Name ______________________________ Date ____________

Planting the Seeds
Exploring Cubic Functions

Vocabulary

Provide an example of each key term. Sketch a graph, if necessary.

1. relative minimum:
   Answers should include either a numeric or graphic example that shows the lowest point of a particular section of a graph.

2. relative maximum:
   Answers should include either a numeric or graphic example that shows the highest point of a particular section of a graph.

3. cubic function:
   Answers should include a polynomial function of degree 3, such as $f(x) = x^3 + 2x^2 - 5x + 7$.

4. multiplicity:
   Answers should include a polynomial function with a particular number as a zero repeated 2 or 3 times, such as $f(x) = x(2x)(3x)$ where the zero, $x = 0$, has a multiplicity of 3.

Problem Set

Complete the table. Include an expression for the volume. Circle the relative maximum or minimum, if there is one.

1.

Height of Box (in.)	Width of Box (in.)	Length of Box (in.)	Volume of Box (cu. in.)
0	8	10	0
1	6	8	48
1.5	5	7	52.5
2	4	6	48
3	2	4	24
4	0	2	0
h	$8 - 2h$	$10 - 2h$	$h(8 - 2h)(10 - 2h)$

2.

Radius of Cylinder (in.)	Height of Cylinder (in.)	Base Area of Cylinder (sq. in.)	Volume of Cylinder (cu. in.)
0	0	0	0
−1	−3	3.14	−9.42
1	3	3.14	9.42
2	6	12.56	75.36
3	9	28.26	254.34
4	12	50.24	602.88
r	$3r$	$3.14r^2$	$(3r)(3.14r^2)$

There is no relative minimum or maximum.

3.

Height of Cube (cm)	Width of Cube (cm)	Length of Cube (cm)	Volume of Cube (cu. in.)
−2	−2	−2	−8
0	0	0	0
1	1	1	1
3	3	3	27
5	5	5	125
10	10	10	1000
s	s	s	$(s)(s)(s)$

There is no relative minimum or maximum.

5

4.

Width of Tank (m)	Height of Tank (m)	Length of Tank (m)	Volume of Tank (cu. m)
10	80	−20	−16,000
20	60	10	12,000
30	40	40	48,000
37	26	61	58,682
40	20	70	56,000
50	0	100	0
w	$100 - 2w$	$3w - 50$	$w(100 - 2w)(3w - 50)$

Name ____ Date ____________

5.

Height of Square Pyramid (ft)	Side of Base Length (ft)	Area of Base (sq. ft)	Volume of Square Pyramid (cu. ft)
−4	−2	4	−5.33
0	0	0	0
3	1.5	2.25	2.25
6	3	9	18
9	4.5	20.25	60.75
12	6	36	144
p	$\frac{1}{2}p$	$\frac{1}{4}p^2$	$\frac{1}{3}\left(\frac{1}{4}p^2\right)(p)$

There is no relative minimum or maximum.

6.

Length of Base (dm)	Height of Base (dm)	Length of Triangular Prism (sq. dm)	Volume of Triangular Prism (cu. dm)
−0.5	−1	−10	−2.5
0	0	−5	0
0.3	0.6	−2	−0.18
0.5	1	0	0
1	2	5	5
2	4	15	60
b	$2b$	$10b - 5$	$\frac{1}{2}(b)(2b)(10b - 5)$

Determine the product of three linear factors. Verify graphically that the expressions are equivalent.

7. $3x(x + 3)(x - 2)$

$$3x(x + 3)(x - 2) = 3x(x^2 - 2x + 3x - 6)$$
$$= 3x(x^2 + x - 6)$$
$$= 3x^3 + 3x^2 - 18x$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

8. $(2x - 1)(2x + 1)(x + 4)$

$$\begin{aligned}(2x - 1)(2x + 1)(x + 4) &= (4x^2 + 2x - 2x - 1)(x + 4)\\ &= (4x^2 - 1)(x + 4)\\ &= 4x^3 + 16x^2 - x - 4\end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

9. $(4x - 7)^3$

$$\begin{aligned}(4x - 7)^3 &= (4x - 7)(4x - 7)(4x - 7)\\ &= (16x^2 - 28x - 28x + 49)(4x - 7)\\ &= (16x^2 - 56x + 49)(4x - 7)\\ &= 64x^3 - 112x^2 - 224x^2 + 392x + 196x - 343\\ &= 64x^3 - 336x^2 + 588x - 343\end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

5

10. $(10 - 3x)(7 + x)(8 + 6x)$

$$\begin{aligned}(10 - 3x)(7 + x)(8 + 6x) &= (70 + 10x - 21x - 3x^2)(8 + 6x)\\ &= (70 - 11x - 3x^2)(8 + 6x)\\ &= 560 + 420x - 88x - 66x^2 - 24x^2 - 18x^3\\ &= -18x^3 - 90x^2 + 332x + 560\end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

Name ____________________ Date ________

11. $\left(\frac{1}{2}x\right)\left(\frac{2}{3}x\right)\left(\frac{1}{4}x - 1\right)$

$$\left(\frac{1}{2}x\right)\left(\frac{2}{3}x\right)\left(\frac{1}{4}x - 1\right) = \left(\frac{1}{3}x^2\right)\left(\frac{1}{4}x - 1\right)$$
$$= \frac{1}{12}x^3 - \frac{1}{3}x^2$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

12. $0.25x(12x - 1)(8 - 3x)$

$$0.25x(12x - 1)(8 - 3x) = (3x^2 - 0.25x)(8 - 3x)$$
$$= 24x^2 - 9x^3 - 2x + 0.75x^2$$
$$= -9x^3 + 24.75x^2 - 2x$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

Determine the product of linear and quadratic factors. Verify graphically that the expressions are equivalent.

13. $x(x^2 + 3x - 4)$

$$x(x^2 + 3x - 4) = x^3 + 3x^2 - 4x$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

14. $(2x - 9)(4x^2 - 5x - 12)$

$$(2x - 9)(4x^2 - 5x - 12) = 8x^3 - 10x^2 - 24x - 36x^2 + 45x + 108$$
$$= 8x^3 - 46x^2 + 21x + 108$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

5

15. $7x(x + 5)^2$

$$\begin{aligned} 7x(x + 5)^2 &= 7x(x + 5)(x + 5) \\ &= 7x(x^2 + 5x + 5x + 25) \\ &= 7x(x^2 + 10x + 25) \\ &= 7x^3 + 70x^2 + 175x \end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

16. $(x^2 + 1)(8 - x)$

$$\begin{aligned} (x^2 + 1)(8 - x) &= 8x^2 - x^3 + 8 - x \\ &= -x^3 + 8x^2 - x + 8 \end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

17. $(-2.3 + 1.1x + 0.9x^2)(4.5x - 3.8)$

$$\begin{aligned} (-2.3 + 1.1x + 0.9x^2)(4.5x - 3.8) &= -10.35x + 8.74 + 4.95x^2 - 4.18x + 4.05x^3 - 3.42x^2 \\ &= 4.05x^3 + 1.53x^2 - 14.53x + 8.74 \end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

5

18. $\left(-\frac{3}{4}x^2 + \frac{1}{8}\right)\left(\frac{1}{4} - \frac{7}{8}x\right)$

$$\begin{aligned} \left(-\frac{3}{4}x^2 + \frac{1}{8}\right)\left(\frac{1}{4} - \frac{7}{8}x\right) &= -\frac{3}{16}x^2 + \frac{21}{32}x^3 + \frac{1}{32} - \frac{7}{64}x \\ &= \frac{21}{32}x^3 - \frac{3}{16}x^2 - \frac{7}{64}x + \frac{1}{32} \end{aligned}$$

The graph of the original expression and the graph of the final expression are the same. So the expressions are equivalent.

Name ______________________________ Date ______________

Polynomial Power
Power Functions

Vocabulary

Choose the term from the box that best completes each statement.

even function	end behavior	symmetric about a point
power function	symmetric about a line	odd function

1. A function is symmetric about a line if the line divides the graph into two identical parts.

2. The end behavior of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity.

3. A(n) odd function has a graph symmetric about the origin, thus $f(x) = -f(-x)$.

4. A function is symmetric about a point if each point on the graph has a point the same distance from the central point but in the opposite direction.

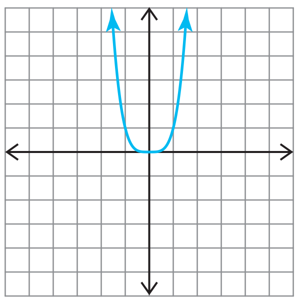
5. A(n) even function has a graph symmetric about the y-axis, thus $f(x) = f(-x)$.

6. A(n) power function is a function of the form $P(x) = ax^n$, where n is a non-negative integer.

Problem Set

Sketch the graph of $f(x)$ and describe the end behavior of each graph.

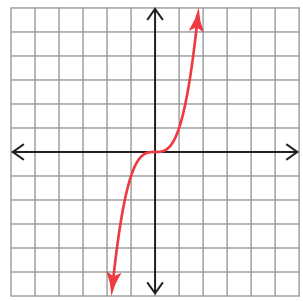
1. x^4



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

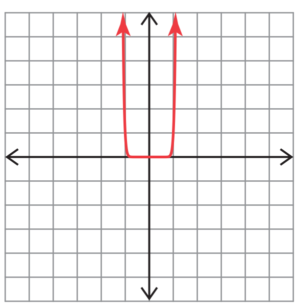
2. x^3



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

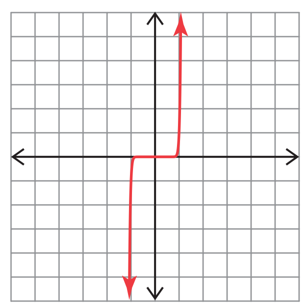
3. x^{20}



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to \infty$

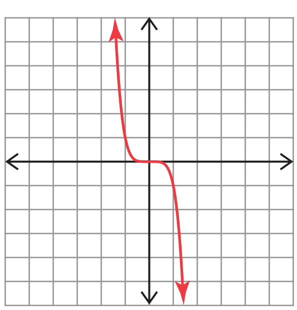
4. x^{25}



As $x \to \infty$, $f(x) \to \infty$

As $x \to -\infty$, $f(x) \to -\infty$

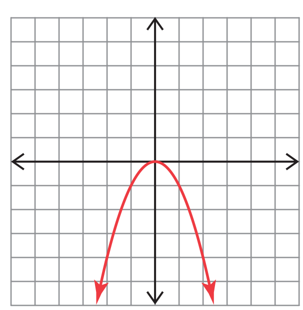
5. $-x^5$



As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

6. $-x^2$



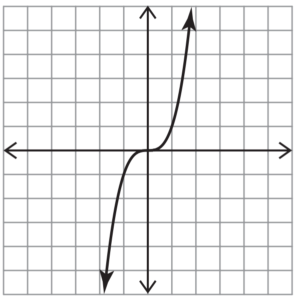
As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to -\infty$

Name ______________________________ Date ____________

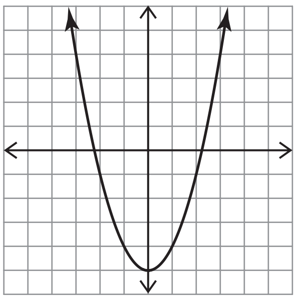
Determine whether the function represented by each graph is even, odd, or neither. Explain your reasoning.

7.



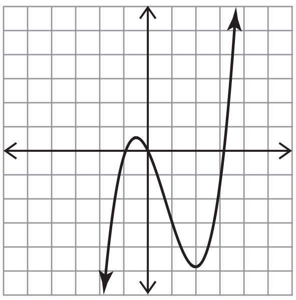
The function is odd because it is symmetrical about the origin.

8.



