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Sequence—Not Just Another Glittery Accessory
Arithmetic and Geometric Sequences

Vocabulary

Choose the term from the box that best completes each statement.

arithmetic sequence	geometric sequence
finite sequence	infinite sequence

1. A(n) geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant.

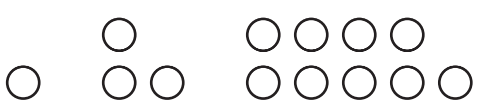
2. If a sequence terminates it is called a(n) finite sequence.

3. A(n) arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant.

4. If a sequences goes on forever it is called a(n) infinite sequence.

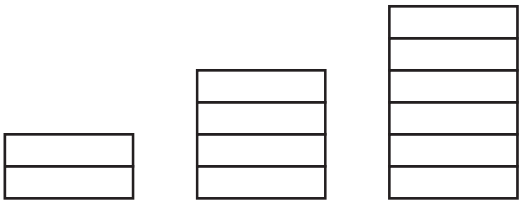
Problem Set

Analyze each sequence and identify whether the sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, identify the common difference. If the sequence is geometric, identify the common ratio.

1. 

The sequence is geometric.

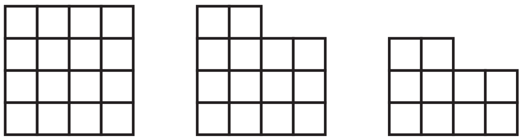
$r = 3$

2. 

The sequence is arithmetic.

$d = 2$

3.



The sequence is neither arithmetic or geometric.

4. $-4, -7, -10, -13, \ldots$

The sequence is arithmetic.

$d = -3$

5. 3, 5, 9, 15

The sequence is neither arithmetic nor geometric.

6. $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}, \ldots$

The sequence is geometric.

$r = -\frac{1}{3}$

Create your own sequence given the type indicated. Include the first four terms.

7. finite arithmetic sequence

Answers will vary.

$-3, -7, -11, -15$

8. infinite arithmetic sequence

Answers will vary.

2, 2.5, 3, 3.5 . . .

9. finite geometric sequence

Answers will vary.

$0.2, -0.4, 0.8, -1.6$

10. infinite geometric sequence

Answers will vary.

$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \ldots$

11. finite sequence that is neither arithmetic or geometric

Answers will vary.

$-1, 0, -1, 0$

12. infinite sequence that is neither arithmetic or geometric

Answers will vary.

1, 1, 2, 2 . . .

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Identify each sequence as arithmetic or geometric. Write a recursive formula for each sequence. Use the formula to determine the next term of each sequence.

13. 5, 3, 1, −1, . . .

The sequence is arithmetic.

$a_n = a_{n-1} + d$

$d = -2$

$a_n = a_{n-1} + -2$

$a_4 = -1$

$a_5 = a_4 + -2$

$= -1 + -2$

$= -3$

14. −2, −4, −8, −16, . . .

The sequence is geometric.

$g_n = g_{n-1} \cdot r$

$r = 2$

$g_n = g_{n-1} \cdot 2$

$g_4 = -16$

$g_5 = g_4 \cdot 2$

$= -16 \cdot 2$

$= -32$

15. $\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \ldots$

The sequence is geometric.

$g_n = g_{n-1} \cdot r$

$r = -1$

$g_n = g_{n-1} \cdot -1$

$g_4 = -\frac{1}{4}$

$g_5 = g_4 \cdot -1$

$= -\frac{1}{4} \cdot -1$

$= \frac{1}{4}$

16. 0.2, 0.4, 0.6, 0.8, 1, . . .

The sequence is arithmetic.

$a_n = a_{n-1} + d$

$d = 0.2$

$a_n = a_{n-1} + 0.2$

$a_5 = 1$

$a_6 = a_5 + 0.2$

$= 1 + 0.2$

$= 1.02$

17. 7, 9.3, 11.6, 13.9, 16.2, . . .

The sequence is arithmetic.

$a_n = a_{n-1} + d$

$d = 2.3$

$a_n = a_{n-1} + 2.3$

$a_5 = 16.2$

$a_6 = a_5 + 2.3$

$= 16.2 + 2.3$

$= 18.5$

18. $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \ldots$

The sequence is geometric.

$g_n = g_{n-1} \cdot r$

$r = \frac{1}{10}$

$g_n = g_{n-1} \cdot \frac{1}{10}$

$g_3 = \frac{1}{1000}$

$g_4 = g_3 \cdot \frac{1}{10}$

$= \frac{1}{1000} \cdot \frac{1}{10}$

$= \frac{1}{10{,}000}$

Identify each sequence as arithmetic or geometric. Write an explicit formula for each sequence. Use the formula to determine the 35th term of each sequence.

19. 1, 2, 4, 8, 16, . . .

The sequence is geometric.

$g_1 = 1, r = 2$

$g_n = g_1 \cdot r^{n-1}$

$g_{35} = 1 \cdot 2^{35-1}$

$g_{35} = 2^{34}$

$\approx 1.72 \times 10^{10}$

20. 3, 6, 12, 24, . . .

The sequence is geometric.

$g_1 = 3, r = 2$

$g_n = g_1 \cdot r^{n-1}$

$g_{35} = 3 \cdot 2^{35-1}$

$g_{35} = 3 \cdot 2^{34}$

$\approx 5.15 \times 10^{10}$

21. 3.1, 1.1, −0.9, . . .

The sequence is arithmetic.

