log\left(\frac{x}{y^2}\right)$

18. $3\log_4 x + \log_4 y - \log_4 z$

$3\log_4 x + \log_4 y - \log_4 z = \log_4\left(\frac{x^3y}{z}\right)$

19. $6\log_2 x - 2\log_2 x$

$6\log_2 x - 2\log_2 x = \log_2\left(\frac{x^6}{x^2}\right) = \log_2(x^4)$

20. $\log 3 + 2\log 7 - \log 6$

$\log 3 + 2\log 7 - \log 6 = \log\left(\frac{3(7)^2}{6}\right)$

$= \log\left(\frac{147}{6}\right) = \log 24.5 = 1.3892$

21. $\log x + 3\log y - \frac{1}{2}\log z$

$\log x + 3\log y - \frac{1}{2}\log z = \log\left(\frac{xy^3}{z^{\frac{1}{2}}}\right)$

22. $7\log_3 x - (2\log_3 x + 5\log_3 y)$

$7\log_3 x - (2\log_3 x + 5\log_3 y)$

$= \log_3\left(\frac{x^7}{x^2y^5}\right) = \log_3\left(\frac{x^5}{y^5}\right)$

23. $2\ln(2x + 3) - 4\ln(y - 2)$

$2\ln(2x + 3) - 4\ln(y - 2) = \ln\left(\frac{(2x + 3)^2}{(y - 2)^4}\right)$

24. $\ln(x - 7) - 2(\ln x + \ln y)$

$\ln(x - 7) - 2(\ln x + \ln y) = \ln\left(\frac{x - 7}{x^2y^2}\right)$

Suppose $w = \log_b 2$, $x = \log_b 3$, $y = \log_b 7$, and $z = \log_b 11$. Write an algebraic expression for each logarithmic expression.

25. $\log_b 33$

$\log_b 33 = \log_b(3 \cdot 11)$

$= \log_b 3 + \log_b 11$

$= x + z$

26. $\log_b 98$

$\log_b 98 = \log_b(7^2 \cdot 2)$

$= 2\log_b 7 + \log_b 2$

$= 2y + w$

27. $\log_b\left(\frac{2}{3}\right)$

$\log_b\left(\frac{2}{3}\right) = \log_b 2 - \log_b 3$

$= w - x$

28. $\log_b\left(\frac{7}{8}\right)$

$\log_b\left(\frac{7}{8}\right) = \log_b 7 - \log_b(2^3)$

$= \log_b 7 - 3\log_b 2$

$= y - 3w$

29. $\log_b 1.5$

$\log_b 1.5 = \log_b\left(\frac{3}{2}\right)$

$= \log_b 3 - \log_b 2$

$= x - w$

30. $\log_b 2.75$

$\log_b 2.75 = \log_b\left(\frac{11}{2^2}\right)$

$= \log_b 11 - 2\log_b 2$

$= z - 2w$

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What's Your Strategy?
Solving Exponential Equations

Vocabulary

1. Define the Change of Base Formula and explain how it is used.

The Change of Base Formula states: $\log_b c = \frac{\log_a c}{\log_a b}$, where $a, b, c > 0$ and $a, b \neq 1$. It allows you to calculate an exact value for a logarithm by rewriting it in terms of a different base.

Problem Set

Solve each exponential equation by using the Change of Base Formula.

1. $6^{x-5} = 24$

$6^{x-5} = 24$

$x - 5 = \log_6 24$

$x - 5 = \frac{\log 24}{\log 6}$

$x - 5 \approx 1.774$

$x \approx 6.774$

2. $12^{x+4} = 65$

$12^{x+4} = 65$

$x + 4 = \log_{12} 65$

$x + 4 = \frac{\log 65}{\log 12}$

$x + 4 \approx 1.680$

$x \approx -2.32$

3. $7^{3x} = 15$

$7^{3x} = 15$

$3x = \log_7 15$

$3x = \frac{\log 15}{\log 7}$

$3x \approx 1.392$

$x \approx 0.464$

4. $8^{5x} = 71$

$8^{5x} = 71$

$5x = \log_8 71$

$5x = \frac{\log 71}{\log 8}$

$5x \approx 2.050$

$x \approx 0.41$

5. $4^{x+3} - 7 = 32$

$$4^{x+3} - 7 = 32$$
$$4^{x+3} = 39$$
$$x + 3 = \log_4 39$$
$$x + 3 = \frac{\log 39}{\log 4}$$
$$x + 3 \approx 2.643$$
$$x \approx -0.357$$

6. $11^{x-4} + 8 = 59$

$$11^{x-4} + 8 = 59$$
$$11^{x-4} = 51$$
$$x - 4 = \log_{11} 51$$
$$x - 4 = \frac{\log 51}{\log 11}$$
$$x - 4 \approx 1.640$$
$$x \approx 5.640$$

7. $4\left(\frac{2}{3}\right)^{3x} = 248$

$$4\left(\frac{2}{3}\right)^{3x} = 248$$
$$\left(\frac{2}{3}\right)^{3x} = 62$$
$$3x = \log_{\frac{2}{3}} 62$$
$$3x = \frac{\log 62}{\log\left(\frac{2}{3}\right)}$$
$$3x \approx -10.179$$
$$x \approx -3.393$$