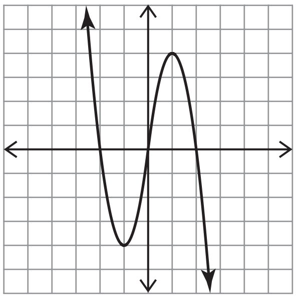
The graph is even because it is symmetrical about the *y*-axis.

9.



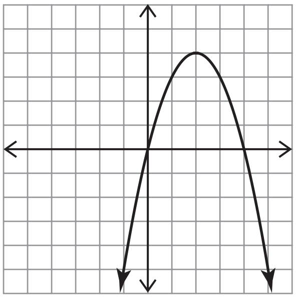
The function is neither even nor odd because it has no symmetry.

10.



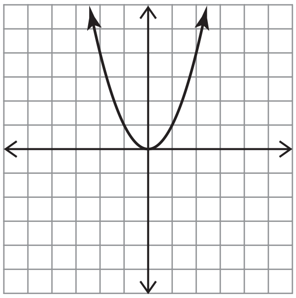
The function is odd because it is symmetrical about the origin.

11.



The function is neither even nor odd because it is not symmetrical about the *y*-axis or the origin.

12.



The function is even because it is symmetrical about the *y*-axis.

Determine algebraically whether each function is even, odd, or neither.

13. $f(x) = x^3 - 4x + 3$

$f(x) = x^3 - 4x + 3$

$f(-x) = (-x)^3 - 4(-x) + 3$

$f(-x) = -x^3 + 4x - 3$

$-f(x) = -(x^3 - 4x + 3)$

$-f(x) = -x^3 + 4x - 3$

$f(x) \neq f(-x)$ or $-f(x)$ thus $f(x)$ is neither even nor odd.

14. $f(x) = 2x^4 - x^2 + 9$

$f(x) = 2x^4 - x^2 + 9$

$f(-x) = 2(-x)^4 - (-x)^2 + 9$

$f(-x) = 2x^4 - x^2 + 9$

$f(x) = f(-x)$ thus $f(x)$ is even.

15. $f(x) = 5x^2 + 13$

$f(x) = 5x^2 + 13$

$f(-x) = 5(-x)^2 + 13$

$f(-x) = 5x^2 + 13$

$f(x) = f(-x)$ thus $f(x)$ is even.

Name ______________________________ Date ______________

16. $f(x) = 4x^3 - 6$

$f(x) = 4x^3 - 6$

$f(-x) = 4(-x)^3 - 6$

$f(-x) = -4x^3 - 6$

$-f(x) = -(4x^3 + 6)$

$-f(x) = -4x^3 - 6$

$f(-x) = -f(x)$ thus $f(x)$ is odd.

17. $f(x) = 3x^5 - x$

$f(x) = 3x^5 - x$

$f(-x) = 3(-x)^5 - (-x)$

$f(-x) = -3x^5 + x$

$-f(x) = -(3x^5 - x)$

$-f(x) = -3x^5 + x$

$f(-x) = -f(x)$ thus $f(x)$ is odd.

18. $f(x) = -2x^7 + 5x^3 + x$

$f(x) = -2x^7 + 5x^3 + x$

$f(-x) = -2(-x)^7 + 5(-x)^3 + (-x)$

$f(-x) = 2x^7 - 5x^3 - x$

$-f(x) = -(-2x^7 + 5x^3 + x)$

$-f(x) = 2x^7 - 5x^3 - x$

$f(-x) = -f(x)$ thus $f(x)$ is odd.

5

Name ______________________________ Date ____________

Function Makeover
Transformations and Symmetry of Polynomial Functions

Vocabulary

Provide an example of each term.

1. polynomial function

Answers will vary.

Examples should be in the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x^1 + a_0$

2. quartic function

Answers will vary.

Example: $f(x) = x^4 + x^3 + x^2 + x + 1$

3. quintic function

Answers will vary.

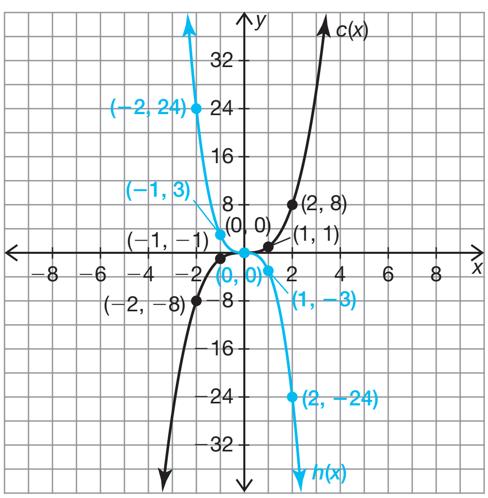
Example: $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

Problem Set

Use reference points and symmetry to complete the table of values for each function. Then, graph the function on the coordinate plane. State whether the function is odd, even, or neither.

1. $c(x) = x^3$; $h(x) = -3c(x)$

Reference Points on $c(x)$	→	Corresponding Points on $h(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	(1, −3)
(2, 8)	→	(2, −24)



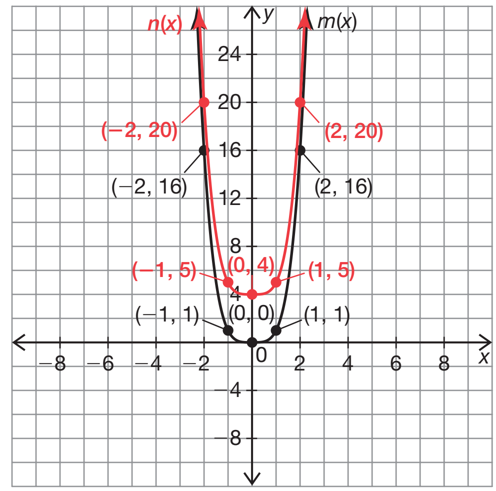
The function $h(x)$ is an odd function.

5

2. $m(x) = x^4$; $n(x) = m(x) + 4$

Reference Points on $m(x)$	$\rightarrow$	Corresponding Points on $n(x)$
(0, 0)	$\rightarrow$	(0, 4)
(1, 1)	$\rightarrow$	(1, 5)
(2, 16)	$\rightarrow$	(2, 20)

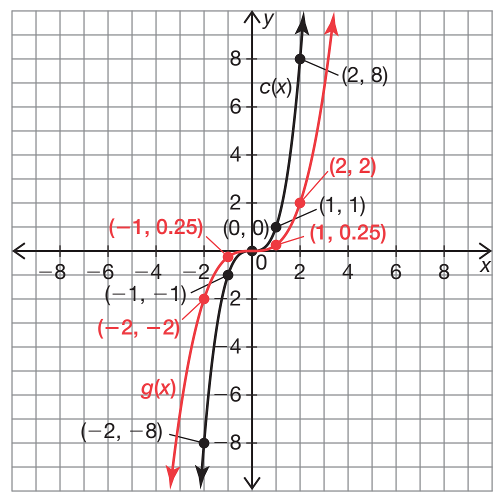
The function $n(x)$ is an even function.



3. $c(x) = x^3$; $g(x) = \frac{1}{4}c(x)$

Reference Points on $c(x)$	$\rightarrow$	Corresponding Points on $g(x)$
(0, 0)	$\rightarrow$	(0, 0)
(1, 1)	$\rightarrow$	(1, 0.25)
(2, 8)	$\rightarrow$	(2, 2)

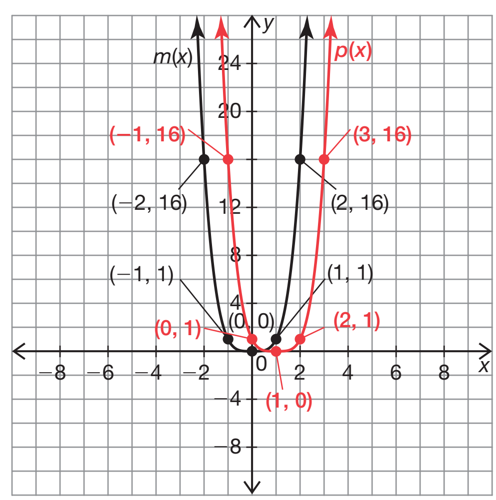
The function $g(x)$ is an odd function.



4. $m(x) = x^4$; $p(x) = m(x - 1)$

Reference Points on $m(x)$	$\rightarrow$	Corresponding Points on $p(x)$
(0, 0)	$\rightarrow$	(0, 1)
(1, 1)	$\rightarrow$	(2, 1)
(2, 16)	$\rightarrow$	(3, 16)

The function $p(x)$ is neither even nor odd.

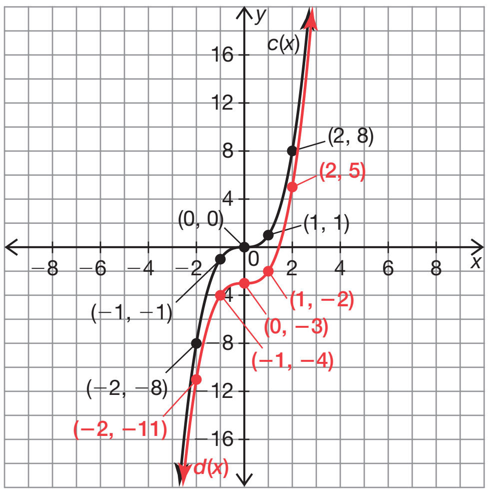


Name ____________________ Date ________

5. $c(x) = x^3$; $d(x) = c(x) - 3$

Reference Points on $c(x)$	→	Corresponding Points on $d(x)$
(0, 0)	→	(0, −3)
(1, 1)	→	(1, −2)
(2, 8)	→	(2, 5)

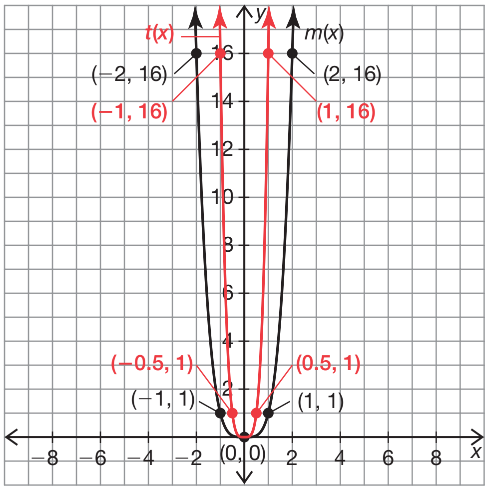
The function $d(x)$ is neither even nor odd.



6. $m(x) = x^4$; $t(x) = m(-2x)$

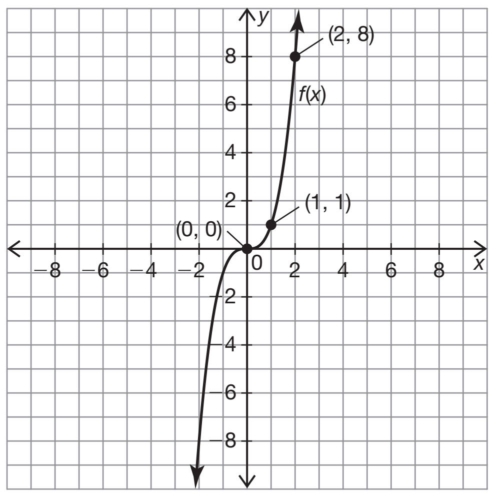
Reference Points on $m(x)$	→	Corresponding Points on $t(x)$
(0, 0)	→	(0, 0)
(1, 1)	→	$\left(-\frac{1}{2}, 1\right)$
(2, 16)	→	(−1, 16)

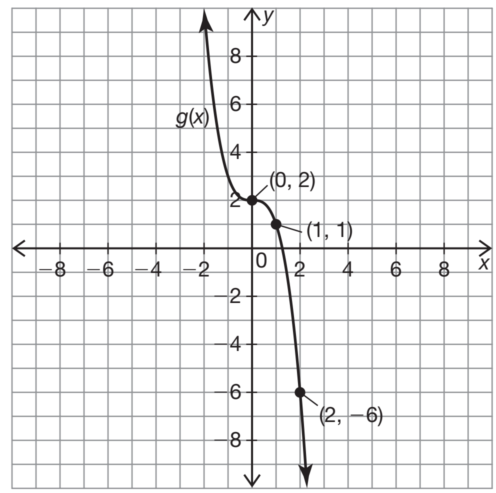
The function $t(x)$ is an even function.



