Name ____________________ Date ________

Recharge It!
Normal Distributions

Vocabulary

Write the term that best completes each statement.

1. A normal curve models a theoretical data set that is said to have a
   normal distribution.

2. Continuous data are data which can take any numerical value within a range.

3. A bell shaped curve that is symmetric about the mean of a data set is a
   normal curve.

4. Data whose possible values are countable and often finite are
   discrete data.

5. The mean of a population is often represented with the symbol μ.

6. The standard deviation of data is a measure of how spread out the data are and its often used symbol is σ.

Problem Set

Label each of the following data as either continuous or discrete.

1. The number of vacation days for an employee during a year Discrete

2. The 100-yard sprint times for a freshman swimmer Continuous

3. The distance travelled by migratory birds one winter Continuous

4. The number of defective cell phones in a batch of 5,000 Discrete

5. The number of empty seats on a transcontinental flight Discrete

6. The high temperature for Nome, Alaska during the Iditarod Continuous

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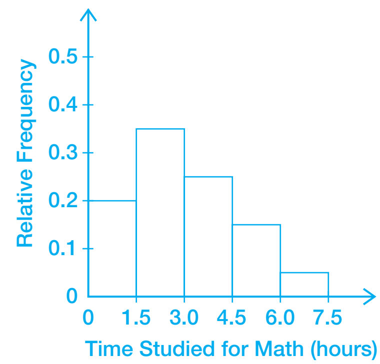
Sketch a relative frequency histogram for each distribution of sample data and determine if it is a normal or non-normal distribution.

7.

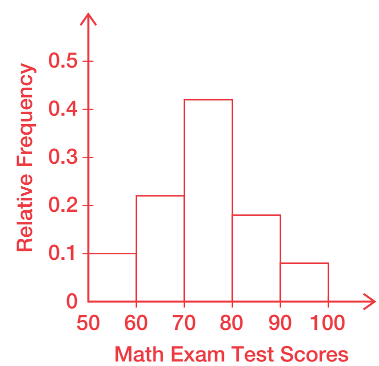
Time Studied for Math Exam (hours)	Relative Frequency
0–1.5	0.20
1.5–3.0	0.35
3.0–4.5	0.25
4.5–6.0	0.15
6.0–7.5	0.05



The distribution is non-normal because it is neither bell-shaped nor symmetrical.

8.

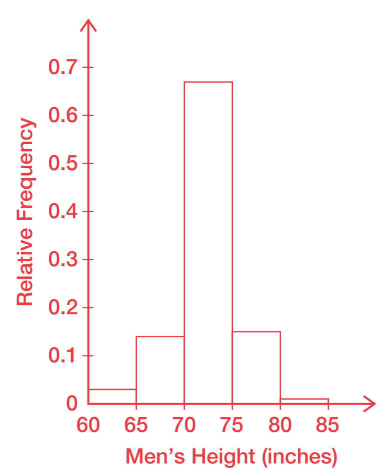
Math Exam Test Scores	Relative Frequency
50–60	0.10
60–70	0.22
70–80	0.42
80–90	0.18
90–100	0.08



The distribution is normal because it is bell-shaped and fairly symmetrical.

9.

Men's Height (inches)	Relative Frequency
60–65	0.03
65–70	0.14
70–75	0.67
75–80	0.15
80–85	0.01



The distribution is normal because it is bell-shaped and fairly symmetrical.

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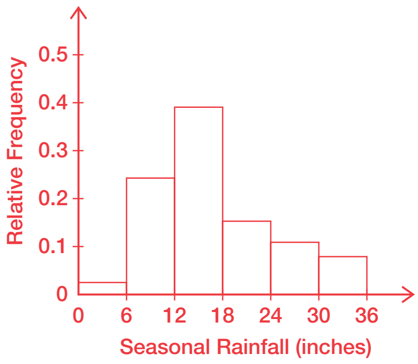
10.

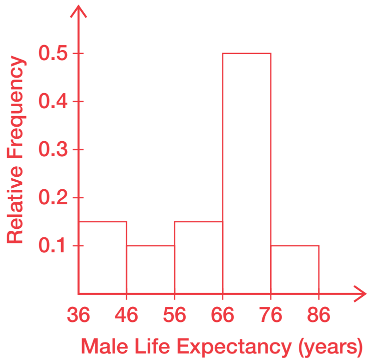
Seasonal Rainfall (inches)	Relative Frequency
0–6.0	0.025
6.0–12.0	0.243
12.0–18.0	0.391
18.0–24.0	0.153
24.0–30.0	0.109
30.0–36.0	0.079



The distribution is non-normal because it is neither bell-shaped nor symmetrical.

11.