8. $9\left(\frac{3}{5}\right)^{2x} = 999$

$$9\left(\frac{3}{5}\right)^{2x} = 999$$
$$\left(\frac{3}{5}\right)^{2x} = 111$$
$$2x = \log_{\frac{3}{5}} 111$$
$$2x = \frac{\log 111}{\log\left(\frac{3}{5}\right)}$$
$$2x \approx -9.219$$
$$x \approx -4.610$$

9. $2(5)^{2x+1} + 4 = 18$

$$2(5)^{2x+1} + 4 = 18$$
$$2(5)^{2x+1} = 14$$
$$(5)^{2x+1} = 7$$
$$2x + 1 = \log_5 7$$
$$2x + 1 = \frac{\log 7}{\log 5}$$
$$2x + 1 \approx 1.209$$
$$2x \approx 0.209$$
$$x \approx 0.105$$

10. $-8(2)^{x-9} - 5 = -77$

$$-8(2)^{x-9} - 5 = -77$$
$$-8(2)^{x-9} = -72$$
$$(2)^{x-9} = 9$$
$$x - 9 = \log_2 9$$
$$x - 9 = \frac{\log 9}{\log 2}$$
$$x - 9 \approx 3.170$$
$$x \approx 12.170$$

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Solve each exponential equation using properties of logarithms.

11. $7^{x-2} = 28$

$$7^{x-2} = 28$$
$$(x - 2)\log 7 = \log 28$$
$$x - 2 = \frac{\log 28}{\log 7}$$
$$x - 2 \approx 1.712$$
$$x \approx 3.712$$

12. $15^{x+2} = 60$

$$15^{x+2} = 60$$
$$(x + 2)\log 15 = \log 60$$
$$x + 2 = \frac{\log 60}{\log 15}$$
$$x + 2 \approx 1.512$$
$$x \approx -0.488$$

13. $5^{4x} = 32$

$$5^{4x} = 32$$
$$4x\log 5 = \log 32$$
$$4x = \frac{\log 32}{\log 5}$$
$$4x \approx 2.153$$
$$x \approx 0.538$$

14. $9^{3x} = 124$

$$9^{3x} = 124$$
$$3x\log 9 = \log 124$$
$$3x = \frac{\log 124}{\log 9}$$
$$3x \approx 2.194$$
$$x \approx 0.731$$

15. $3^{x+7} - 5 = 63$

$$3^{x+7} - 5 = 63$$
$$3^{x+7} = 68$$
$$(x + 7)\log 3 = \log 68$$
$$x + 7 = \frac{\log 68}{\log 3}$$
$$x + 7 \approx 3.841$$
$$x \approx -3.159$$

16. $14^{x-6} + 3 = 73$

$$14^{x-6} + 3 = 73$$
$$14^{x-6} = 70$$
$$(x - 6)\log 14 = \log 70$$
$$x - 6 = \frac{\log 70}{\log 14}$$
$$x - 6 \approx 1.610$$
$$x \approx 7.610$$

17. $6\left(\frac{4}{7}\right)^{2x} = 348$

$$6\left(\frac{4}{7}\right)^{2x} = 348$$
$$\left(\frac{4}{7}\right)^{2x} = 58$$
$$2x \log\left(\frac{4}{7}\right) = \log 58$$
$$2x = \frac{\log 58}{\log\left(\frac{4}{7}\right)}$$
$$2x \approx -7.256$$
$$x \approx -3.628$$

18. $8\left(\frac{5}{8}\right)^{4x} = 448$

$$8\left(\frac{5}{8}\right)^{4x} = 448$$
$$\left(\frac{5}{8}\right)^{4x} = 56$$
$$4x \log\left(\frac{5}{8}\right) = \log 56$$
$$4x = \frac{\log 56}{\log\left(\frac{5}{8}\right)}$$
$$4x \approx -8.565$$
$$x \approx -2.141$$

19. $3(4)^{3x-6} + 2 = 35$

$$3(4)^{3x-6} + 2 = 35$$
$$3(4)^{3x-6} = 33$$
$$(4)^{3x-6} = 11$$
$$(3x - 6) \log 4 = \log 11$$
$$3x - 6 = \frac{\log 11}{\log 4}$$
$$3x - 6 \approx 1.730$$
$$3x \approx 7.730$$
$$x \approx 2.577$$

20. $-7(3)^{x+6} - 8 = -162$

$$-7(3)^{x+6} - 8 = -162$$
$$-7(3)^{x+6} = -154$$
$$(3)^{x+6} = 22$$
$$(x + 6) \log 3 = \log 22$$
$$x + 6 = \frac{\log 22}{\log 3}$$
$$x + 6 \approx 2.814$$
$$x \approx -3.186$$

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Solve each exponential equation. Explain why you chose the method that you used.

21. $11^{x-4} = 343$

$$11^{x-4} = 343$$
$$(x - 4) \log 11 = \log 343$$
$$x - 4 = \frac{\log 343}{\log 11}$$
$$x - 4 \approx 2.435$$
$$x \approx 6.435$$

I took the log of both sides, because 343 cannot be written as a power of 11.

22. $5^{2x+1} = 3125$

Answers will vary.

$$5^{2x+1} = 3125$$
$$5^{2x+1} = 5^5$$
$$2x + 1 = 5$$
$$2x = 4$$
$$x = 2$$

I used common bases, because 3125 can be written as 5^5.