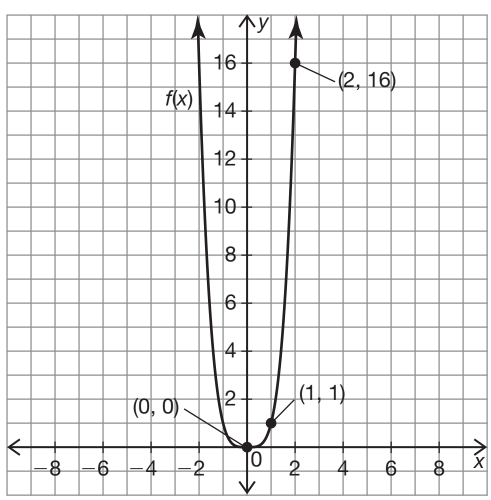
Analyze the graphs of $f(x)$ and $g(x)$. Write an equation for $g(x)$ in terms of $f(x)$.

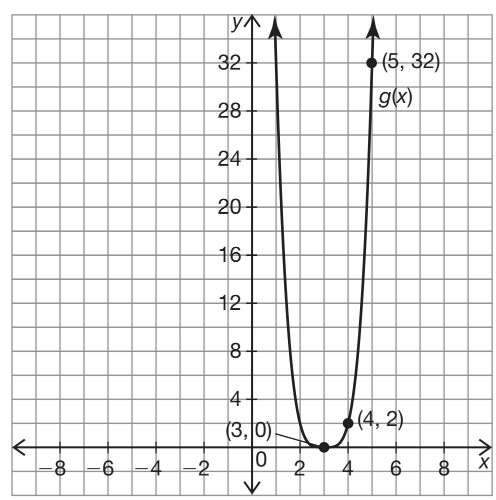
7. $g(x) = -f(x) + 2$





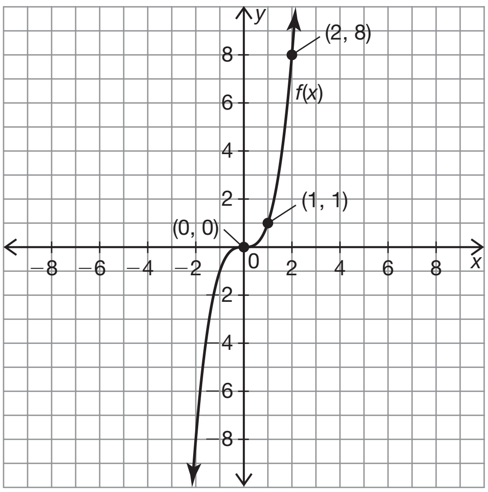
8. $g(x) = 2f(x - 3)$

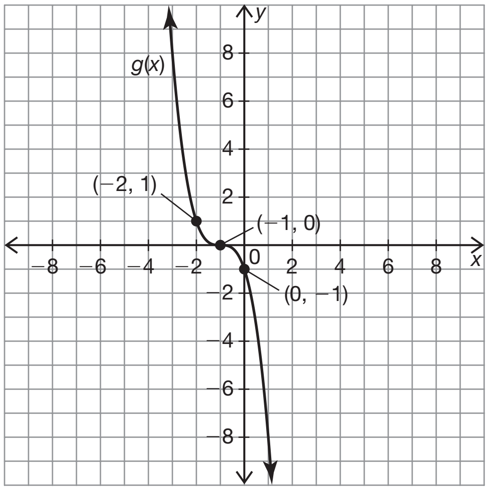




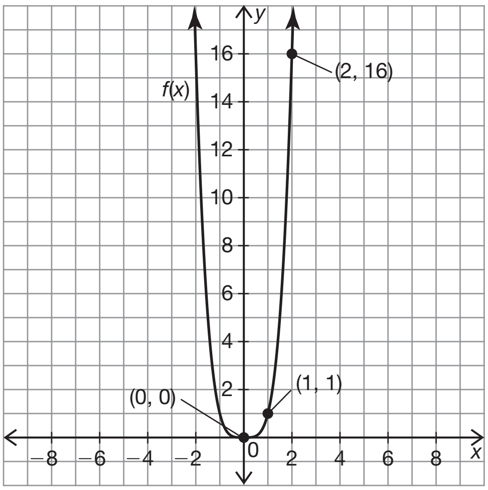
Name ____ Date ____________

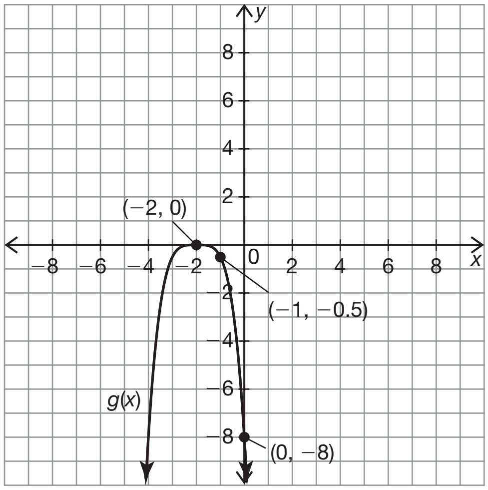
9. $g(x) = f(-x - 1)$



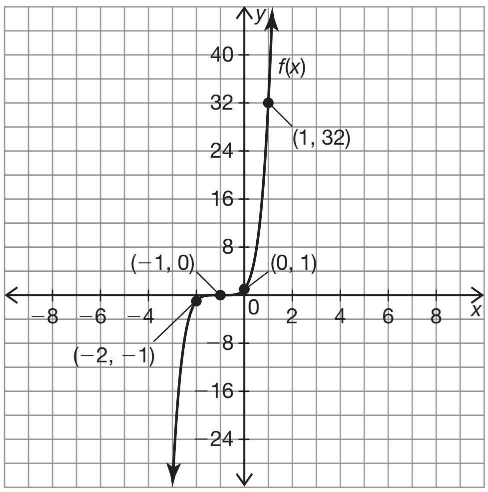


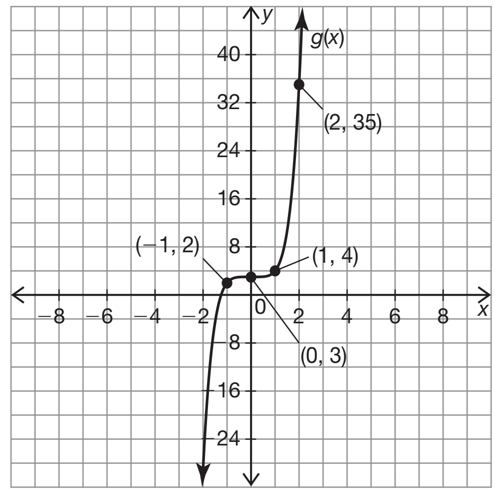
10. $g(x) = -0.5f(x + 2)$



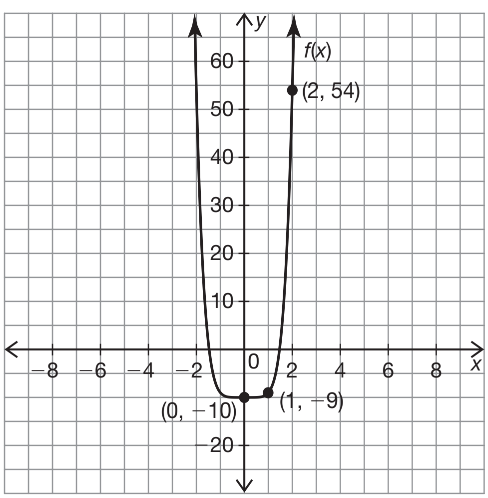


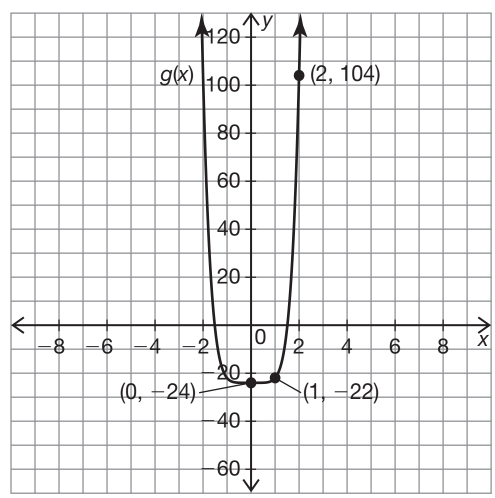
11. $g(x) = f(x - 1) + 3$





12. $g(x) = 2f(x) - 4$





Name ______________________________ Date ____________

The equation for a polynomial function $p(x)$ is given. The equation for the transformed function $m(x)$ in terms of $p(x)$ is also given. Describe the transformation(s) performed on $p(x)$ that produced $m(x)$. Then, write a specific equation for $m(x)$.

13. $p(x) = x^4$; $m(x) = -p(0.5x) + 2$

$(x, y) \rightarrow (2x, -y + 2)$

The graph of the function $m(x)$ is stretched horizontally by a factor of 2, translated 2 units up and reflected about the line $y = 2$.

$$\begin{aligned} m(x) &= -p(0.5x) + 2 \\ &= -(0.5x)^4 + 2 \end{aligned}$$

14. $p(x) = x^3$; $m(x) = 4p(x - 3) - 5$

$(x, y) \rightarrow (x + 3, 4y - 5)$

The graph of the function $m(x)$ is translated 3 units to the right and 5 units down. It has also been stretched vertically by a factor of 4.

$$\begin{aligned} m(x) &= 4p(x - 3) - 5 \\ &= 4(x - 3)^3 - 5 \\ &= 4(x^3 - 9x^2 + 27x - 27) - 5 \\ &= 4x^3 - 36x^2 + 108x - 113 \end{aligned}$$

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15. $p(x) = x^5$; $m(x) = 0.5p(-x) + 4$

$(x, y) \rightarrow (-x, 0.5y + 4)$

The graph of the function $m(x)$ is vertically compressed by a factor of 0.5, translated 4 units up and reflected about the y-axis.

$$\begin{aligned} m(x) &= 0.5p(-x) + 4 \\ &= 0.5(-x)^5 + 4 \\ &= -0.5x^5 + 4 \end{aligned}$$

16. $p(x) = x^3$; $m(x) = -p(x + 5)$

$(x, y) \to (x - 5, -y)$

The graph of the function $m(x)$ is translated 5 units to the left and reflected about the x-axis.

$$\begin{aligned} m(x) &= -p(x + 5) \\ &= -(x + 5)^3 \\ &= -\left((x^2 + 10x + 25)(x + 5)\right) \\ &= -x^3 - 15x^2 - 75x - 125 \end{aligned}$$

17. $p(x) = x^4$; $m(x) = 2p(-x -2)$

$(x, y) \to (-x + 2, 2y)$

The graph of the function $m(x)$ is reflected about the y-axis, translated 2 units to the right and vertically stretched by a factor of 2.