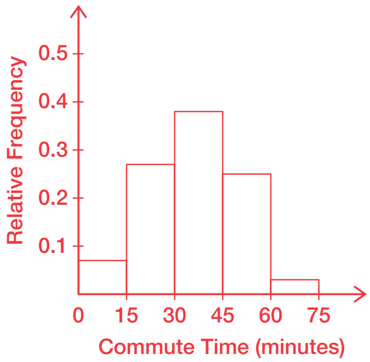
Male Life Expectancy (years)	Relative Frequency
36–46	0.15
46–56	0.10
56–66	0.15
66–76	0.50
76–86	0.10



The distribution is non-normal because it is neither bell-shaped nor symmetrical.

12.

Commute Time (minutes)	Relative Frequency
0–15	0.07
15–30	0.27
30–45	0.38
45–60	0.25
60–75	0.03



The distribution is normal because it is bell-shaped and fairly symmetrical.

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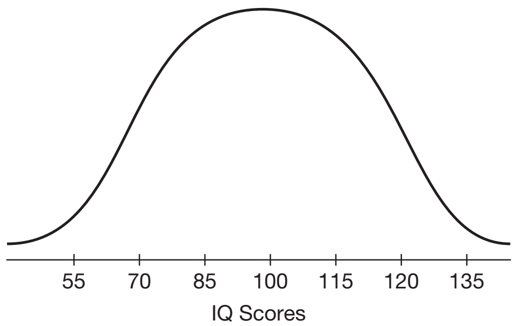
Identify the mean and standard deviation for each normal distribution below.

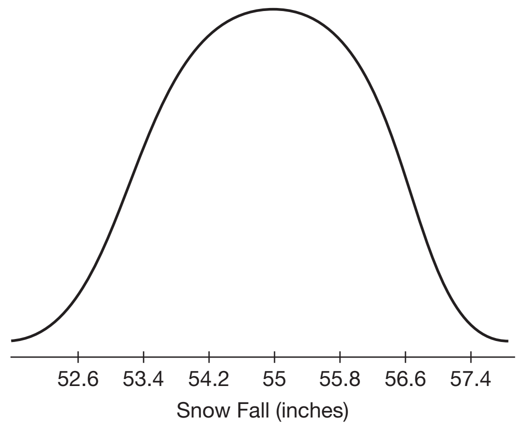
13.



mean = 100

standard deviation = 15

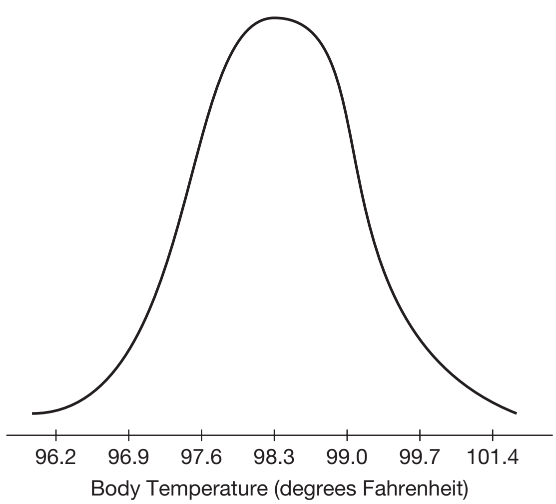
14.



mean = 55 inches

standard deviation = 0.8 inch

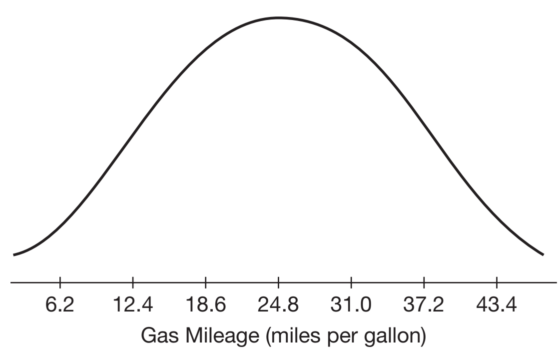
15.



mean = 98.3 degrees

standard deviation = 0.7 degree

16.



mean = 24.8 miles per gallon

standard deviation = 6.2 miles per gallon

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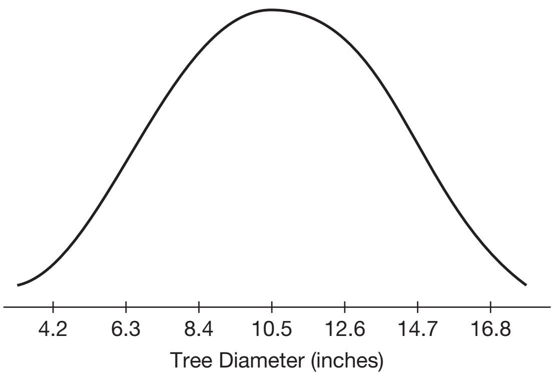
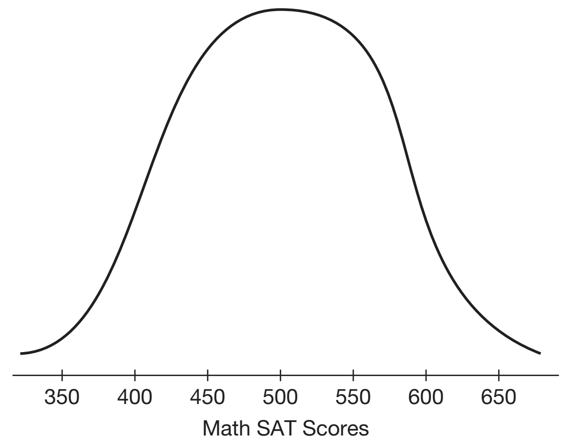
17.



mean = 10.5 inches

standard deviation = 2.1 inches

18.

