Chapter 7 "Review of Formulas"

Note Title 4/7/2010

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## QUICK REVIEW

**CONCEPTS** | **EXAMPLES**

### 7.1 Oblique Triangles and the Law of Sines

1

**Law of Sines**

In any triangle $ABC$, with sides $a$, $b$, and $c$,

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad \frac{a}{\sin A} = \frac{c}{\sin C}, \quad \text{and} \quad \frac{b}{\sin B} = \frac{c}{\sin C}.$$

In triangle $ABC$, find $c$ if $A = 44°$, $C = 62°$, and $a = 12.00$ units. Then find its area.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{12.00}{\sin 44°} = \frac{c}{\sin 62°}$$

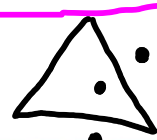
$$c = \frac{12.00 \sin 62°}{\sin 44°} \approx 15.25 \text{ units}$$

2

**Area of a Triangle**

In any triangle $ABC$, the area is half the product of the lengths of two sides and the sine of the angle between them.

$$\mathcal{A} = \frac{1}{2}bc \sin A, \quad \mathcal{A} = \frac{1}{2}ab \sin C, \quad \mathcal{A} = \frac{1}{2}ac \sin B$$



SAS

For triangle $ABC$ above,

$$\mathcal{A} = \frac{1}{2}ac \sin B$$

$$= \frac{1}{2}(12.00)(15.25) \sin 74° \qquad B = 180° - 44° - 62°$$

$$\approx 87.96 \text{ sq units.}$$

CONCEPTS | EXAMPLES

## 7.2 The of the Law of Sines

**Ambiguous Case**

If we are given the lengths of two sides and the angle opposite one of them, for example, $A$, $a$, and $b$ in triangle $ABC$, then it is possible that zero, one, or two such triangles exist. If $A$ is acute, $h$ is the altitude from $C$, and

1. $a < h < b$, then there is no triangle.
2. $a = h$ and $h < b$, then there is one triangle (a right triangle).
3. $a \geq b$, then there is one triangle.
4. $h < a < b$, then there are two triangles.

If $A$ is obtuse and

1. $a \leq b$, then there is no triangle.
2. $a > b$, then there is one triangle.

See the table on page 313 that illustrates the possible outcomes.

Solve triangle $ABC$, given $A = 44.5°$, $a = 11.0$ in., and $c = 7.0$ in.

Find angle $C$.

SSA

$$\frac{\sin C}{7.0} = \frac{\sin 44.5°}{11.0}$$

$$\sin C \approx .4460$$

$$C \approx 26.5°$$

Another angle with this sine value is

$$180° - 26.5° = 153.5°.$$

However, $153.5° + 44.5° > 180°$, so there is only one triangle.

$$B = 180° - 44.5° - 26.5°$$

$$B = 109°$$

Using the law of sines again,

$$b \approx 14.8 \text{ in.}$$

## 7.3 The Law of Cosines

3

**Law of Cosines**

In any triangle $ABC$, with sides $a$, $b$, and $c$,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C.$$

In triangle $ABC$, find $C$ if $a = 11$ units, $b = 13$ units, and $c = 20$ units. Then find its area.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$20^2 = 11^2 + 13^2 - 2(11)(13)\cos C$$

$$400 = 121 + 169 - 286\cos C$$

$$\frac{400 - 121 - 169}{-286} = \cos C$$

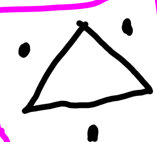
$$C = \cos^{-1}\left(\frac{400 - 121 - 169}{-286}\right)$$