$$\begin{aligned} m(x) &= 2p(-x - 2) \\ &= 2(-x - 2)^4 \\ &= 2((x^2 + 4x + 4)(x^2 + 4x + 4)) \\ &= 2(x^4 + 8x^3 + 24x^2 + 32x + 16) \\ &= 2x^4 + 16x^3 + 48x^2 + 64x + 32 \end{aligned}$$

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18. $p(x) = x^5$; $m(x) = p(x + 4) - 1$

$(x, y) \to (x - 4, y - 1)$

The graph of the function $m(x)$ is translated 4 units to the left and 1 unit down.

$$\begin{aligned} m(x) &= p(x + 4) - 1 \\ &= (x + 4)^5 - 1 \\ &= (x^2 + 8x + 16)(x^2 + 8x + 16)(x + 4) \\ &= (x^2 + 8x + 16)(x^3 + 12x^2 + 48x + 64) \\ &= x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024 \end{aligned}$$

Name ______________________________ Date ______________

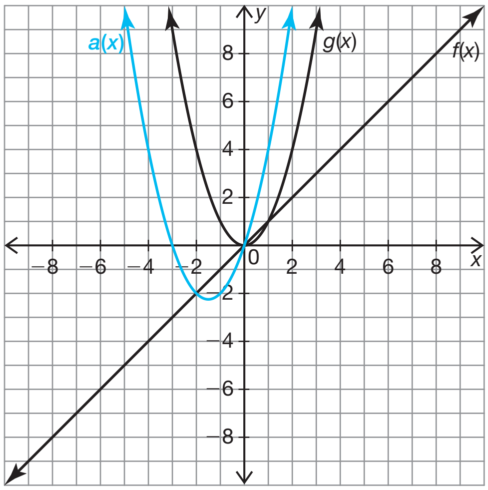
Use each basic power function shown to complete the table of values and sketch $a(x)$ on the coordinate plane. Then, write a specific equation for $a(x)$.

$f(x) = x$	$g(x) = x^2$	$h(x) = x^3$	$j(x) = x^4$	$k(x) = x^5$

19. $a(x) = g(x) + 3f(x)$

x	g(x)	f(x)	a(x)
−2	4	−2	−2
−1	1	−1	−2
0	0	0	0
1	1	1	4
2	4	2	10

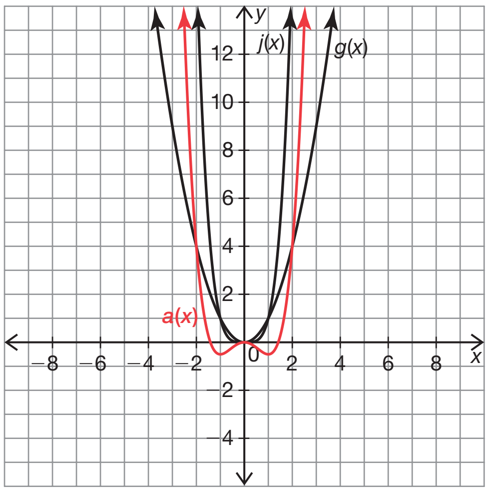
$a(x) = x^2 + 3x$



20. $a(x) = 0.5j(x) - g(x)$

x	j(x)	g(x)	a(x)
−2	16	4	4
−1	1	1	−0.5
0	0	0	0
1	1	1	−0.5
2	16	4	4

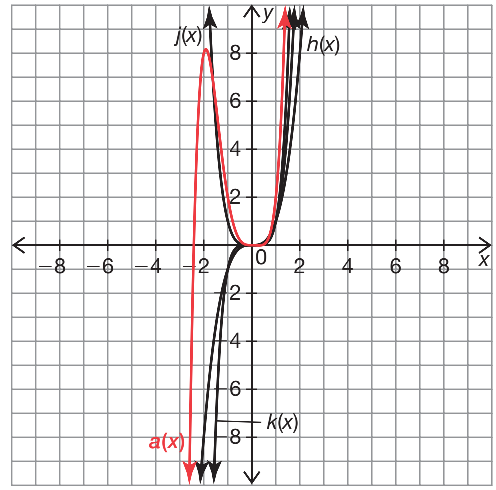
$a(x) = 0.5x^4 - x^2$



21. $a(x) = k(x) + 2j(x) - h(x)$

x	$k(x)$	$j(x)$	$h(x)$	$a(x)$
−3	−243	81	−27	−54
−2	−32	16	−8	8
−1	−1	1	−1	2
0	0	0	0	0
1	1	1	1	2
2	32	16	8	56
3	243	81	27	378

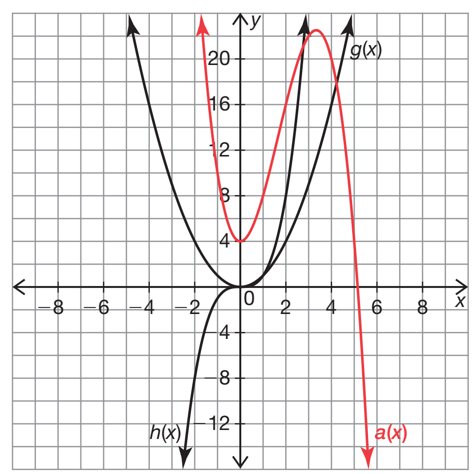
$a(x) = x^5 + 2x^4 - x^3$



22. $a(x) = -h(x) + 5g(x) + 4$

x	$h(x)$	$g(x)$	$a(x)$
−2	−8	4	32
−1	−1	1	10
0	0	0	4
1	1	1	8
2	8	4	16

$a(x) = -x^3 + 5x^2 + 4$



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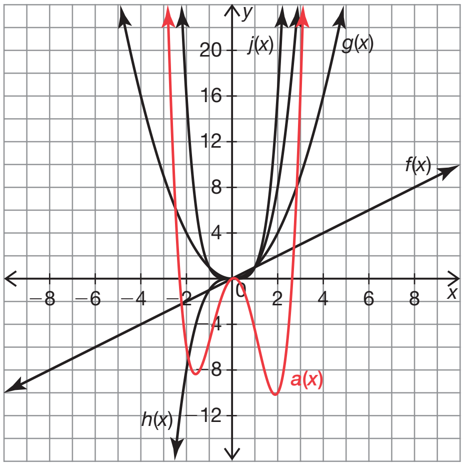
Name ____ Date _______________

23. $a(x) = j(x) - 0.5h(x) - 6g(x) + f(x)$

x	$j(x)$	$h(x)$	$g(x)$	$f(x)$	$a(x)$
−3	81	−27	9	−3	37.5
−2	16	−8	4	−2	−6
−1	1	−1	1	−1	−5.5
0	0	0	0	0	0
1	1	1	1	1	−4.5
2	16	8	4	2	−10
3	81	27	9	3	16.5

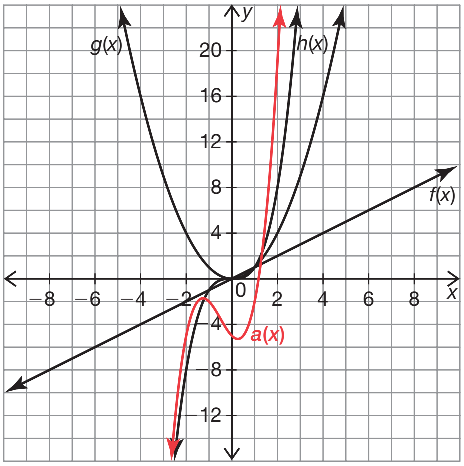
$a(x) = x^4 - 0.5x^3 - 6x^2 + x$



24. $a(x) = 2h(x) + 3g(x) - 2f(x) - 5$

x	$h(x)$	$g(x)$	$f(x)$	$a(x)$
−3	−27	9	−3	−26
−2	−8	4	−2	−5
−1	−1	1	−1	−2
0	0	0	0	−5
1	1	1	1	−2
2	8	4	2	19
3	27	9	3	70

$a(x) = 2x^3 + 3x^2 - 2x - 5$



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Name ____________________ Date ________

Polynomial DNA
Key Characteristics of Polynomial Functions

Vocabulary

Define each term in your own words.

1. absolute maximum
   The absolute maximum is the highest point of the entire graph.

2. absolute minimum
   The absolute maximum is the lowest point of the entire graph.

3. extrema
   Extrema are the set of absolute maximums, absolute minimums, relative maximums, and relative minimums.

Problem Set

List the number of possible extrema for each polynomial.

1. 3^{rd} degree polynomial
   A 3^{rd} degree polynomial can have 0 or 2 extrema.

2. 4^{th} degree polynomial
   A 4^{th} degree polynomial can have 1 or 3 extrema.

3. 8^{th} degree polynomial
   An 8^{th} degree polynomial can have 1, 3, 5, or 7 extrema.

4. 15^{th} degree polynomial
   A 15^{th} degree polynomial can have 0, 2, 4, 6, 8, 10, 12, or 14 extrema.

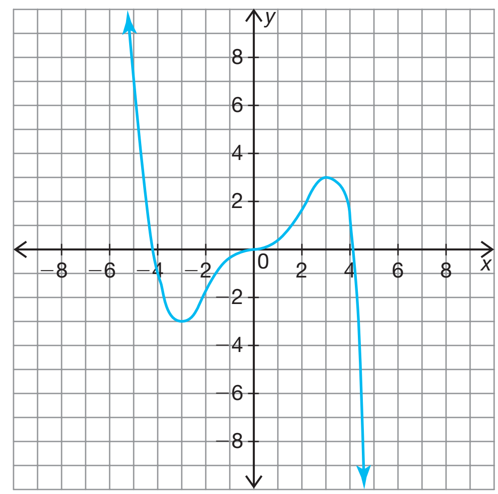
5. 20^{th} degree polynomial
   A 20^{th} degree polynomial can have 1, 3, 5, 7, 9, 11, 13, 15, 17, or 19 extrema.

6. 37^{th} degree polynomial
   A 37^{th} degree polynomial can have 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, or 36 extrema.

Use the coordinate plane to sketch a graph with the given characteristics. If the graph is not possible to sketch, explain why.

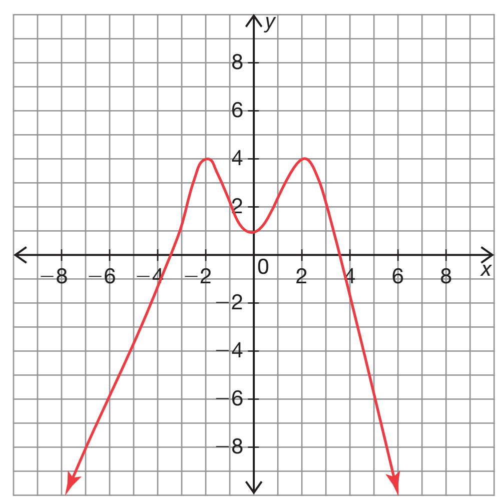
7. Characteristics:

- degree 5
- starts in quadrant I
- ends in quadrant IV
- relative maximum at $x = 3$
- relative minimum at $x = -3$



8. Characteristics:

- even degree
- increases to $x = -2$, then decreases to $x = 0$, then increases to $x = 2$, then decreases
- relative minimum at $y = 1$
- two absolute maximums at $y = 4$



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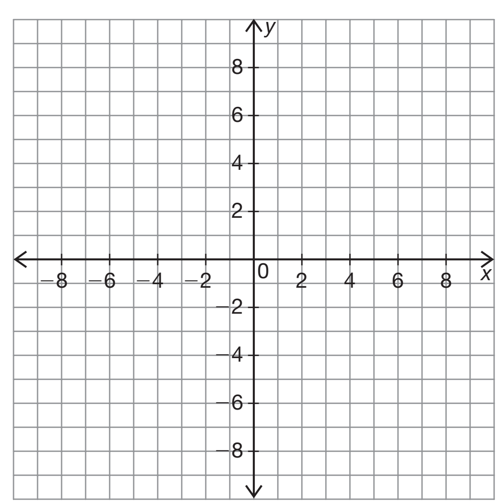
9. Characteristics:

- degree 3
- negative *a*-value
- *y*-intercept at $y = -4$
- *x*-intercepts at $x = -5$, $x = -2$, $x = 1$, and $x = 3$

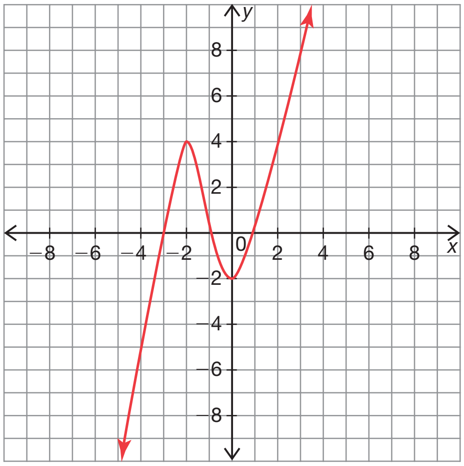
This is not possible because a third degree polynomial can have at most 3 *x*-intercepts. It cannot have 4 *x*-intercepts.