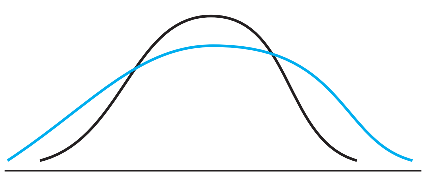


mean = 500

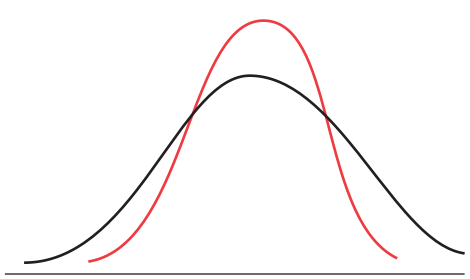
standard deviation = 50

Draw a second normal curve on the same axes as the first with the new properties.

19 The mean is the same and the standard deviation is larger.



20. The mean is the same and the standard deviation is smaller.



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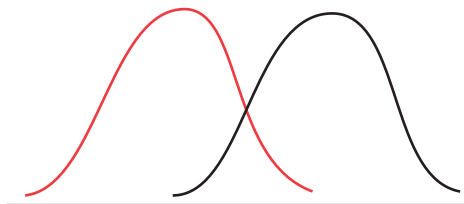
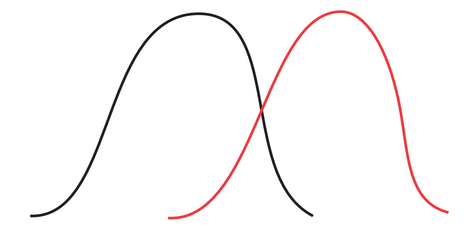
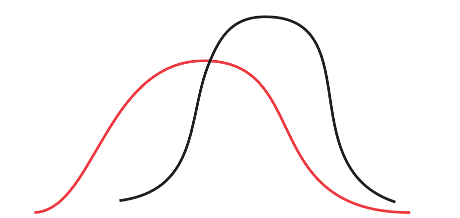
21. The mean is smaller and the standard deviation is the same.



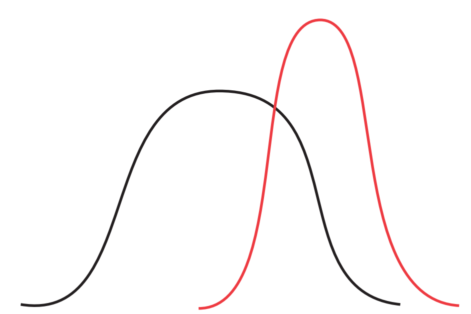
22. The mean is larger and the standard deviation is the same.



23. The mean is smaller and the standard deviation is larger.



24. The mean is larger and the standard deviation is smaller.



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The Empirical Rule for Normal Distributions

Vocabulary

Write a definition for each term in your own words.

1. standard normal distribution

   The standard normal distribution is a normal distribution with a mean value of 0 and a standard deviation of 1.

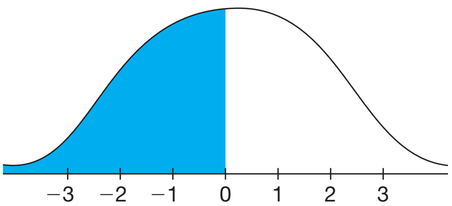
2. Empirical Rule for Normal Distributions

   The Empirical Rule for Normal Distributions states that approximately 68% of the data in a normal distribution is within one standard deviation of the mean, 95% is within two standard deviations of the mean, and 99.7% is within three standard deviations of the mean.

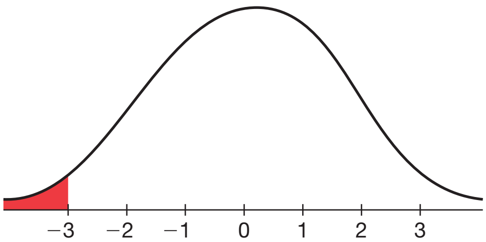
Problem Set

Shade the corresponding region under each standard normal curve.