23. $16\left(\frac{8}{9}\right)^{5x} = 752$

Answers will vary.

$$16\left(\frac{8}{9}\right)^{5x} = 752$$
$$\left(\frac{8}{9}\right)^{5x} = 47$$
$$5x \log\left(\frac{8}{9}\right) = \log 47$$
$$5x = \frac{\log 47}{\log\left(\frac{8}{9}\right)}$$
$$5x \approx -32.688$$
$$x \approx 6.538$$

I took the log of both sides, because 752 cannot be written as a power of $\frac{8}{9}$.

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24. $23^{5x} = 736$

Answers will vary.

$23^{5x} = 736$

$5x = \log_{23} 736$

$5x = \frac{\log 736}{\log 23}$

$5x \approx 2.105$

$x \approx 0.421$

I used the Change of Base Formula, because 736 cannot be written as a power of 23.

25. $\left(\frac{7}{12}\right)^{4x} = 49$

Answers will vary.

$\left(\frac{7}{12}\right)^{4x} = 49$

$4x = \log_{\frac{7}{12}} 49$

$4x = \frac{\log 49}{\log\left(\frac{7}{12}\right)}$

$4x \approx -7.220$

$x \approx -1.805$

I used the Change of Base Formula, because 49 cannot be written as a power of $\frac{7}{12}$.

26. $14^{5x-12} - 8 = 2736$

Answers will vary.

$14^{5x-12} - 8 = 2736$

$14^{5x-12} = 2744$

$14^{5x-12} = 14^3$

$5x - 12 = 3$

$5x = 15$

$x = 3$

I used common bases, because 2744 can be written as 14^3.

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Logging On

Solving Logarithmic Equations

Problem Set

Solve each logarithmic equation. Check your answer(s).

1. $\log_2 (x^2 - x) = 1$

$\log_2 (x^2 - x) = 1$

$2^1 = x^2 - x$

$0 = x^2 - x - 2$

$0 = (x + 1)(x - 2)$

$x = -1, 2$

Check:

$\log_2 ((-1)^2 - (-1)) \stackrel{?}{=} 1$

$\log_2 (1 + 1) \stackrel{?}{=} 1$

$\log_2 2 = 1$

$\log_2 (2^2 - 2) \stackrel{?}{=} 1$

$\log_2 (4 - 2) \stackrel{?}{=} 1$

$\log_2 2 = 1$

2. $\log_{15} (x^2 - 2x) = 1$

$\log_{15} (x^2 - 2x) = 1$

$15^1 = x^2 - 2x$

$0 = x^2 - 2x - 15$

$0 = (x + 3)(x - 5)$

$x = -3, 5$

Check:

$\log_{15} ((-3)^2 - 2(-3)) \stackrel{?}{=} 1$

$\log_{15} (9 + 6) \stackrel{?}{=} 1$

$\log_{15} 15 = 1$

$\log_{15} (5^2 - 2(5)) \stackrel{?}{=} 1$

$\log_{15} (25 - 10) \stackrel{?}{=} 1$

$\log_{15} 15 = 1$

3. $\log_6 (x^2 + 5x) = 2$

$\log_6 (x^2 + 5x) = 2$

$6^2 = x^2 + 5x$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x + 9)(x - 4)$

$x = -9, 4$

Check:

$\log_6 ((-9)^2 + 5(-9)) \stackrel{?}{=} 2$

$\log_6 (81 - 45) \stackrel{?}{=} 2$

$\log_6 36 = 2$

$\log_6 (4^2 + 5(4)) \stackrel{?}{=} 2$

$\log_6 (16 + 20) \stackrel{?}{=} 2$

$\log_6 36 = 2$

4. $\log_2 (x^2 + 6x) = 4$

$\log_2 (x^2 + 6x) = 4$

$2^4 = x^2 + 6x$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x + 8)(x - 2)$

$x = -8, 2$

Check:

$\log_2 ((-8)^2 + 6(-8)) \stackrel{?}{=} 4$

$\log_2 (64 - 48) \stackrel{?}{=} 4$

$\log_2 16 = 4$

$\log_2 (2^2 + 6(2)) \stackrel{?}{=} 4$

$\log_2 (4 + 12) \stackrel{?}{=} 4$

$\log_2 16 = 4$

5. $\log_4 (x^2 - 12x) = 3$

$\log_4 (x^2 - 12x) = 3$

$4^3 = x^2 - 12x$

$64 = x^2 - 12x$

$0 = x^2 - 12x - 64$

$0 = (x + 4)(x - 16)$

$x = -4, 16$

Check:

$\log_4 ((-4)^2 - 12(-4)) \stackrel{?}{=} 3$

$\log_4 (16 + 48) \stackrel{?}{=} 3$

$\log_4 64 = 3$

$\log_4 (16^2 - 12(16)) \stackrel{?}{=} 3$

$\log_4 (256 - 192) \stackrel{?}{=} 3$

$\log_4 64 = 3$

6. $\log_{10} (x^2 + 15x) = 2$

$\log_{10} (x^2 + 15x) = 2$

$10^2 = x^2 + 15x$

$100 = x^2 + 15x$

$0 = x^2 + 15x - 100$

$0 = (x + 20)(x - 5)$

$x = -20, 5$

Check:

$\log_{10} ((-20)^2 + 15(-20)) \stackrel{?}{=} 2$

$\log_{10} (400 - 300) \stackrel{?}{=} 2$

$\log_{10} 100 = 2$

$\log_{10} (5^2 + 15(5)) \stackrel{?}{=} 2$

$\log_{10} (25 + 75) \stackrel{?}{=} 2$

$\log_{10} 100 = 2$

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7. $\log_3 (3x^2 + 18x) = 4$

$\log_3 (3x^2 + 18x) = 4$

$3^4 = 3x^2 + 18x$

$81 = 3x^2 + 18x$

$0 = 3x^2 + 18x - 81$

$0 = 3(x^2 + 6x - 27)$

$0 = 3(x + 9)(x - 3)$

$x = -9, 3$

Check:

$\log_3 (3(-9)^2 + 18(-9)) \stackrel{?}{=} 4$

$\log_3 (243 - 162) \stackrel{?}{=} 4$

$\log_3 81 = 4$

$\log_3 (3(3)^2 + 18(3)) \stackrel{?}{=} 4$

$\log_3 (27 + 54) \stackrel{?}{=} 4$

$\log_3 81 = 4$

8. $\log_4 (2x^2 - 28x) = 3$

$\log_4 (2x^2 - 28x) = 3$

$4^3 = 2x^2 - 28x$

$64 = 2x^2 - 28x$

$0 = 2x^2 - 28x - 64$

$0 = 2(x^2 - 14x - 32)$

$0 = 2(x + 2)(x - 16)$

$x = -2, 16$

Check:

$\log_4 (2(-2)^2 - 28(-2)) \stackrel{?}{=} 3$

$\log_4 (8 + 56) \stackrel{?}{=} 3$

$\log_4 64 = 3$

$\log_4 (2(16)^2 - 28(16)) \stackrel{?}{=} 3$

$\log_4 (512 - 448) \stackrel{?}{=} 3$

$\log_4 64 = 3$

Use the properties of logarithms to solve each logarithmic equation. Check your answer(s).

9. $2 \log_3 x - \log_3 8 = \log_3 (x - 2)$

$$2 \log_3 x - \log_3 8 = \log_3 (x - 2)$$
$$\log_3 x^2 - \log_3 8 = \log_3 (x - 2)$$
$$\log_3 \left(\frac{x^2}{8}\right) = \log_3 (x - 2)$$
$$\frac{x^2}{8} = x - 2$$
$$x^2 = 8x - 16$$
$$x^2 - 8x + 16 = 0$$
$$(x - 4)^2 = 0$$
$$x = 4$$

Check:

$$2 \log_3 4 - \log_3 8 \stackrel{?}{=} \log_3 (4 - 2)$$
$$\log_3 16 - \log_3 8 \stackrel{?}{=} \log_3 2$$
$$\log_3 \left(\frac{16}{8}\right) \stackrel{?}{=} \log_3 2$$
$$\log_3 2 = \log_3 2$$

10. $\log_4 (x + 3) + \log_4 x = 1$

$$\log_4 (x + 3) + \log_4 x = 1$$
$$\log_4 (x(x + 3)) = 1$$
$$x(x + 3) = 4^1$$
$$x^2 + 3x = 4$$
$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x = -4, 1$$

Check:

−4 is an extraneous solution.

$$\log_4 (1 + 3) + \log_4 1 \stackrel{?}{=} 1$$
$$\log_4 4 + \log_4 1 \stackrel{?}{=} 1$$
$$1 + 0 \stackrel{?}{=} 1$$
$$1 = 1$$

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11. $\log(2x^2 + 3) + \log 2 = \log 10x$

$$\log(2x^2 + 3) + \log 2 = \log 10x$$
$$\log(2(2x^2 + 3)) = \log 10x$$
$$2(2x^2 + 3) = 10x$$
$$2x^2 + 3 = 5x$$
$$2x^2 - 5x + 3 = 0$$
$$(2x - 3)(x - 1) = 0$$
$$x = \frac{3}{2}, 1$$

Check:

$$\log\left[2\left(\frac{3}{2}\right)^2 + 3\right] + \log 2 \stackrel{?}{=} \log 10\left(\frac{3}{2}\right)$$
$$\log\left(\frac{15}{2}\right) + \log 2 \stackrel{?}{=} \log 15$$
$$\log\left(\frac{15}{2} \times 2\right) \stackrel{?}{=} \log 15$$
$$\log 15 = \log 15$$

$$\log\left[2(1)^2 + 3\right] + \log 2 \stackrel{?}{=} \log(10(1))$$
$$\log 5 + \log 2 \stackrel{?}{=} \log 10$$
$$\log((5)2) \stackrel{?}{=} \log 10$$
$$\log 10 = \log 10$$

12. $\log_2 x + \log_2 (x - 6) = 4$

$$\log_2 x + \log_2 (x - 6) = 4$$
$$\log_2 (x(x - 6)) = 4$$
$$x(x - 6) = 2^4$$
$$x^2 - 6x = 16$$
$$x^2 - 6x - 16 = 0$$
$$(x - 8)(x + 2) = 0$$
$$x = 8, -2$$

Check:

-2 is an extraneous solution.

$$\log_2 8 + \log_2 (8 - 6) \stackrel{?}{=} 4$$
$$\log_2 8 + \log_2 2 \stackrel{?}{=} 4$$
$$3 + 1 \stackrel{?}{=} 4$$
$$4 = 4$$

13. $2\log_5 x - \log_5 4 = \log_5 (8 - x)$

$2\log_5 x - \log_5 4 = \log_5 (8 - x)$

$\log_5 (x^2) - \log_5 4 = \log_5 (8 - x)$

$\log_5 \left(\frac{x^2}{4}\right) = \log_5 (8 - x)$

$\frac{x^2}{4} = 8 - x$

$x^2 = 32 - 4x$

$x^2 + 4x - 32 = 0$

$(x + 8)(x - 4) = 0$

$x = -8, 4$

Check:

-8 is an extraneous solution.