Name ____ Date ______________

10. Characteristics:

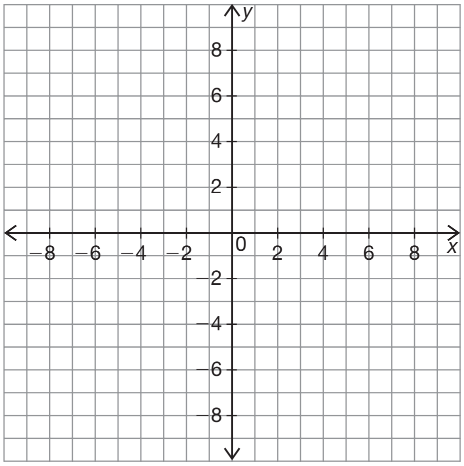
- as $x \to \infty$, $f(x) \to \infty$
  as $x \to -\infty$, $f(x) \to -\infty$
- y-intercept at $y = -2$
- three x-intercepts
- two relative extrema



11. Characteristics:

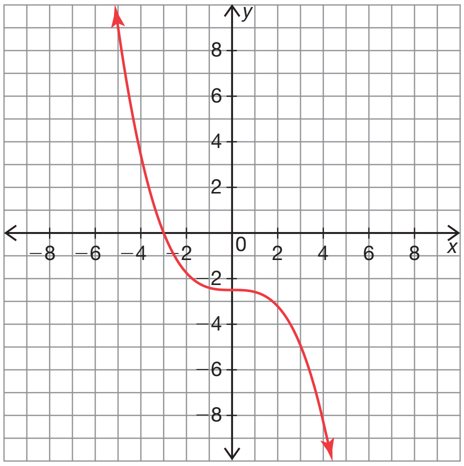
- even degree
- positive a-value
- six x-intercepts
- absolute maximum at $y = 1$
- relative minimum at $y = -4$

This is not possible because an even degree function with a positive a-value will go towards positive infinity at both ends, so it cannot have an absolute maximum.



12. Characteristics:

- always decreasing
- y-intercept at $y = -2.5$
- x-intercept at $x = -3$



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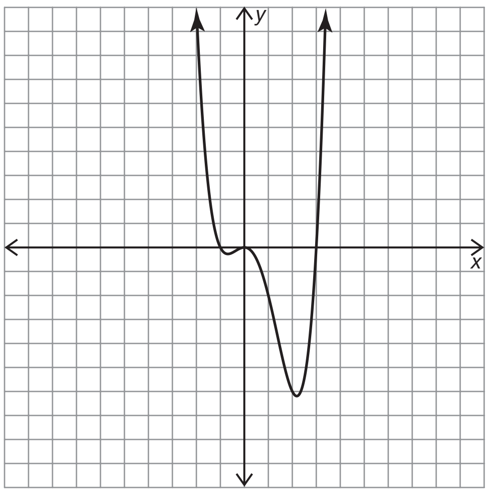
Circle the function(s) that could model each graph. Describe your reasoning for either eliminating or choosing each function.

13. 

$f(x) = x^4 - 2x^3 - 3x^2$

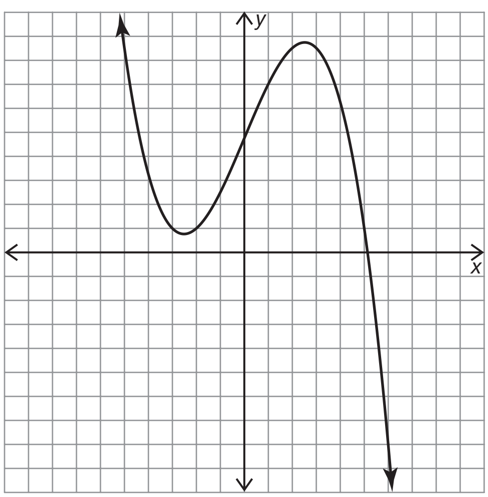
I chose this function because it represents an even degree polynomial with a positive *a*-value.

$f(x) = -2x^4 - 3x^2 - x$

I eliminated this function because the graph represents an even degree polynomial. This function has a negative *a*-value.

$f(x) = 2(x - 2)(x + 3)(x + 1)$

I eliminated this function because the graph represents an even degree function and this function is an odd degree.

14. 

$f(x) = 4x^6 + 2x^3 - 1$

I eliminated this function because the graph represents an odd degree polynomial and this function is an even degree.

$f(x) = (x + 2)(x - 5)(x + 3) + 2$

I eliminated this function because the graph represents an odd degree polynomial with a negative *a*-value and this function has a positive *a*-value.

$f(x) = -0.25(x + 2)(x - 5)(x + 3) + 2$

I chose this function because the graph represents an odd degree polynomial with a negative *a*-value.

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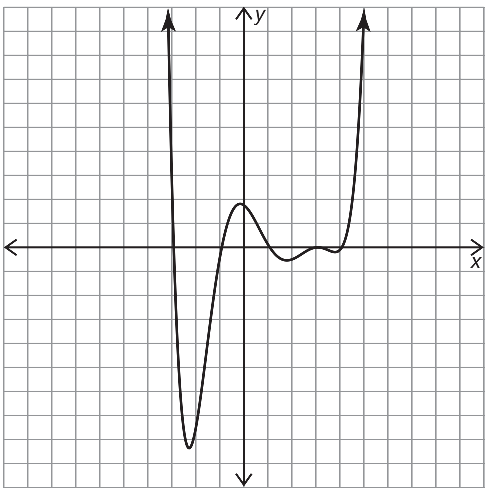
Name ____________________ Date ____________

15.



$f(x) = -2x^6 - 13x^5 + 20x$

I eliminated this function because the graph represents an even degree polynomial with a positive *a*-value and this function has a negative *a*-value.

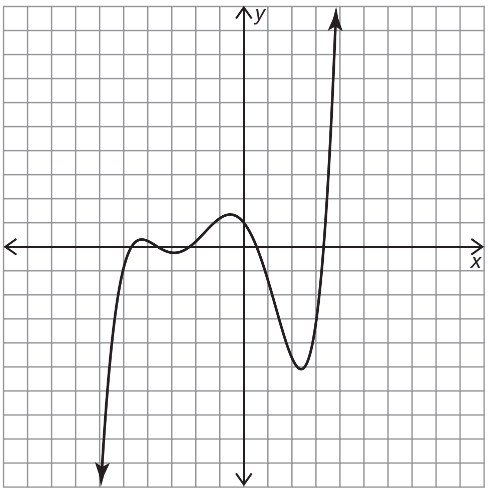
$f(x) = 2x^6 - 13x^5 + 26x^4 - 7x^3 - 28x^2 + 20x$

I chose this function because it represents an even function with a positive *a*-value.

$f(x) = 2x(x + 7)(x - 4)(x + 3)(x - 2) - 3$

I eliminated this function because the graph represents an even degree polynomial with 5 extrema and this function is degree 5 which cannot have more than 4 extrema.

16.



$f(x) = 3x^5 + 20x^4 - 10x^3 - 240x^2 - 250x + 200$

I chose this function because it represents an odd function with a positive *a*-value.

$f(x) = (2x - 3)(x + 4)(x - 10)(x + 14) + 20$

I eliminated this function because the graph represents an odd function and this function is an even degree.

$f(x) = -3x^7 + 15x^6 - 20x^2 + 125x - 150$

I eliminated this function because the graph represents an odd function with a positive *a*-value and this function has a negative *a*-value.

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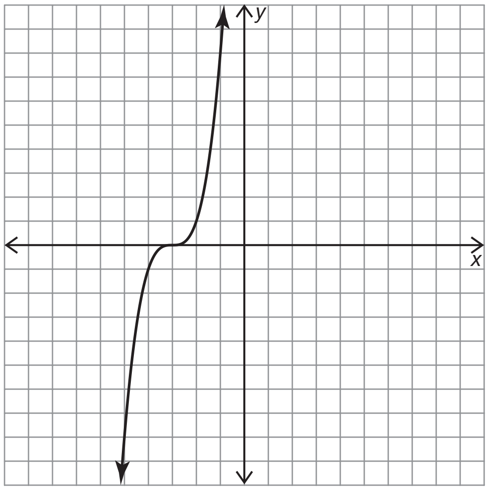
17. 

$f(x) = -x^3 + 2x^2 - x + 3$

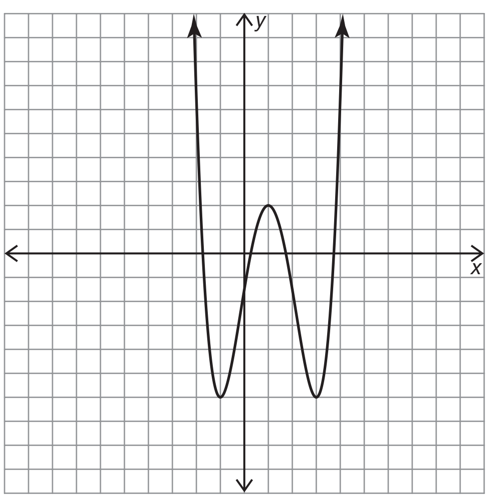
I eliminated this function because the graph represents an odd function with a positive *a*-value and this function has a negative *a*-value.

$f(x) = \frac{1}{2}x(x + 3)^3$

I eliminated this function because the graph represents an odd function and this function is an even degree.

$f(x) = (x + 3)^3$

I chose this function because it represents an odd function with a positive *a*-value.

18. 

$f(x) = x^4 - 4x^3 - 2x^2 + 12x - 3$

I chose this function because it represents an even function greater than degree 2 with a positive *a*-value.

$f(x) = 2(x + 3)(x + 4)$

I eliminated this function because the graph represents a function with 4 roots and this function only has 2.

$f(x) = -2x^5 + x^4 - 3x^3 + 12$

I eliminated this function because the graph represents an even function with a positive *a*-value and this function is odd degree with a negative *a*-value.

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Name ______________________________ Date ____________

That Graph Looks a Little Sketchy

Building Cubic and Quartic Functions

Problem Set

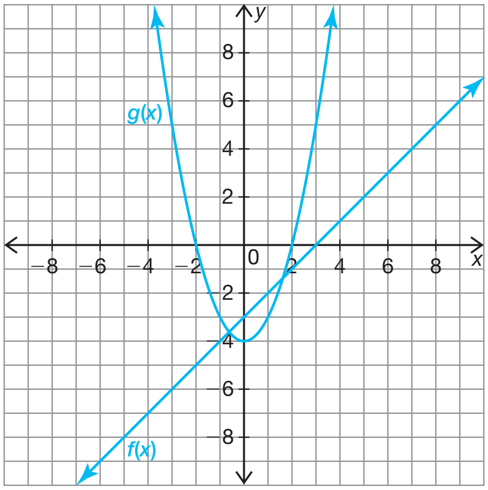
Sketch a set of functions whose product builds a cubic function with the given characteristics. Explain your reasoning.