1. The region that represents data less than the mean



2. The region that represents data less than three standard deviations below the mean



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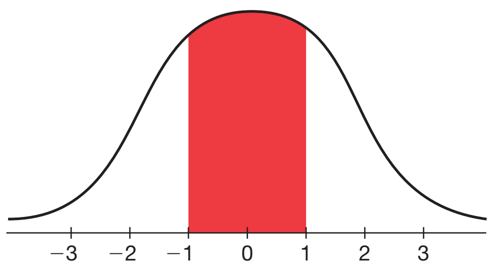
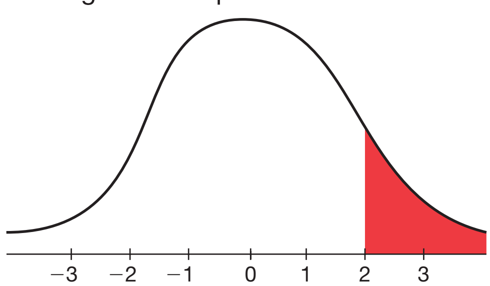
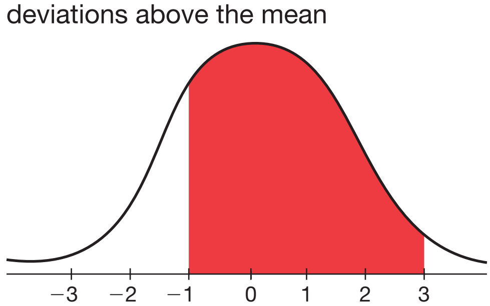
3. The region that represents data within one standard deviation of the mean



4. The region that represents data more than two standard deviations above the mean



5. The region that represents data between one standard deviation below the mean and three standard deviations above the mean



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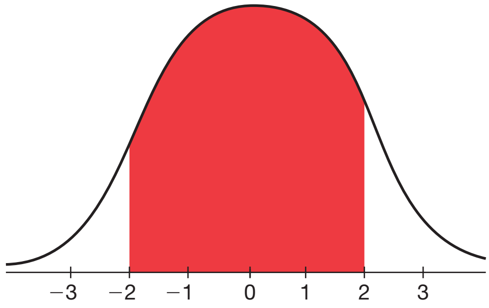
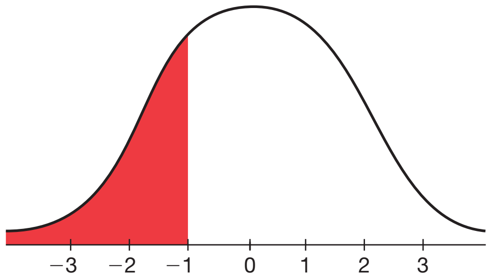
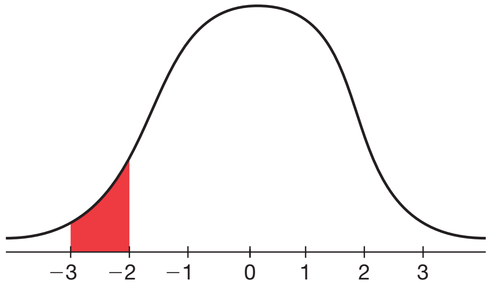
6. The region that represents data within two standard deviations of the mean



7. The region that represents data more than one standard deviation below the mean



8. The region that represents data between two and three standard deviations below the mean



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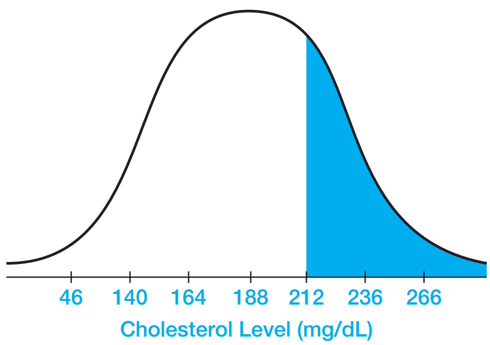
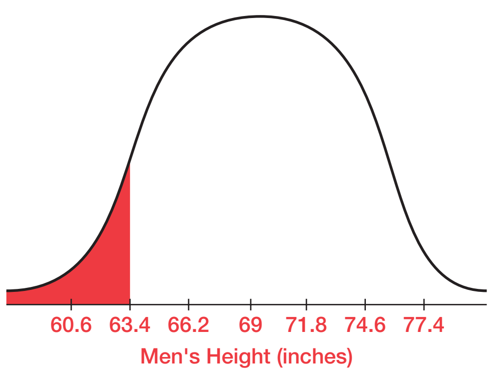
Estimate the percent of data within the specified intervals of each normal distribution. Shade the corresponding region under the normal curve, label the tick marks on the horizontal axis, and label the horizontal axis. Use the Empirical Rule for Normal Distributions.

9. Determine the percent of adult women with a cholesterol level higher than 212 mg/dL, given that the mean cholesterol level of adult women is 188 mg/dL with a standard deviation of 24 mg/dL.



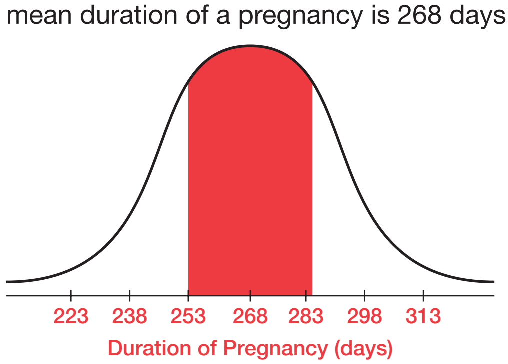
Approximately 16% of adult women have cholesterol levels higher than 212 mg/dL.