$2\log_5 4 - \log_5 4 \stackrel{?}{=} \log_5 (8 - 4)$

$\log_5 (4^2) - \log_5 4 \stackrel{?}{=} \log_5 4$

$\log_5 16 - \log_5 4 \stackrel{?}{=} \log_5 4$

$\log_5 \left(\frac{16}{4}\right) \stackrel{?}{=} \log_5 4$

$\log_5 4 = \log_5 4$

14. $\log_2 3 + \log_2 (3x^2 + 4) = \log_2 (39x)$

$\log_2 3 + \log_2 (3x^2 + 4) = \log_2 (39x)$

$\log_2 (3(3x^2 + 4)) = \log_2 (39x)$

$3(3x^2 + 4) = 39x$

$3x^2 + 4 = 13x$

$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

$x = \frac{1}{3}, 4$

Check:

$\log_2 3 + \log_2 \left(3\left(\frac{1}{3}\right)^2 + 4\right) \stackrel{?}{=} \log_2 39\left(\frac{1}{3}\right)$

$\log_2 \left(3\left(3\left(\frac{1}{9}\right) + 4\right)\right) \stackrel{?}{=} \log_2 13$

$\log_2 \left(3\left(\frac{1}{3} + 4\right)\right) \stackrel{?}{=} \log_2 13$

$\log_2 (1 + 12) \stackrel{?}{=} \log_2 13$

$\log_2 13 = \log_2 13$

$\log_2 3 + \log_2 (3(4)^2 + 4) \stackrel{?}{=} \log_2 39(4)$

$\log_2 (3(3(16) + 4)) \stackrel{?}{=} \log_2 156$

$\log_2 (3(48 + 4)) \stackrel{?}{=} \log_2 156$

$\log_2 (3(52)) \stackrel{?}{=} \log_2 156$

$\log_2 156 = \log_2 156$

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15. $\ln\left(x^2 + \frac{15}{2}\right) + \ln 2 = \ln(11x)$

$\ln\left(x^2 + \frac{15}{2}\right) + \ln 2 = \ln(11x)$

$\ln\left(2\left(x^2 + \frac{15}{2}\right)\right) = \ln(11x)$

$\ln(2x^2 + 15) = \ln(11x)$

$2x^2 + 15 = 11x$

$2x^2 - 11x + 15 = 0$

$(2x - 5)(x - 3) = 0$

$x = \frac{5}{2}, 3$

Check:

$\ln\left(\left(\frac{5}{2}\right)^2 + \frac{15}{2}\right) + \ln 2 \stackrel{?}{=} \ln\left(11\left(\frac{5}{2}\right)\right)$

$\ln\left(2\left(\frac{25}{4} + \frac{15}{2}\right)\right) \stackrel{?}{=} \ln\left(\frac{55}{2}\right)$

$\ln\left(\frac{25}{2} + \frac{30}{2}\right) \stackrel{?}{=} \ln\left(\frac{55}{2}\right)$

$\ln\left(\frac{55}{2}\right) = \ln\left(\frac{55}{2}\right)$

$\ln\left(3^2 + \frac{15}{2}\right) + \ln 2 \stackrel{?}{=} \ln(11(3))$

$\ln\left(2\left(9 + \frac{15}{2}\right)\right) \stackrel{?}{=} \ln 33$

$\ln(18 + 15) \stackrel{?}{=} \ln 33$

$\ln 33 = \ln 33$

16. $\log_4\left(\frac{1}{5}x^2 - 6\right) - \log_4\left(\frac{1}{5}\right) = \log_4 x$

$\log_4\left(\frac{1}{5}x^2 - 6\right) - \log_4\left(\frac{1}{5}\right) = \log_4 x$

$\log_4\left(\frac{\frac{1}{5}x^2 - 6}{\frac{1}{5}}\right) = \log_4 x$

$\log_4\left(5\left(\frac{1}{5}x^2 - 6\right)\right) = \log_4 x$

$\log_4(x^2 - 30) = \log_4 x$

$x^2 - 30 = x$

$x^2 - x - 30 = 0$

$(x + 5)(x - 6) = 0$

$x = -5, 6$

Check:

-5 is an extraneous solution.

$\log_4\left(\frac{1}{5}(6)^2 - 6\right) - \log_4\left(\frac{1}{5}\right) \stackrel{?}{=} \log_4 6$

$\log_4\left(\frac{1}{5}(36) - 6\right) - \log_4\left(\frac{1}{5}\right) \stackrel{?}{=} \log_4 6$

$\log_4\left(\frac{36}{5} - 6\right) - \log_4\left(\frac{1}{5}\right) \stackrel{?}{=} \log_4 6$

$\log_4\left(\frac{36}{5} - \frac{30}{5}\right) - \log_4\left(\frac{1}{5}\right) \stackrel{?}{=} \log_4 6$

$\log_4\left(\frac{6}{5}\right) - \log_4\left(\frac{1}{5}\right) \stackrel{?}{=} \log_4 6$

$\log_4\left(\frac{\frac{6}{5}}{\frac{1}{5}}\right) \stackrel{?}{=} \log_4 6$

$\log_4 6 = \log_4 6$

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So When Will I Use This?