1. zeros: $x = -2$, $x = 2$, and $x = 3$

The graphs can be three linear functions or a linear function and a quadratic function. The following functions representing one possible solution are shown.

$f(x) = x - 3$

$g(x) = x^2 - 4$

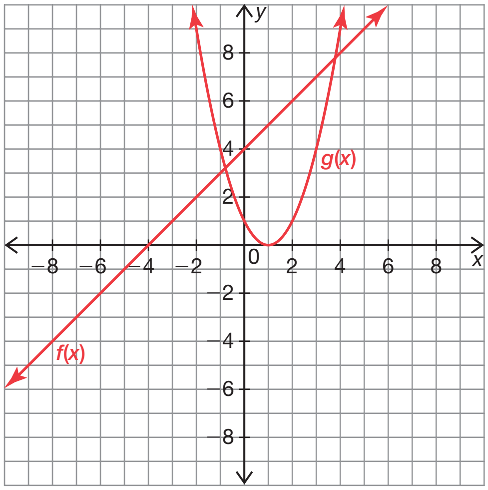


2. zeros: $x = 1$ (multiplicity 2) and $x = -4$

The graphs can be three linear functions, two of which have an x-intercept at $x = 1$. It can also be a linear function with an x-intercept at $x = -4$ and a quadratic function with a vertex at $x = 1$. The following functions representing one possible solution are shown.

$f(x) = x + 4$

$g(x) = (x - 1)^2$



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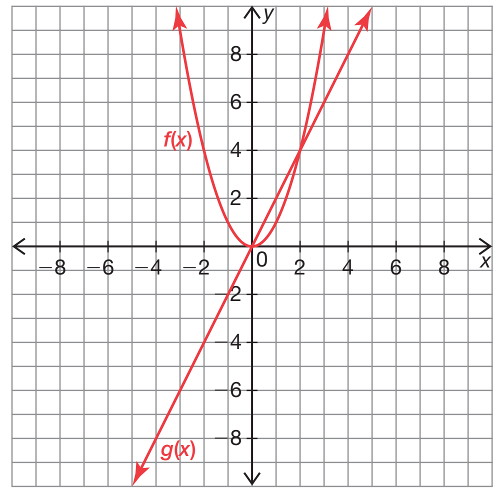
3. The cubic function is in Quadrants I and III only

The cubic function will increase from left to right, so the sketch may show either all positive functions or two negative. All x-intercepts are at 0 because the cubic function needs to pass through the origin. The following functions representing one possible solution are shown.

$f(x) = (-x)^2$

$g(x) = 2x$



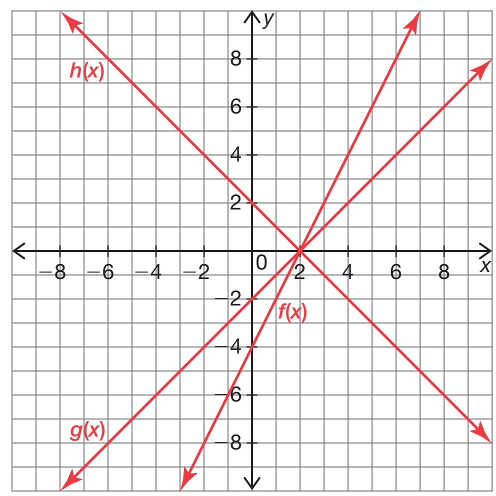
4. zero: $x = 2$ (multiplicity 3)

The graphs can be either 3 linear functions or a linear function and a quadratic function. The x-intercepts must all be $x = 2$. The following functions representing one possible solution are shown.

$f(x) = 2x - 4$

$g(x) = x - 2$

$h(x) = -x + 2$



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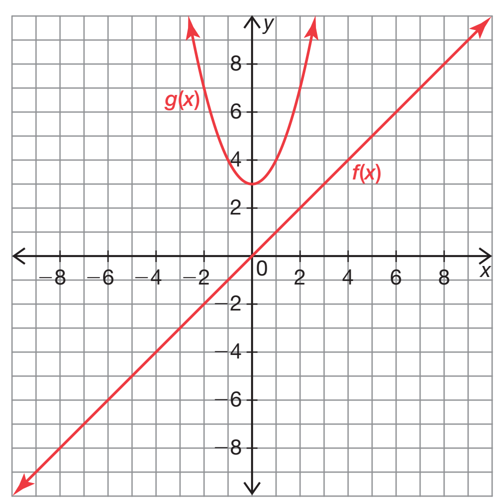
5. two imaginary zeros and a real zero $x = 0$

The graph must be a linear function with a real root $x = 0$ and a quadratic function above the x-axis because the roots must be imaginary. The following functions representing one possible solution are shown.

$f(x) = x$

$g(x) = x^2 + 3$



Name ____________________ Date ____________

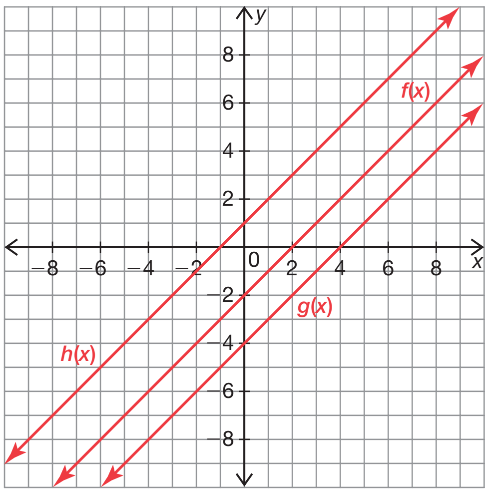
6. y-intercept of $(0, 8)$

The graphs can be either 3 linear functions or a linear function and a quadratic function. The product of constant terms must be 8. This guarantees that the cubic function will have a y-intercept of $(0, 8)$. The following functions representing one possible solution are shown.

$f(x) = x - 2$

$g(x) = x - 4$

$h(x) = x + 1$



Write two different cubic functions (as a product of functions) with the given characteristics.

7. zeros: $x = -3$, $x = 0$, $x = 1$

Two possible correct answers: $f(x) = x(x + 3)(x - 1)$ and $g(x) = 3x(2x + 6)(4x - 4)$

8. zeros: $x = 0$, $x = -3i$, $x = 3i$

Two possible correct answers: $f(x) = 4x(x^2 + 9)$ and $g(x) = x(3x^2 + 27)$

9. zeros: $x = 4$ (multiplicity 2) and $x = -2$

Two possible correct answers: $f(x) = (x - 4)^2(x + 2)$ and $g(x) = 7(6x - 24)^2(5x + 10)$

10. for an input value of 2, an output of 24

Two possible correct answers: $f(x) = (x)(x)(x + 4)$ and $g(x) = (x)(2x)\left(\frac{3}{2}x\right)$

11. zeros: $x = -5$, $x = -1$, $x = 2$ and a y-intercept of $(0, -20)$

Two possible correct answers: $f(x) = 2(x + 5)(x + 1)(x - 2)$ and $g(x) = (-2x - 10)(x + 1)(-x + 2)$

12. zero: $x = 2$ (multiplicity 3)

Two possible correct answers: $f(x) = (x - 2)(x - 2)(x - 2)$ and $g(x) = (2x - 4)(-x + 2)(3x - 6)$

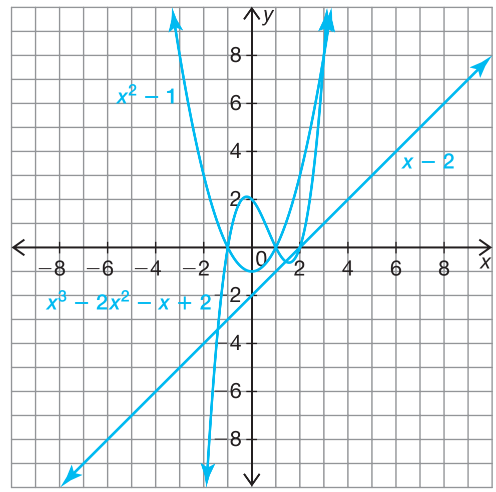
5

Sketch the graph of the cubic function that is the product of the functions shown. Then, determine the product of the functions algebraically. Verify your sketch by graphing the product on a graphing calculator.

13. $(x - 2)(x^2 - 1)$

$(x - 2)(x^2 - 1) = x^3 - x - 2x^2 + 2$

$= x^3 - 2x^2 - x + 2$



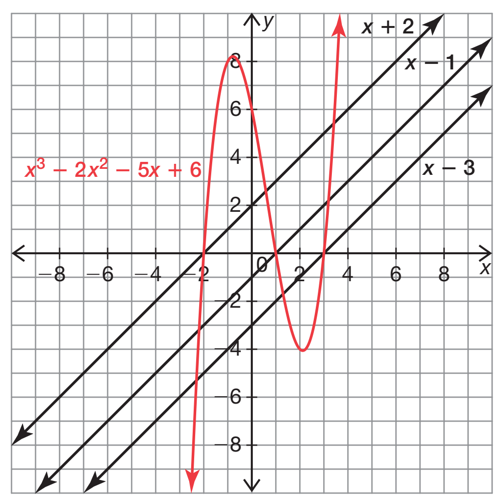
14. $(x + 2)(x - 1)(x - 3)$

$(x + 2)(x - 1)(x - 3) = (x^2 - x + 2x - 2)(x - 3)$

$= (x^2 + x - 2)(x - 3)$

$= x^3 - 3x^2 + x^2 - 3x - 2x + 6$

$= x^3 - 2x^2 - 5x + 6$



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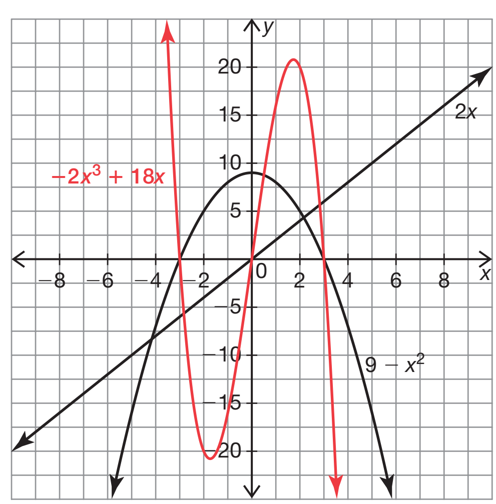
15. $2x(9 - x^2)$