10. Determine the percent of adult men who are shorter than 63.4 inches, given that the average adult man is 69 inches tall with a standard deviation of 2.8 inches.



Approximately 2.5% of adult men are shorter than 63.4 inches.

11. Determine the percent of pregnancies with a duration between 253 and 283 days, given that the mean duration of a pregnancy is 268 days and the standard deviation is 15 days.



Approximately 68% of pregnancies last between 253 and 283 days.

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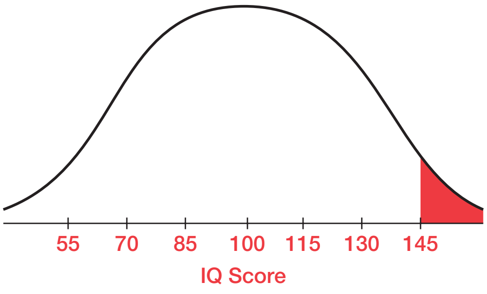
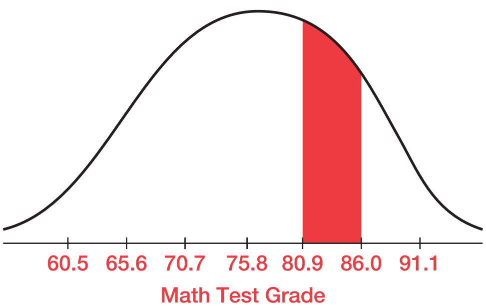
12. Determine the percent of people that have an IQ greater than 145, given that the mean IQ score is 100 and the standard deviation is 15.



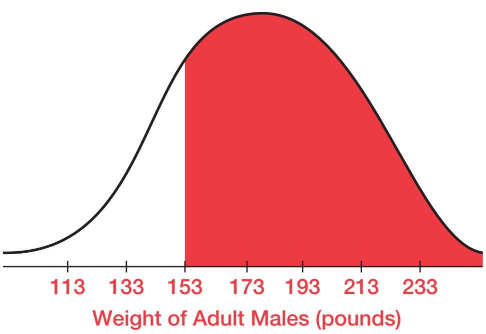
Approximately 0.15% of people have an IQ greater than 145.

13. Determine the percent of students who will get a grade between 80.9 and 86 on an upcoming math test, given that the professor's tests are normally distributed with a mean of 75.8 and a standard deviation of 5.1.



Approximately 13.5% of students will get a grade between 80.9 and 86.

14. Determine the percent of adult males who weigh more than 153 pounds, given that the mean weight for adult males is 173 pounds and the standard deviation is 20 pounds.



Approximately 84% of adult males weight more than 153 pounds.

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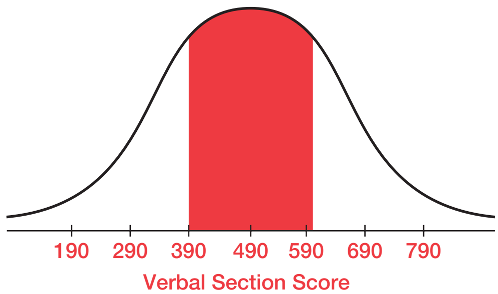
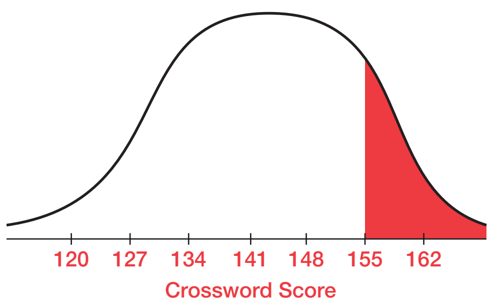
15 Determine the percent of students who score between a 390 and 590 on the verbal section of a standardized test, given that the mean score is 490 and the standard deviation is 100.



Approximately 68% of students score between a 390 and 590 on the verbal section of a standardized test.

16. Determine the percent of players who score more than 155 points in a crossword game, given that the mean score is 141 and the standard deviation is 7.



Approximately 2.5% of players score more than 155 points in the crossword game.

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Below the Line, Above the Line, and Between the Lines
Z-Scores and Percentiles

Vocabulary

Explain how the two terms below are related by identifying similarities and differences.

1. *z*-score and percentile

The *z*-score and percentile are similar in that they are both used to identify specific items on a normal distribution. They are both values on the horizontal axis of a standard normal curve or a normal curve. They are different in that the *z*-score describes a data value's distance from the mean in terms of standard deviation units, while the percentile is the data value for which a certain percent of the data is below that value.

Problem Set

Calculate each percent using a *z*-score table. The weights of bags of chips are normally distributed with a mean of 31 grams and a standard deviation of 4 grams.

