Integrated Math 3 **Chapter 5: Review** Name:________________________
Date:___________ Period:_____

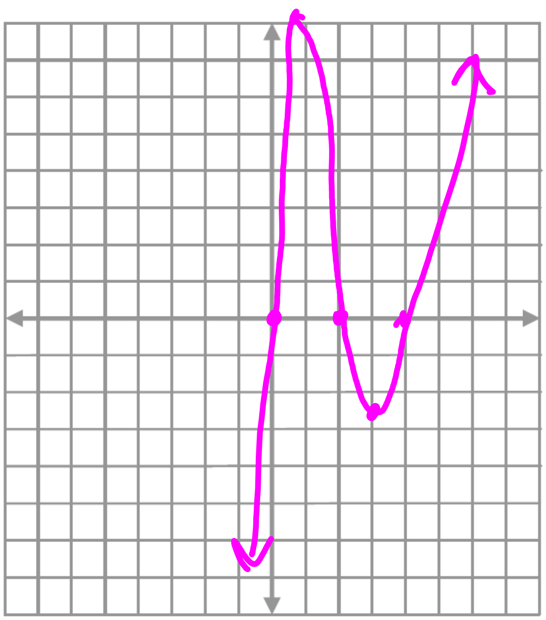
Section 5.1 Cubic Functions (2 Days)

→ I can identify the differences between the graph of a quadratic function and the graph of a cubic function
→I can model volume with a cubic function
→ I can identify characteristics of the graph of a cubic function, including domain & range, relative extrema, and intercepts

1. A rectangular prism has a length of x cm, width of $(4-2x)$ cm, and a height of $(6-2x)$ cm. Determine the volume of the prism, $V(x)$, in standard form by finding the product of the linear factors.

$V(x) = x(4-2x)(6-2x)$ $V(x) = 4x^3 - 20x^2 + 24x$

2. Sketch a graph of the function $V(x)$. Determine all key characteristics.



Domain of the function: $(-\infty, \infty)$

Domain in the context of the problem: $(0, 2)$

Range of the function: $(-\infty, \infty)$

Range in the context of the problem: $(0, 8.45]$

X-intercepts: $(0,0)$, $(2,0)$, $(3,0)$

Y-intercept: $(0,0)$

There is a relative minimum of -2.53 at x= 2.55.

There is a relative maximum of 8.45 at x= .78.

The graph is increasing/decreasing on the interval of $(-\infty, .78)$ then it is increasing/decreasing on the interval of $(.78, 2.55)$ then it is increasing/decreasing on the interval of $(2.55, \infty)$

3. What is the maximum volume of the function? What are the dimensions of the prism at this volume? max. vol = 8.45 cm^3

at .78 cm x 2.44 cm x 4.44 cm

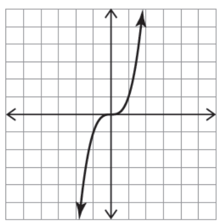
Integrated Math 3 **Chapter 5: Review** Name:________________________
Date:___________ Period:_____

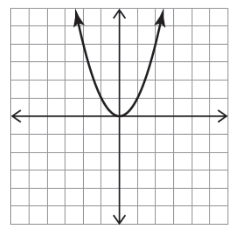
Section 5.2 Power Functions (3 Days)

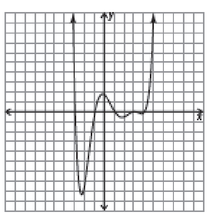
→I can look at the degree of the polynomial function and its leading coefficient and identify the end behavior of the graph (use limits to define end behavior)
→ I can identify whether a graph represents an even or odd degree polynomial function
→ I can determine if a function is even or odd depending on the symmetry of its graph
→ I can use the equation of a function to determine if it is even or odd algebraically

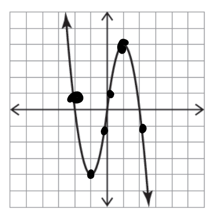
4. Use limits to determine end behavior of the function.
   a. $f(x) = 3x^2$ $\lim_{x\to\infty} f(x)=\infty$ $\lim_{x\to-\infty} f(x)=\infty$
   b. $g(x) = -4x^3$ $\lim_{x\to\infty} f(x)=-\infty$ $\lim_{x\to-\infty} f(x)=\infty$
   c. $h(x) = +12x^5$ $\lim_{x\to\infty} f(x)=\infty$ $\lim_{x\to-\infty} f(x)=-\infty$
   d. $j(x) = -6x^8$ $\lim_{x\to\infty} f(x)=-\infty$ $\lim_{x\to-\infty} f(x)=-\infty$