Applications of Exponential and Logarithmic Equations

Problem Set

The amount of a radioactive isotope remaining can be modeled using the formula $A = A_0e^{-kt}$, where t represents the time in years, A represents the amount of the isotope remaining in grams after t years, A_0 represents the original amount of the isotope in grams, and k is the decay constant. Use this formula to solve each problem.

1. Strontium-90 is a radioactive isotope with a half-life of about 29 years. Calculate the decay constant for Strontium-90. Then find the amount of 100 grams of Strontium-90 remaining after 120 years.

$A = A_0e^{-kt}$

$\frac{1}{2}A_0 = A_0e^{-k(29)}$

$\frac{1}{2} = e^{-29k}$

$\ln\left(\frac{1}{2}\right) = -29k$

$k \approx 0.0239$

The decay constant for Strontium-90 is about 0.0239.

$A = 100e^{-0.0239(120)}$

$A \approx 5.681$

After 120 years, there would be about 5.681 grams remaining.

2. Radium-226 is a radioactive isotope with a half-life of about 1622 years. Calculate the decay constant for Radium-226. Then find the amount of 20 grams of Radium-226 remaining after 500 years.

$A = A_0e^{-kt}$

$\frac{1}{2}A_0 = A_0e^{-k(1622)}$

$\frac{1}{2} = e^{-1622k}$

$\ln\left(\frac{1}{2}\right) = -1622k$

$k \approx 4.273 \times 10^{-4}$

The decay constant for Radium-226 is about 4.273×10^{-4}.

$A = 20e^{-4.273 \times 10^{-4}(500)}$

$A \approx 16.152$

After 500 years, there would be about 16.152 grams remaining.

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3. Carbon-14 is a radioactive isotope with a half-life of about 5730 years. Calculate the decay constant for Carbon-14. Then find the amount of 6 grams of Carbon-14 that will remain after 22,000 years.

$$A = A_0e^{-kt}$$

$$\frac{1}{2}A_0 = A_0e^{-k(5730)}$$

$$\frac{1}{2} = e^{-5730k}$$

$$\ln\left(\frac{1}{2}\right) = -5730k$$

$$k \approx 1.210 \times 10^{-4}$$

The decay constant for Carbon-14 is about 1.210×10^{-4}.

$$A = 6e^{-1.210\times10^{-4}(22,000)}$$

$$A \approx 0.419$$

After 22,000 years, there would be about 0.419 gram remaining.

4. Cesium-137 is a radioactive isotope with a half-life of about 30 years. Calculate the decay constant for Cesium-137. Then calculate the percentage of a Cesium-137 sample remaining after 100 years.

$$A = A_0e^{-kt}$$

$$\frac{1}{2}A_0 = A_0e^{-k(30)}$$

$$\frac{1}{2} = e^{-30k}$$

$$\ln\left(\frac{1}{2}\right) = -30k$$

$$k \approx 0.0231$$

The decay constant for Cesium-137 is about 0.0231.

$$A = A_0e^{-0.0231t}$$

$$A = A_0e^{-0.0231(100)}$$

$$\frac{A}{A_0} \approx 0.0993$$

After 100 years, there will be about 9.93% of the Cesium-137 remaining.

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5. Uranium-232 is a radioactive isotope with a half-life of about 69 years. Calculate the decay constant for Uranium-232. Then calculate the percentage of a Uranium-232 sample remaining after 200 years.

$$A = A_0 e^{-kt}$$

$$\frac{1}{2}A_0 = A_0 e^{-k(69)}$$

$$\frac{1}{2} = e^{-69k}$$

$$\ln\left(\frac{1}{2}\right) = -69k$$

$$k \approx 0.0100$$

The decay constant for Uranium-232 is about 0.0100.

$$A = A_0 e^{-0.0100t}$$

$$A = A_0 e^{-0.0100(200)}$$

$$\frac{A}{A_0} \approx 0.135$$

After 200 years, there will be about 13.5% of the Uranium-232 remaining.

6. Rubidium-87 is a radioactive isotope with a half-life of about 4.7×10^7 years. Calculate the decay constant for Rubidium-87. Then calculate the percentage of a Rubidium-87 sample remaining after 1,000,000 years.

$$A = A_0 e^{-kt}$$

$$\frac{1}{2}A_0 = A_0 e^{-k(4.7\times10^7)}$$

$$\frac{1}{2} = e^{(-4.7\times10^7)k}$$

$$\ln\left(\frac{1}{2}\right) = (-4.7 \times 10^7)k$$

$$k \approx 1.475 \times 10^{-8}$$

The decay constant for Rubidium-87 is about 1.475×10^{-8}.

$$A = A_0 e^{-(1.475\times10^{-8})t}$$

$$A = A_0 e^{-(1.475\times10^{-8}(1,000,000)}$$

$$\frac{A}{A_0} \approx 0.985$$

After 1,000,000 years, there will be about 98.5% of the Rubidium-87 remaining.