1. Calculate the percent of bags that weigh more than 33 grams

Approximately 30.85% of bags of chips weigh more than 33 grams.

$$z = \frac{33 - 31}{4} = \frac{2}{4} = 0.5$$

About 69.15% of bags weigh less than 33 grams, so 100 − 69.15, or 30.85% of bags weigh more than 33 grams.

2. Calculate the percent of bags that weigh less than 24 grams

About 4.01% of bags weigh less than 24 grams.

I calculated the *z*-score and then used it to look up the percent in the *z*-score table.

$$z = \frac{24 - 31}{4} = -\frac{7}{4} = -1.75$$

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3. The percent of bags that weigh between 26.5 grams and 35.5 grams

Approximately 74.16% of bags of chips weigh between 26.5 grams and 35.5 grams.

$$z = \frac{26.5 - 31}{4} = -\frac{4.5}{4} \approx -1.13$$

$$z = \frac{35.5 - 31}{4} = \frac{4.5}{4} \approx 1.13$$

About 12.92% of bags weigh less than 26.5 grams, and about 87.08% of bags weigh less than 35.5 grams. Therefore, about 87.08 − 12.92 or 74.16% of bags weigh between 26.5 grams and 35.5 grams.

4. The percent of bags that weigh more than 40 grams

Approximately 1.22% of bags of chips weigh more than 40 grams.

$$z = \frac{40 - 31}{4} = \frac{9}{4} = 2.25$$

About 98.78% of bags weigh less than 40 grams, so 100 − 98.78 or 1.22% of bags weigh more than 40 grams.

5. The percent of bags that will be discarded because they weigh less than two standard deviations below the mean

About 2.28% of bags will be discarded because they weigh less than two standard deviations below the mean.

I calculated the *z*-score and then used it to look up the percent in the *z*-score table.

$$z = \frac{23 - 31}{4} = -\frac{8}{4} = -2.0$$

6. The percent of bags that weigh less than 37.25 grams

About 94.06% of bags weigh less than 37.25 grams.

I calculated the *z*-score and then used it to look up the percent in the *z*-score table.

$$z = \frac{37.25 - 31}{4} = \frac{6.25}{4} \approx 1.56$$

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Determine each percent using a graphing calculator. The systolic blood pressure for women is normally distributed with a mean of 120 mmHg and a standard deviation of 12mmHg.

7. Determine the percent of women with a systolic blood pressure less than 120 mmHg.

Approximately 50% of women have a systolic blood pressure less than 120 mmHg.

normalcdf (0, 120, 120, 12)

8. Determine the percent of women with a systolic blood pressure in between 125 and 140 mmHg.

Approximately 29.07% of women have a systolic blood pressure between 125 and 140 mmHg.

normalcdf (125, 140, 120, 12)

9. Determine the percent of women with a systolic blood pressure less than 93 mmHg.

Approximately 1.22% of women have a systolic blood pressure less than 93 mmHg.

normalcdf (0, 93, 120, 12)

10. If a doctor would like a woman's systolic blood pressure to be within one standard deviation of the mean, determine the percent of women who meet this criterion.

Approximately 68.27% of women have a systolic blood pressure within one standard deviation of the mean.

normalcdf (108, 132, 120, 12)

11. A woman's systolic blood pressure is considered high if it is greater than 140 mmHg. Determine the percent of women who have high systolic blood pressure.

Approximately 4.78% of women have a high systolic blood pressure.

normalcdf (140, 10000, 120, 12)

12. Determine the percent of women with a systolic blood pressure more than two standard deviations below the mean.

Approximately 2.28% of women have a systolic blood pressure more than two standard deviations below the mean.

normalcdf (0, 96, 120, 12)

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Calculate each percentile using a *z*-score table. The heights of women are normally distributed with a mean of 64 inches and a standard deviation of 2.7 inches.

13. Determine the 10th percentile for women's heights.

The 10th percentile for women's heights is approximately 60.6 inches.

The percent value in the *z*-score table that is closest to 10% is 0.1003. The *z*-score for this percent value is −1.26.

$$-1.26 = \frac{x - 64}{2.7}$$

$$-3.4 \approx x - 64$$

$$60.6 \approx x$$

14. Determine the 60th percentile for women's heights.

The 60th percentile for women's heights is approximately 64.68 inches.

The percent value in the *z*-score table that is closest to 60% is 0.5987. The *z*-score for this percent value is 0.25.

$$0.25 = \frac{x - 64}{2.7}$$

$$0.68 \approx x - 64$$

$$64.68 \approx x$$

15. Determine the height that separates the top 10% of heights from the rest.

The height that separates the top 10% from the rest is approximately 67.46 inches.

The percent value in the *z*-score table that is closest to 90% is 0.8997. The *z*-score for this percent value is 1.28.

$$1.28 = \frac{x - 64}{2.7}$$

$$3.46 \approx x - 64$$

$$67.46 \approx x$$

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Name ____________________ Date ____________

16. Determine the woman's height that would separate the shortest 1% of heights from the rest.

The woman's height that would separate the shortest 1% from the rest is approximately 57.98 inches.