5. Use end behavior to determine whether the polynomial function is an even or odd DEGREE function.

a. 
odd degree
$\lim_{x\to\infty} f(x)=\infty$
$\lim_{x\to-\infty} f(x)=-\infty$

c. 
even degree
$\lim_{x\to\infty} f(x)=\infty$
$\lim_{x\to-\infty} f(x)=\infty$

b. 
even degree
$\lim_{x\to\infty} f(x)=\infty$
$\lim_{x\to-\infty} f(x)=\infty$

d. 
odd degree
$\lim_{x\to\infty} f(x)=-\infty$
$\lim_{x\to-\infty} f(x)=\infty$

6. Determine whether each function **<u>from question 5</u>** is even or odd based on symmetry of the graph.
   a. Even/Odd/Neither
   Explain: Graph is symmetric about the origin
   b. Even/Odd/Neither
   Explain:______________________________
   c. Even/Odd/Neither
   Explain: Graph is symmetric about the y-axis.
   d. Even/Odd/Neither
   Explain: Graph is symmetric about the origin.

7. Determine whether the function is odd, even, or neither algebraically.
   a. $f(x) = 3x^4 - 2x^2 - 6$

   $f(-x) = 3x^4 - 2x^2 - 6$ even

   b. $g(x) = -5x^5 + 2x^4 - 3x^2 + x - 3$

   $g(-x) = 5x^5 + 2x^4 - 3x^2 - x - 3$ neither

   c. $h(x) = -3x^5 + 3x^3 - 7x$

   $h(-x) = 3x^5 - 3x^3 + 7x$ odd

Section 5.3 Transformation and Symmetry of Polynomial Functions (1 Day)

→ I can use my knowledge of translation, reflection, and dilation to graph transformations of polynomial functions
→ I can use the symmetry of a graph to determine if the function is even or odd

8. Describe all transformations from the parent graph $f(x) = x^3$ to the function
   a. $g(x) = -\frac{1}{2}(x-2)^3 + 6$

   vertical shrink by factor of 1/2
   reflection over x-axis
   horizontal shift 2 units to the right

   vertical shift 6 units up

   b. $h(x) = 5(x+3)^3 - 2$

   vertical stretch by a factor of 5
   horizontal shift 3 units left
   vertical shift 2 units down

Section 5.4 Key Characteristics of Polynomial Functions (2 Days)

→I can determine the possible number of extrema of a polynomial function depending on its degree
→I can use the Fundamental Theorem of Algebra along with knowledge of multiple and imaginary roots to sketch graphs of polynomial functions of a given degree with a given number of zeros.

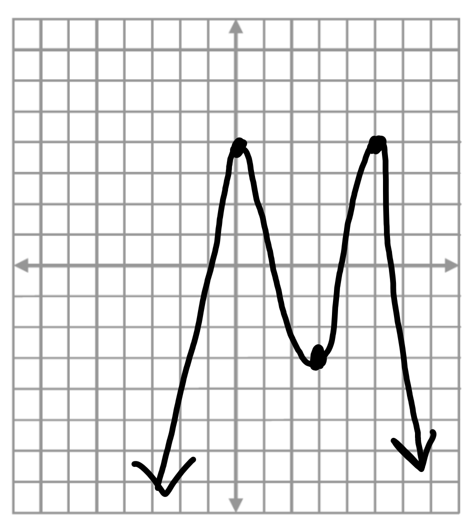
9. Determine the possible number of extrema for each polynomial.
   a. 7th degree polynomial

   0, 2, 4, 6

   b. 10th degree polynomial

   1, 3, 5, 7, 9

10. Sketch a graph with the following characteristics.
    a. Even degree
    b. Increases to $x = 0$, then decreases to $x = 3$, then increases to $x = 5$, then decreases
    c. Relative minimum of -3
    d. Two absolute maximums of 4



Integrated Math 3 **Chapter 5: Review** Name:_________________________
Date:____________ Period:_____

<u>Section 5.5 Building Cubic and Quartic Functions (1 Day)</u>

→I can identify the number of real, imaginary, and multiple roots of a simple polynomial function
→I can identify the type of polynomial function given a table of values (using first, second, third differences)
→I know that multiplying polynomials yields a new polynomial function (e.g. multiplying a linear function by a cubic function yields a quartic function)

11. Determine the type of function based on common differences.

x	$f(x)$	First Differences	Second Differences
−2	12		
		-9	
−1	3		6
		-3	
0	0		6
		3	
1	3		6
		9	
2	12		

Type of function: quadratic

x	$g(x)$	First Differences	Second Differences
−2	−6		
		3	
−1	−3		0
		3	
0	0		0
		3	
1	3		0
		3	
2	6		

Type of function: linear

12. If $h(x) = f(x) \cdot g(x)$, what type of function is $h(x)$? How many real and imaginary zeros does $h(x)$ have?

h(x) is a cubic function with 3 real zeros (all at x=0) because of multiplicity.