Use the given exponential equation to answer each question. Show your work.

7. The number of students exposed to the measles at a school can be modeled by the equation $S = 10e^{0.15t}$, where S represents the number of students exposed after t days. How many students were exposed after eight days?

$S = 10e^{0.15t}$

$= 10e^{(0.15 \cdot 8)}$

$= 10e^{1.2}$

≈ 33.20116923

Approximately 33 students were exposed after eight days.

8. The minnow population in White Mountain Lake each year can be modeled by the equation $M = 700(10^{0.2t})$, where M represents the minnow population t years from now. What will the minnow population be in 15 years?

$M = 700(10^{0.2t})$

$= 700(10^{(0.2 \cdot 15)})$

$= 700(10^3)$

$= 700(1000)$

$= 700{,}000$

There will be 700,000 minnows in 15 years.

9. Aiden invested $600 in a savings account with continuous compound interest. The equation $V = 600e^{0.05t}$ can be used to predict the value, V, of Aiden's account after t years. What would the value of Aiden's account be after five years?

$V = 600e^{0.05t}$

$= 600e^{(0.05 \cdot 5)}$

$= 600e^{0.25}$

≈ 770.41525

The value of Aiden's account would be about $770.42 after five years.

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10. The rabbit population on Hare Island can be modeled by the equation $R = 60e^{0.09t}$, where R represents the rabbit population t years from now. How many years from now will the rabbit population of Hare Island be 177 rabbits?

$$R = 60e^{0.09t}$$

$$177 = 60e^{0.09t}$$

$$2.95 = e^{0.09t}$$

$$\ln 2.95 = \ln e^{0.09t}$$

$$\ln 2.95 = 0.09t \ln e$$

$$\frac{\ln 2.95}{\ln e} = 0.09t$$

$$1.08180517 \approx 0.09t$$

$$12.02005745 \approx t$$

In a little longer than 12 years from now, there will be 177 rabbits on Hare Island.

11. A disease is destroying the elm tree population in the Dutch Forest. The equation $N = 16(10^{0.15t})$ can be used to predict the number of elm trees, N, killed by the disease t years from now. In how many years from now will 406 elm trees have been killed by the disease?

$$N = 16(10^{0.15t})$$

$$406 = 16(10^{0.15t})$$

$$25.375 = 10^{0.15t}$$

$$\log 25.375 = \log 10^{0.15t}$$

$$\log 25.375 = 0.15t \log 10$$

$$\log 25.375 = 0.15t\ (1)$$

$$1.404406051 \approx 0.15t$$

$$9.362707006 \approx t$$

In approximately 9.4 years, 406 elm trees will have been killed by the disease.

12. Manuel invested money in a savings account with continuous compound interest. The equation $V = 10{,}000e^{0.03t}$ can be used to determine the value, V, of the account after t years. In how many years will the value of the account be \$12,000?

$$V = 10{,}000e^{0.03t}$$

$$12{,}000 = 10{,}000e^{0.03t}$$

$$1.2 = e^{0.03t}$$

$$\ln 1.2 = \ln e^{0.03t}$$

$$\ln 1.2 = 0.03t \ln e$$

$$\ln 1.2 = 0.03t(1)$$

$$0.1823215568 \approx 0.03t$$

$$6.077385226 \approx t$$

In approximately 6.1 years, the account will have a value of \$12,000.

Use the formula $M = \log\left(\frac{I}{I_0}\right)$, where M is the magnitude of an earthquake on the Richter scale, I_0 represents the intensity of a zero-level earthquake the same distance from the epicenter, and I is the number of times more intense an earthquake is than a zero-level earthquake, to solve each problem. A zero-level earthquake has a seismographic reading of 0.001 millimeter at a distance of 100 kilometers from the center.

13. An earthquake southwest of Chattanooga, Tennessee in 2003 had a seismographic reading of 79.43 millimeters registered 100 kilometers from the center. What was the magnitude of the Tennessee earthquake of 2003 on the Richter scale?

$$M = \log\left(\frac{I}{I_0}\right)$$

$$M = \log\left(\frac{79.43}{0.001}\right)$$

$$M \approx 4.9$$

The Tennessee earthquake of 2003 measured 4.9 on the Richter scale.

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14. An earthquake in Illinois in 2008 had a seismographic reading of 158.5 millimeters registered 100 kilometers from the center. What was the magnitude of the Illinois earthquake of 2008 on the Richter scale?

$$M = \log\left(\frac{I}{I_0}\right)$$

$$M = \log\left(\frac{158.5}{0.001}\right)$$

$$M \approx 5.2$$

The Illinois earthquake of 2008 measured 5.2 on the Richter scale.

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15. An earthquake off the northern coast of California in 2005 had a seismographic reading of 15,849 millimeters registered 100 kilometers from the center. What was the magnitude of the California earthquake in 2005 on the Richter scale?

$M = \log\left(\frac{I}{I_0}\right)$

$M = \log\left(\frac{15{,}849}{0.001}\right)$

$M \approx 7.2$

The California earthquake of 2005 measured 7.2 on the Richter scale.

16. The devastating earthquake in Haiti in 2010 had a magnitude of 7.0 on the Richter scale. What was its seismographic reading in millimeters 100 kilometers from the center?