The percent value in the *z*-score table that is closest to 1% is 0.0099. The *z*-score for this percent value is −2.33.

$$-2.23 = \frac{x - 64}{2.7}$$

$$-6.02 \approx x - 64$$

$$57.98 \approx x$$

17. Determine the 45th percentile for women's heights.

The woman's height that represents the 45th percentile is approximately 63.65 inches.

The percent value in the *z*-score table that is closest to 45% is 0.4483. The *z*-score for this percent value is −0.13.

$$-0.13 = \frac{x - 64}{2.7}$$

$$-0.35 \approx x - 64$$

$$63.65 \approx x$$

18. A doctor has determined that a girl will probably be in the top 4% of women's heights when she is grown. Determine the probable interval for the girl's height.

The girl will probably be taller than approximately 68.73 inches.

The percent value in the *z*-score table that is closest to 96% is 0.9599. The *z*-score for this percent value is 1.75.

$$1.75 = \frac{x - 64}{2.7}$$

$$4.73 \approx x - 64$$

$$68.73 \approx x$$

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Determine each percentile using a graphing calculator. The scores on the ACT test are normally distributed with a mean of 20.9 and a standard deviation of 4.8.

19. Determine the 30th percentile for the ACT scores.

The ACT score that represents the 30th percentile is approximately 18.4.

invnorm(0.30, 20.9, 4.8) ≈ 18.4

20. Determine the 85th percentile for the ACT scores.

The ACT score that represents the 85th percentile is approximately 25.9.

invnorm(0.85, 20.9, 4.8) ≈ 25.9

21. Determine the score separating the lowest 8% of scores from the rest.

The ACT score that separates the lowest 8% of scores from the rest is approximately 14.2.

invnorm(0.08, 20.9, 4.8) ≈ 14.2

22. Greg scored in the top 3% of ACT test scores. Determine the cutoff score for the top 3%.

The cutoff score for the top 3% is 29.9. Greg scored above a 29.9 on the ACT exam.

invnorm(0.97, 20.9, 4.8) ≈ 29.9

23. A university only considers admitting students who scored in the top 20%. Determine the cutoff score that the university uses to consider students for admission.

The university considers admitting students with scores above 24.9.

invnorm(0.80, 20.9, 4.8) ≈ 24.9

24. Determine the score that separates the top 75% of scores from the rest.

A score of 17.7 separates the top 75% of scores from the rest.

invnorm(0.25, 20.9, 4.8) ≈ 17.7

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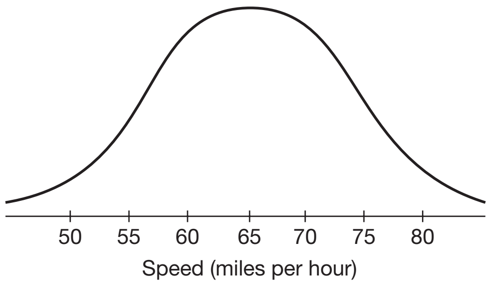
You Make the Call

Normal Distributions and Probability

Problem Set

Calculate each probability. A local police force determined that drivers' speeds on a stretch of road in the county is normally distributed with a mean of 65 miles per hour and a standard deviation of 5 miles per hour.



1. Determine the probability of randomly selecting a driver who is traveling between 60 and 65 miles per hour.

The probability of randomly selecting a driver who is traveling between 60 and 65 miles per hour is approximately 34%.

I used a graphing calculator and entered normalcdf(60, 65, 65, 5).

2. Determine the probability of randomly selecting a driver who is traveling faster than 80 miles per hour.

The probability of randomly selecting a driver who is traveling faster than 80 miles per hour is approximately 0.013%.

I used a graphing calculator and entered normalcdf(80, 1×10^{99}, 65, 5).

3. Determine the probability of randomly selecting a driver who is traveling slower than 55 miles per hour.

The probability of randomly selecting a driver who is traveling slower than 55 miles per hour is approximately 2.3%.

I used a graphing calculator and entered normalcdf(0, 55, 65, 5).

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4. Determine the probability of randomly selecting a driver who is traveling between 60 and 75 miles per hour.

   The probability of randomly selecting a driver who is traveling between 60 and 75 miles per hour is approximately 81.9%.

   I used a graphing calculator and entered normalcdf(60, 75, 65, 5).

5. Determine the probability of randomly selecting a driver who is traveling faster than 72 miles per hour.

   The probability of randomly selecting a driver who is traveling faster than 72 miles per hour is approximately 8.08%.

   I used a graphing calculator and entered normalcdf(72, 1×10^{99}, 65, 5).

6. Determine the probability of randomly selecting a driver who is traveling slower than 62 miles per hour.

   The probability of randomly selecting a driver who is traveling slower than 62 miles per hour is approximately 27.43%.

   I used a graphing calculator and entered normalcdf(0, 62, 65, 5).