$2x(9 - x^2) = 18x - 2x^3$

$= -2x^3 + 18x$



Name ______________________________ Date ____________

16. $(4 - x)(-x + 2)(3x - 6)$

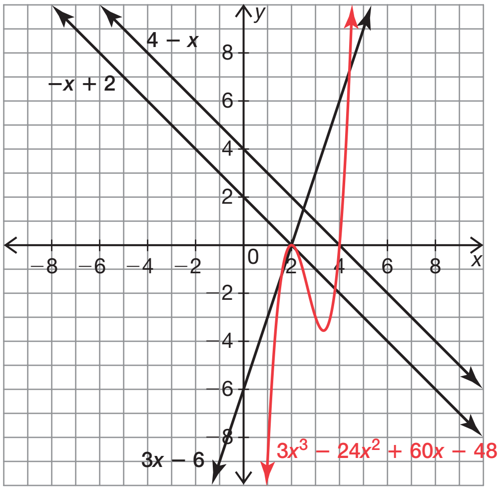
$(4 - x)(-x + 2)(3x - 6)$

$= (-4x + 8 + x^2 - 2x)(3x - 6)$

$= (x^2 - 6x + 8)(3x - 6)$

$= 3x^3 - 6x^2 - 18x^2 + 36x + 24x - 48$

$= 3x^3 - 24x^2 + 60x - 48$



17. $(x + 3)(1 - 4x)(x - 3)$

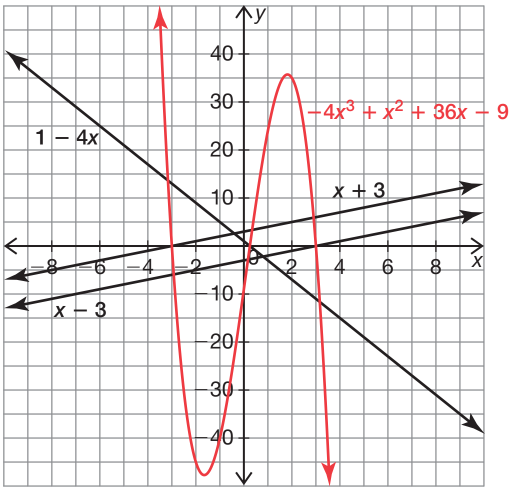
$(x + 3)(1 - 4x)(x - 3)$

$= (x - 4x^2 + 3 - 12x)(x - 3)$

$= (-4x^2 - 11x + 3)(x - 3)$

$= -4x^3 + 12x^2 - 11x^2 + 33x + 3x - 9$

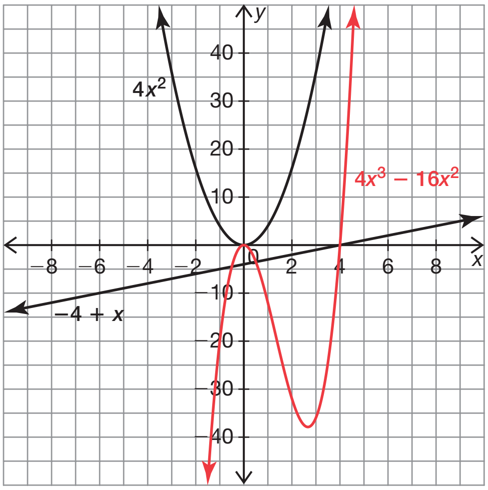
$= -4x^3 + x^2 + 36x - 9$



18. $(-4 + x)(4x^2)$

$(-4 + x)(4x^2) = -16x^2 + 4x^3$

$= 4x^3 - 16x^2$



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Sketch a set of functions whose product builds a quartic function with the given characteristics. Explain your reasoning.

19. four distinct roots and y-intercept of (0, 6)

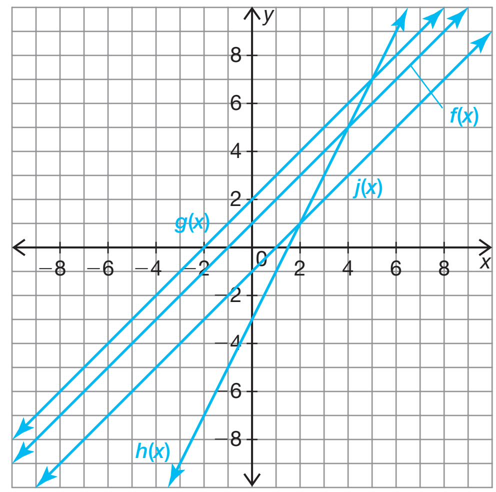
The graphs can be any functions as long as the product of the constant terms is 6. The following functions representing one possible solution are shown.

$f(x) = x + 1$

$g(x) = x + 2$

$h(x) = 2x - 3$

$j(x) = x - 1$



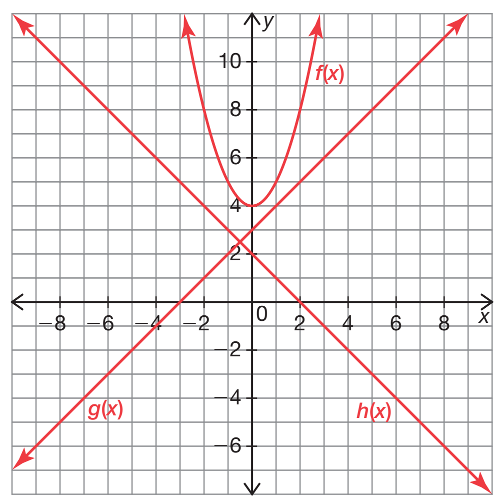
20. two imaginary roots and zeros $x = 2$ and $x = -3$

The imaginary roots come from a quadratic function above or below the x-axis without an x-intercept. The other zeros will come from two linear functions. The following functions representing one possible solution are shown.

$f(x) = x^2 + 4$

$g(x) = x + 3$

$h(x) = -x + 2$

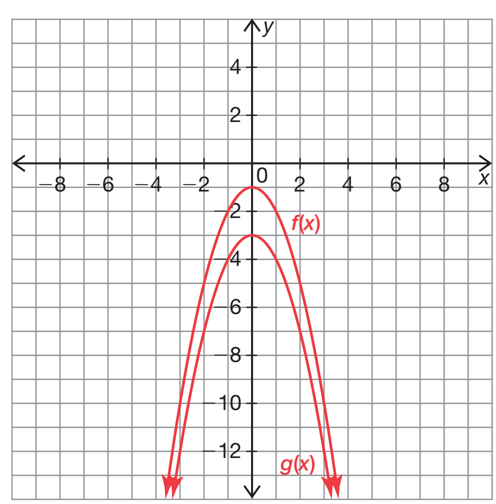


21. located in Quadrants I and II only

In order for the quartic function to be in quadrants I and II only, the product of the outputs must be positive. The following functions representing one possible solution are shown.

$f(x) = -x^2 - 1$

$g(x) = -x^2 - 3$



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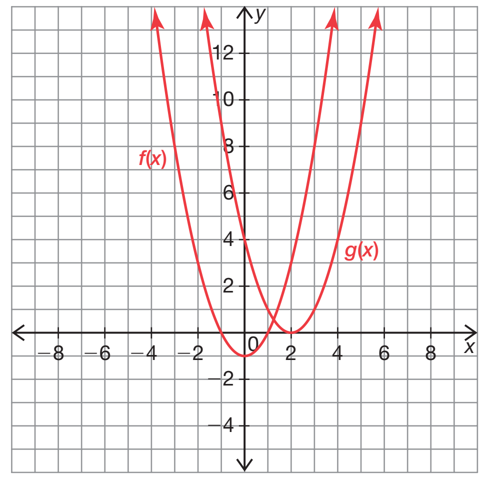
Name ______________________________ Date ____________

22. zeros: $x = -1$, $x = 2$ (multiplicity 2), and $x = 1$

The graphs can be four linear functions or a quadratic function with a vertex at $x = 2$ and a quadratic function with roots of 1 and -1. The following functions representing one possible solution are shown.

$f(x) = x^2 - 1$

$g(x) = (x - 2)^2$

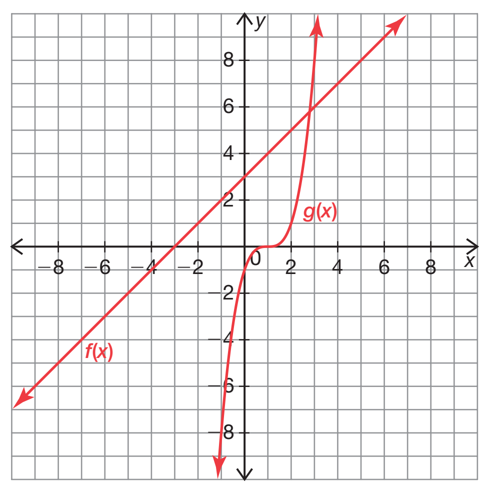


23. zeros: $x = -3$ and $x = 1$ (multiplicity 3)

The graphs can be various combinations of linear, quadratic and cubic functions as long as three roots equal 1 and one root equals -3. The following functions representing one possible solution are shown.

$f(x) = x + 3$

$g(x) = (x + 3)^3$

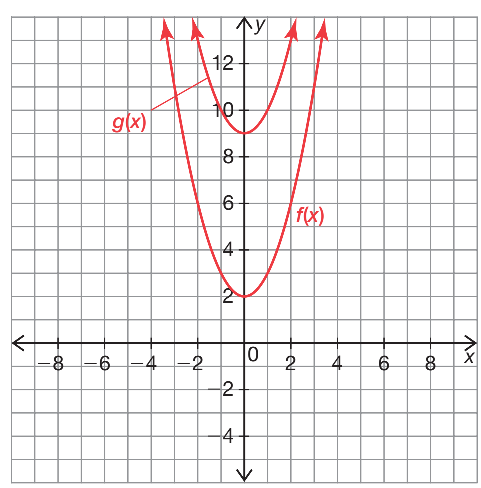


24. four imaginary roots

To get four imaginary roots, no function can have an x-intercept. The following functions representing one possible solution are shown.

$f(x) = x^2 + 2$

$g(x) = x^2 + 9$



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Name ______________________________ Date ____________

Closing Time

The Closure Property

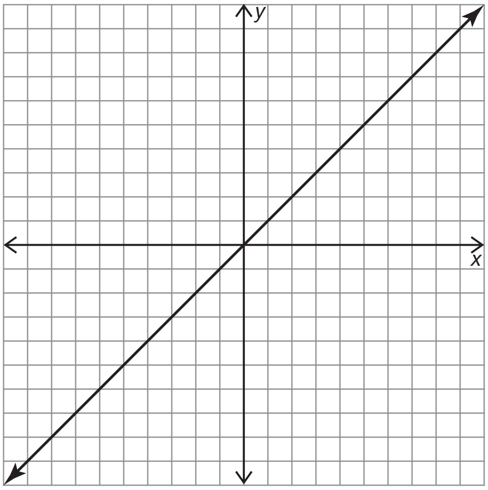
Vocabulary

1. Describe in your own words what it means for a set of numbers or expressions to be *closed* under an operation.

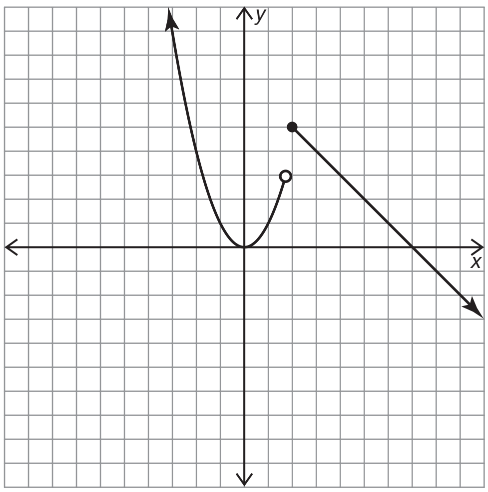
   When an operation is performed on any number or expression in a set and the result is in the same set, it is said to be closed under that operation.

Problem Set

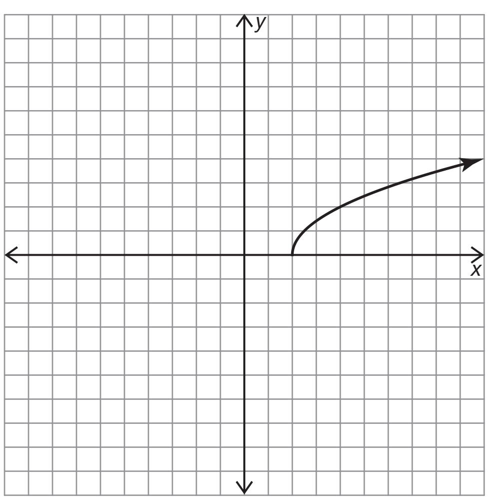
Determine whether each graph represents a polynomial function. Explain your reasoning.