$$C \approx 112.6^\circ$$

4

**Heron's Area Formula**

SSS



If a triangle has sides of lengths $a$, $b$, and $c$, with semiperimeter

$$s = \frac{1}{2}(a + b + c),$$

then the area of the triangle is

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}.$$

The semiperimeter $s$ is

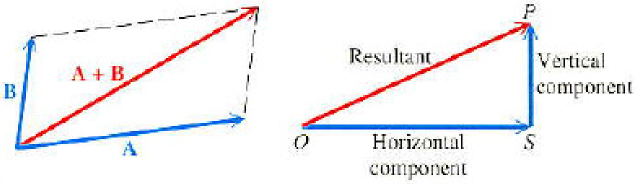
$$s = \frac{1}{2}(11 + 13 + 20) = 22,$$

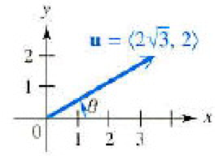
so

$$\mathcal{A} = \sqrt{22(22-11)(22-13)(22-20)} = 66 \text{ sq units}.$$

CONCEPTS | EXAMPLES

## 7.4 Vectors, Operations, and the Dot Product




5 **Magnitude and Direction Angle of a Vector**

The magnitude (length) of vector $\mathbf{u} = \langle a, b\rangle$ is given by

6 $$|\mathbf{u}| = \sqrt{a^2 + b^2}.$$

$$|\mathbf{u}| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{16} = 4$$

The direction angle $\theta$ satisfies $\tan\theta = \frac{b}{a}$, where $a \neq 0$. or $\theta = \tan^{-1}\frac{b}{a}$

Since $\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, it follows that $\theta = 30°$.

**Vector Operations**

For any real numbers $a$, $b$, $c$, $d$, and $k$,

7
$$\langle a, b\rangle + \langle c, d\rangle = \langle a + c, b + d\rangle$$
$$k \cdot \langle a, b\rangle = \langle ka, kb\rangle.$$
If $\mathbf{a} = \langle a_1, a_2\rangle$, then $-\mathbf{a} = \langle -a_1, -a_2\rangle$.
$$\langle a, b\rangle - \langle c, d\rangle = \langle a, b\rangle + (-\langle c, d\rangle) = \langle a - c, b - d\rangle.$$

$$\langle 4, 6\rangle + \langle -8, 3\rangle = \langle -4, 9\rangle$$
$$5\langle -2, 1\rangle = \langle -10, 5\rangle$$
$$-\langle -9, 6\rangle = \langle 9, -6\rangle$$
$$\langle 4, 6\rangle - \langle -8, 3\rangle = \langle 12, 3\rangle$$

8 If $\mathbf{u} = \langle x, y\rangle$ has direction angle $\theta$, then

$$\mathbf{u} = \langle |\mathbf{u}|\cos\theta, |\mathbf{u}|\sin\theta\rangle.$$

For $\mathbf{u}$ defined above,

$$\mathbf{u} = \langle 4\cos 30°, 4\sin 30°\rangle$$
$$= \langle 2\sqrt{3}, 2\rangle$$

9 The dot product of the two vectors $\mathbf{u} = \langle a, b\rangle$ and $\mathbf{v} = \langle c, d\rangle$, denoted $\mathbf{u} \cdot \mathbf{v}$, is given by

$$\mathbf{u} \cdot \mathbf{v} = ac + bd.$$

$$\langle 2, 1\rangle \cdot \langle 5, -2\rangle = 2 \cdot 5 + 1(-2) = 8$$

10 If $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$, where $0° \leq \theta \leq 180°$, then

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta, \quad \text{or} \quad \cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}.$$

$$\theta = \cos^{-1}\frac{u \cdot v}{|u||v|}$$

Find the angle $\theta$ between $\mathbf{u} = \langle 3, 1\rangle$ and $\mathbf{v} = \langle 2, -3\rangle$.

$$\cos\theta = \frac{\langle 3, 1\rangle \cdot \langle 2, -3\rangle}{\sqrt{3^2 + 1^2} \cdot \sqrt{2^2 + (-3)^2}}$$
$$\cos\theta = \frac{6 + (-3)}{\sqrt{10} \cdot \sqrt{13}}$$
$$\cos\theta = \frac{3}{\sqrt{130}}$$
$$\theta = \cos^{-1}\frac{3}{\sqrt{130}} \approx 74.7°$$

$u \cdot v = 0$

11 Orthogonal Vectors are perpendicular or 90°