Calculate each probability. The mean amount of time a customer waits in line at a local bank is 16 minutes with a standard deviation of 3.2 minutes.

7. Determine the probability that a randomly selected customer will wait in line for less than 10 minutes.

   The probability that a randomly selected customer will wait in line for less than 10 minutes is approximately 3.04%.

   I used a graphing calculator and entered normalcdf(0, 10, 16, 3.2).

8. Determine the probability that a randomly selected customer will wait in line for more than 17 minutes.

   The probability that a randomly selected customer will wait in line for more than 17 minutes is approximately 37.73%.

   I used a graphing calculator and entered normalcdf(17, 1×10^{99}, 16, 3.2).

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9. Determine the probability that a randomly selected customer will wait in line between 12.8 and 19.2 minutes.

The probability that a randomly selected customer will wait in line between 12.8 and 19.2 minutes is approximately 68.27%.

I used a graphing calculator and entered normalcdf(12.8, 19.2, 16, 3.2).

10. Determine the probability that a randomly selected customer will wait in line for less than 5 minutes.

The probability that a randomly selected customer will wait in line for less than 5 minutes is approximately 0.029%.

I used a graphing calculator and entered normalcdf(0, 5, 16, 3.2).

11. Determine the probability that a randomly selected customer will leave before being seen, given that the customer will not wait any longer than 25 minutes.

The probability that a randomly selected customer will leave is approximately 0.25%.

I used a graphing calculator and entered normalcdf(25, 1×10^{99}, 16, 3.2).

12. Determine the probability that a randomly selected customer will wait in line between 15 and 23 minutes.

The probability that a randomly selected customer will wait in line between 15 and 23 minutes is approximately 60.83%.

I used a graphing calculator and entered normalcdf(15, 23, 16, 3.2).

Grange dishwashers have a mean life expectancy of 10.5 years with a standard deviation of 0.9 years. Sparkle dishwashers have a mean life expectancy 11 years with a standard deviation of 1.3 years.

13. Determine the dishwasher that is more likely to last less than 9 years.

The Sparkle dishwasher is more likely to last less than 9 years. The probability it will last less than 9 years is 6.2%, while the probability that the Grange dishwasher will last less than 9 years is only 4.78%.

Grange: normalcdf(0, 9, 10.5, 0.9) ≈ 0.0478

Sparkle: normalcdf(0, 9, 11, 1.3) ≈ 0.062

14. Determine which dishwasher is more likely to last more than 11.5 years.

The Sparkle dishwasher is more likely to last more than 11.5 years. The probability it will last more than 11.5 years is 35.03%, while the probability that the Grange dishwasher will last more than 11.5 years is only 13.33%.

Grange: normalcdf(11.5, 1*10^99, 10.5, 0.9) ≈ 0.1333

Sparkle: normalcdf(11.5, 1*10^99, 11, 1.3) ≈ 0.3503

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15. Determine the dishwasher that is more likely to last between 9 and 12 years.

The Grange dishwasher is more likely to last between 9 and 12 years. The probability it will last between 9 and 12 years is 90.44%, while the probability that the Sparkle dishwasher will last between 9 and 12 years is only 71.72%.

Grange: normalcdf(9, 12, 10.5, 0.9) ≈ 0.9044

Sparkle: normalcdf(9, 12, 11, 1.3) ≈ 0.7172

16. Determine the dishwasher that a consumer should buy if they need the dishwasher to last at least 10 years.

A consumer should buy the Sparkle dishwasher if they need it to last at least 10 years. The probability that a Sparkle dishwasher will last at least 10 years is 77.91%, while the probability that the Grange dishwasher will last at least 10 years is only 71.07%.

Grange: normalcdf(10, 1*10^99, 10.5, 0.9) ≈ 0.7107

Sparkle: normalcdf(10, 1*10^99, 11, 1.3) ≈ 0.7791

17. Determine the dishwasher that will have the best chance of lasting at least 15 years. Do either of them have a good chance?

The Sparkle dishwasher has the best chance of lasting at least 15 years. The probability a Sparkle dishwasher will last at least 15 years is 0.105%, while the probability that the Grange dishwasher will last at least 15 years is only 0.0000287%. Both are very low percents so neither dishwasher has a very good chance of lasting at least 15 years.

Grange: normalcdf(15, 1*10^99, 10.5, 0.9) ≈ 0.000000287

Sparkle: normalcdf(15, 1*10^99, 11, 1.3) ≈ 0.00105

18. The Cleantastic dishwasher has a 20% probability of lasting between 12 and 13 years. Determine whether either the Sparkle or Grange dishwasher have a better probability.

Neither dishwasher has a better probability of lasting between 12 and 13 years than the Cleantastic dishwasher. The probability a Grange dishwasher will last between 12 and 13 years is 0.45%, while the probability that the Sparkle dishwasher will last at between 12 and 13 years is 15.89%.

Grange: normalcdf(12, 13, 10.5, 0.9) ≈ 0.045

Sparkle: normalcdf(12, 13, 11, 1.3) ≈ 0.1589

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