1. 

   The graph represents a polynomial function because it is continuous and increases to infinity as x approaches infinity and decreases to infinity as x approaches negative infinity. This is the graph of a linear function.

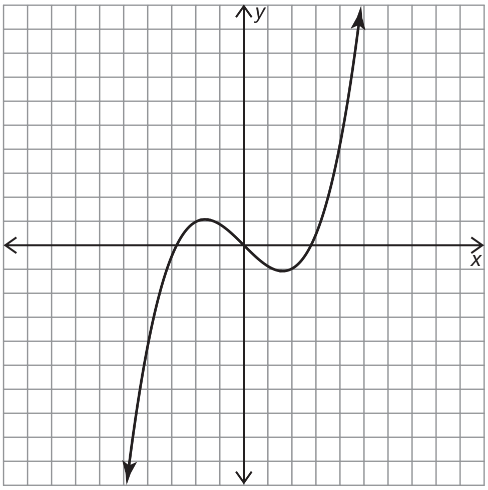
2. 

The graph does not represent a polynomial function because it is not continuous.

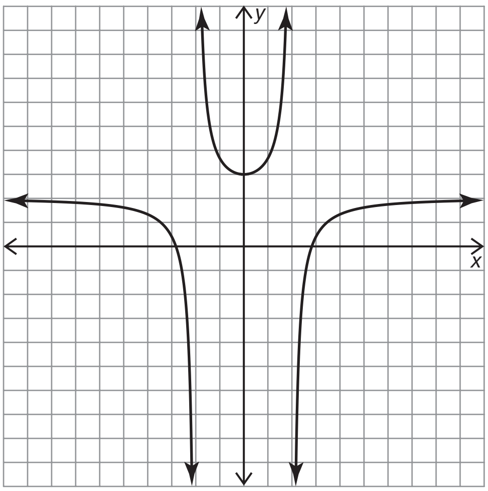
3. 

The graph does not represent a polynomial function because of the end behavior: x stops at $x = 2$ and does not continue towards negative infinity.

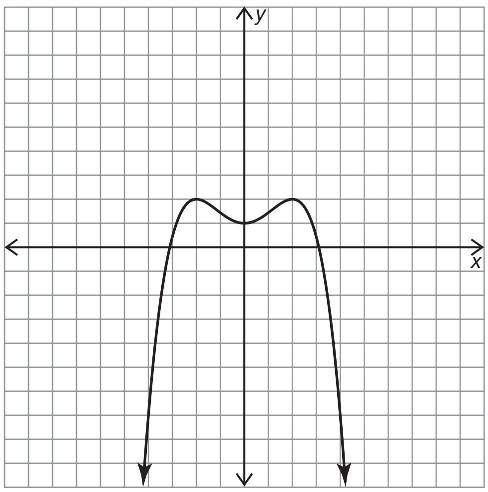
Name ____________________ Date ____________

4. 

The graph represents a polynomial function because it is smooth, continuous, and increases to infinity as x approaches infinity and decreases to infinity as x approaches negative infinity. This is likely a cubic function.

5. 

The graph does not represent a polynomial function because it is not continuous.

6. 

This graph represents a polynomial function because it is smooth, continuous, and decreases to negative infinity as x approaches infinity and decreases to negative infinity as x approaches negative infinity. This is likely a quartic function.

Determine whether each set is closed under the indicated operation. Then, write an example to support your answer.

7. Is the set {even integers} closed under subtraction?

Yes, the set of even integers is closed under subtraction.

Example: $6 - 10 = -4$

Name ______________________________ Date ____________

8. Is the set {odd whole numbers} closed under addition?

No, the set of odd whole numbers is not closed under addition.

Example: $3 + 5 = 8$

9. Is the set {1, 2, 3, 4, 5, 6} closed under multiplication?

No, this set of natural numbers is not closed under multiplication.

Example: $4 \cdot 6 = 24$

10. Is the set {rational numbers} closed under multiplication?

Yes, the set of rational numbers is closed under multiplication.

Example: $\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

11. Is the set {irrational numbers} closed under division?

No, the set of irrational numbers is not closed under division.

Example: $\frac{\sqrt{5}}{\sqrt{5}} = 1$

12. Is the set $\{2\sqrt{3}, 3\sqrt{3}, 5\sqrt{3}\}$ closed under addition?

No, this set of irrational numbers is not closed under addition.

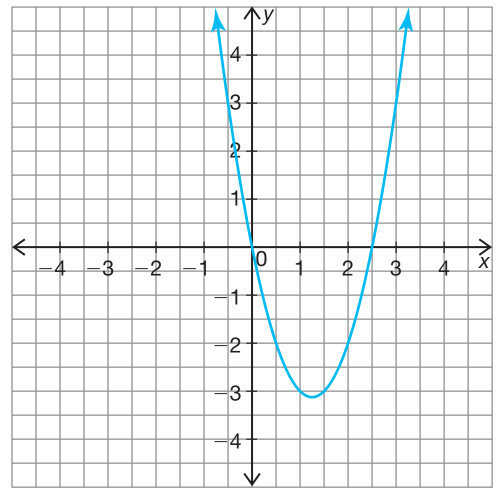
Example: $3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$

Add, subtract, multiply, or divide each set of polynomials to show whether they are closed under the indicated operation. Use a graphing calculator to sketch the graph of the resulting polynomial to verify your answer.

13. Are the polynomials $y_1 = 3x^2 - 5x - 3$ and $y_2 = -x^2 + 3$ closed under addition?

$$\begin{array}{r} 3x^2 - 5x - 3 \\ \underline{+\ -x^2 \qquad\quad + 3} \\ 2x^2 - 5x \qquad \end{array}$$

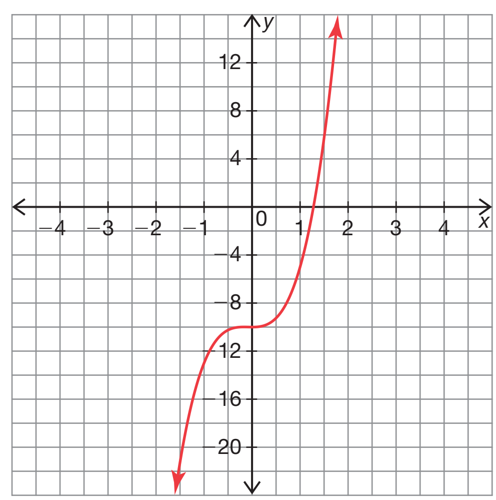
Polynomials are closed under addition because the sum is a polynomial.



14. Are the polynomials $y_1 = x^3 + 3x^2 - 4x + 12$ and $y_2 = -3x^3 + 2x^2 - 4x + 22$ closed under subtraction?

$$\begin{array}{r} x^3 + 3x^2 - 4x + 12 \\ \underline{-(-3x^3 + 2x^2 - 4x + 22)} \\ 4x^3 + \ x^2 \qquad\quad - 10 \end{array}$$

Polynomials are closed under subtraction because the difference is a polynomial.



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Name ______________________________ Date ____________

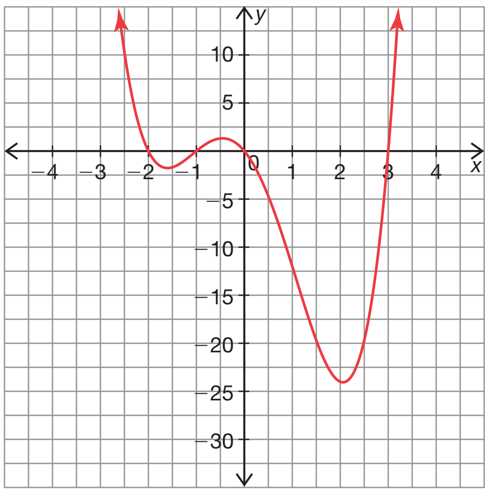
15. Are the polynomials $y_1 = x^2 + 3x + 2$ and $y_2 = x^2 - 3x$ closed under multiplication?

$(x^2 + 3x + 2)(x^2 - 3x)$

$= x^4 - 3x^3 + 3x^3 - 9x^2 + 2x^2 - 6x$

$= x^4 - 7x^2 - 6x$

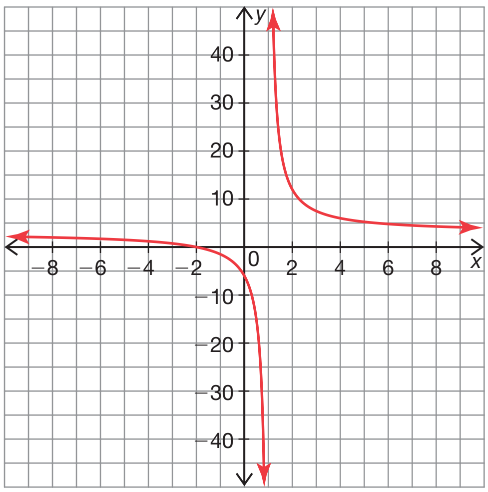
Polynomials are closed under multiplication because the product is a polynomial.



16. Are the polynomials $y_1 = 3x + 6$ and $y_2 = x - 1$ closed under division?

$\frac{3x + 6}{x - 1} = 3R9$

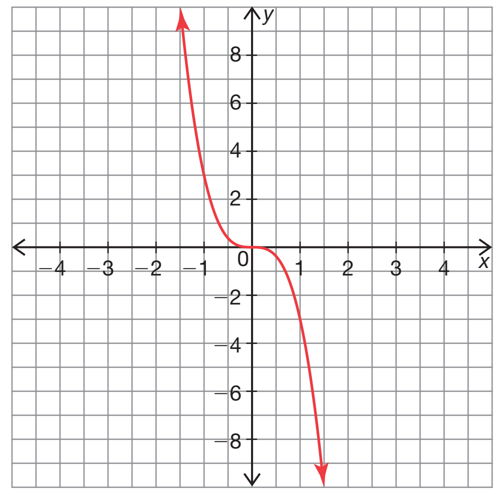
Polynomials are not closed under division because the quotient is not a polynomial.



17. Are the polynomials $y_1 = -x$, $y_2 = -2x$, and $y_3 = -1.5x$ closed under multiplication?

$(-x)(-2x)(-1.5x) = -3x^3$

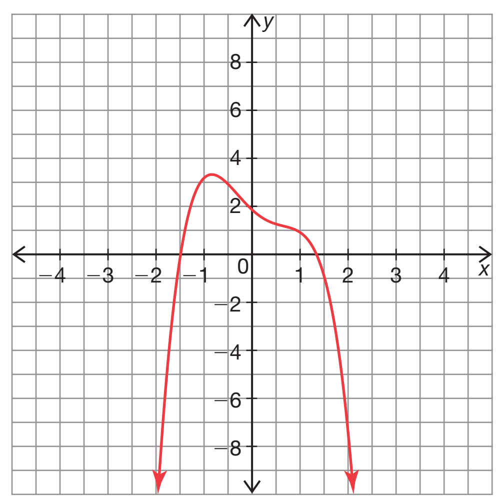
Polynomials are closed under multiplication because the product is a polynomial.



18. Are the polynomials $y_1 = 2x^4 - 5x^3 + 3x^2 - x + 6$ and $y_2 = 3x^4 - 6x^3 + 2x^2 + x + 4$ closed under subtraction?

$$\begin{array}{r} 2x^4 - 5x^3 + 3x^2 - x + 6 \\ -(3x^4 - 6x^3 + 2x^2 + x + 4 \\ \hline -x^4 + x^3 + x^2 - 2x + 2 \end{array}$$

Polynomials are closed under subtraction because the difference is a polynomial.



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