$M = \log\left(\frac{I}{I_0}\right)$

$7.0 = \log\left(\frac{I}{0.001}\right)$

$10^7 = \frac{I}{0.001}$

$I \approx 10{,}000$

The seismographic reading was about 10,000 millimeters.

17. Calculate the value of the seismographic reading for an earthquake of magnitude 6.4 on the Richter scale.

$M = \log\left(\frac{I}{I_0}\right)$

$6.4 = \log\left(\frac{I}{0.001}\right)$

$10^{6.4} = \frac{I}{0.001}$

$I \approx 2512$

The seismographic reading is about 2512 millimeters.

18. Calculate the value of the seismographic reading for an earthquake of magnitude 8.1 on the Richter scale.

$M = \log\left(\frac{I}{I_0}\right)$

$8.1 = \log\left(\frac{I}{0.001}\right)$

$10^{8.1} = \frac{I}{0.001}$

$I \approx 125{,}893$

The seismographic reading was about 125,893 millimeters.

Use the given formula to solve each problem.

19. The formula for the population of a species is $n = k \log(A)$, where n represents the population of a species, A is the area of the region in which the species lives, and k is a constant that is determined by field studies. Based on population samples, an area that is 1000 square miles contains 360 wolves. Calculate the value of k. Then use the formula to find the number of wolves remaining in 15 years if only 300 square miles of this area is still inhabitable.

$n = k \log(A)$

$360 = k \log 1000$

$k = 120$

The value of k is 120.

$n = 120 \log(A) = 120 \log 300 \approx 297$

In 15 years, there will be approximately 297 wolves remaining in the area.

20. The formula for the population of a species is $n = k \log(A)$, where n represents the population of a species, A is the area of the region in which the species lives, and k is a constant that is determined by field studies. Based on population samples, a rainforest that is 100 square miles contains 342 monkeys. Calculate the value of k. Then use the formula to find the number of monkeys remaining in 5 years if only 40 square miles of the rainforest survives due to the current level of deforestation.

$n = k \log(A)$

$342 = k \log 100$

$k = 171$

The value of k is 171.

$n = 171 \log(A)$

$n = 171 \log 40$

$n \approx 274$

In 5 years, there will be approximately 274 monkeys remaining in the area.

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21. The formula $y = a + b \ln t$, where t represents the time in hours, y represents the amount of fresh water produced in t hours, a represents the amount of fresh water produced in one hour, and b is the rate of production, models the amount of fresh water produced from salt water during a desalinization process. In one desalination plant, 15.26 cubic yards of fresh water can be produced in one hour with a rate of production of 31.2. How much fresh water can be produced after 8 hours?

$y = a + b \ln t$

$y = 15.26 + 31.2 \ln 8$

$y \approx 80.14$

About 80.14 cubic yards of fresh water can be produced in eight hours.

22. The formula $y = a + b \ln t$, where t represents the time in hours, y represents the amount of fresh water produced in t hours, a represents the amount of fresh water produced in one hour, and b is the rate of production, models the amount of fresh water produced from salt water during a desalinization process. At a desalination plant, 18.65 cubic yards of fresh water can be produced in one hour with a rate of production of 34.5. How long will it take for the plant to produce 250 cubic yards of fresh water?

$y = a + b \ln t$

$250 = 18.65 + 34.5 \ln t$

$6.7058 \approx \ln t$

$t \approx 817$

It will take about 817 hours to produce 250 cubic yards of fresh water.

23. The relationship between the age of an item in years and its value is given by the equation $t = \frac{\log\left(\frac{V}{C}\right)}{\log(1 - r)}$, where t represents the age of the item in years, V represents the value of the item after t years, C represents the original value of the item, and r represents the yearly rate of appreciation expressed as a decimal. A luxury car was originally purchased for \$110,250 and is currently valued at \$65,200. The average rate of depreciate for this car is 10.3% per year. How old is the car to the nearest tenth of a year?

$$t = \frac{\log\left(\frac{V}{C}\right)}{\log(1 - r)}$$

$$= \frac{\log\left(\frac{65{,}200}{110{,}250}\right)}{\log(1 - 0.103)}$$

$$\approx \frac{\log 0.59138322}{\log 0.897}$$

$$\approx 4.83251024$$

The car is approximately 4.8 years old.

24. The relationship between the age of an item in years and its value is given by the equation $t = \frac{\log\left(\frac{V}{C}\right)}{\log(1 - r)}$, where t represents the age of the item in years, V represents the value of the item after t years, C represents the original value of the item, and r represents the yearly rate of appreciation expressed as a decimal. A 4-year old car was originally purchased for \$35,210. Its current value is \$16,394. What is this car's annual rate of depreciation to the nearest tenth?

$$t = \frac{\log\left(\frac{V}{C}\right)}{\log(1 - r)}$$

$$4 = \frac{\log\left(\frac{16{,}394}{35{,}210}\right)}{\log(1 - r)}$$

$$4 \log(1 - r) \approx \log 0.4656063618$$

$$\log(1 - r) \approx -0.0829952736$$

$$10^{-0.0829952736} \approx 1 - r$$

$$0.8260469394 \approx 1 - r$$

$$-0.1739530606 \approx -r$$

$$0.173953606 \approx r$$

This car's annual rate of depreciation is approximately 17.4%.

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