$a_1 = 3.1, d = -2$

$a_n = a_1 + d(n - 1)$

$a_{35} = 3.1 + [-2(35 - 1)]$

$a_{35} = 3.1 + [-2(34)]$

$= -64.9$

22. 0.2, 1, 1.8, 2.6, . . .

The sequence is arithmetic.

$a_1 = 0.2, d = 0.8$

$a_n = a_1 + d(n - 1)$

$a_{35} = 0.2 + 0.8(35 - 1)$

$a_{35} = 0.2 + 0.8(34)$

$= 27.4$

23. 3, −6, 12, −24, . . .

The sequence is geometric.

$g_1 = 3, r = -2$

$g_n = g_1 \cdot r^{n-1}$

$g_{35} = 3 \cdot (-2)^{35-1}$

$g_{35} = 3 \cdot (-2)^{34}$

$\approx 5.15 \times 10^{10}$

24. $-\frac{1}{4}, -\frac{3}{8}, -\frac{1}{2}, -\frac{5}{8}, \ldots$

The sequence is arithmetic.

$a_1 = -\frac{1}{4}, d = -\frac{1}{8}$

$a_n = a_1 + d(n - 1)$

$a_{35} = -\frac{1}{4} + \left[-\frac{1}{8}(35 - 1)\right]$

$a_{35} = -\frac{1}{4} + \left[-\frac{1}{8}(34)\right]$

$= -\frac{9}{2}$

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This Is Series(ous) Business
Finite Arithmetic Series

Vocabulary

Write a definition for each term in your own words.

1. tessellation

   A tessellation is created when a geometric shape is repeated over a two-dimensional plane such that there are no overlaps and no gaps.

2. series

   A series is the sum of a sequence of numbers.

3. arithmetic series

   An arithmetic series is the sum of an arithmetic sequence.

4. finite series

   A finite series is the sum of a finite number of terms.

5. infinite series

   An infinite series is the sum of an infinite number of terms.

Problem Set

Use sigma notation to rewrite each finite series. Then, calculate the given sum.

1. $4 + 8 + 12 + 16 + 20; S_3$

$$S_3 = \sum_{i=1}^{3} a_i$$
$$= 4 + 8 + 12$$
$$= 24$$

2. $-1 + (-5) + (-9) + (-13); S_4$

$$S_4 = \sum_{i=1}^{4} a_i$$
$$= -1 + (-5) + (-9) + (-13)$$
$$= -28$$

3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}; S_3$

$$S_3 = \sum_{i=1}^{3} a_i$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$
$$= \frac{7}{8}$$

4. $3 + (-2) + 1 + (-4) + (-1); S_5$

$$S_5 = \sum_{i=1}^{5} a_i$$
$$= 3 + (-2) + 1 + (-4) + (-1)$$
$$= -3$$

5. $0.5 + 1 + 1.5 + 2 + 2.5; S_4$

$$S_4 = \sum_{i=1}^{4} a_i$$
$$= 0.5 + 1 + 1.5 + 2$$
$$= 5$$

6. $-1 + 1 + (-1) + 1 + (-1) + 1; S_6$

$$S_6 = \sum_{i=1}^{6} a_i$$
$$= -1 + 1 + (-1) + 1 + (-1) + 1$$
$$= 0$$

Use Gauss's formula to calculate each finite arithmetic series.

7. $3 + 5 + 7 + 9 + 11$

$$S_n = \frac{n(a_1 + a_n)}{2}$$
$$S_5 = \frac{5(3 + 11)}{2}$$
$$= 35$$

8. $-2 + (-3) + (-4) + (-5) + (-6) + (-7)$

$$S_n = \frac{n(a_1 + a_n)}{2}$$
$$S_6 = \frac{6[-2 + (-7)]}{2}$$
$$= -27$$

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9. $\frac{1}{3} + \frac{2}{3} + 1 + \frac{4}{3} + \frac{5}{3} + 2 + \frac{7}{3}$

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_7 = \frac{7\left(\frac{1}{3} + \frac{7}{3}\right)}{2}$

$= \frac{28}{3}$

10. 1.3 + 2.5 + 3.7 + 4.9 + 6.1

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_5 = \frac{5(1.3 + 6.1)}{2}$

$= 18.5$

11. 1 + 2 + 3 + · · · + 51

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_{51} = \frac{51(1 + 51)}{2}$

$= 1326$

12. 3 + 6 + 9 + · · · + 30

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_{10} = \frac{10(3 + 30)}{2}$

$= 165$

Write a function to calculate the sum of the first *n* terms of each arithmetic sequence. Then, determine S_3 by adding the first three terms and by using your function.

13. 1, 2, 3, 4, 5, . . .

$a_1 = 1$ and $d = 1$

$a_n = a_1 + (n - 1)d$

$= 1 + (n - 1)(1)$

$= n$

$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$

$= \frac{n(1 + n)}{2}$

$= \frac{n^2 + n}{2}$

$S_3 = 1 + 2 + 3$

$= 6$

$S_3 = f(3) = \frac{3^2 + 3}{2}$

$= 6$

14. 3, 6, 9, 12, . . .

$a_1 = 3$ and $d = 3$

$a_n = a_1 + (n - 1)d$

$= 3 + (n - 1)(3)$

$= 3n$

$S_3 = 3 + 6 + 9$

$= 18$

$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$

$= \frac{n(3 + 3n)}{2}$

$= \frac{3n^2 + 3n}{2}$

$S_3 = f(3) = \frac{3(3)^2 + 3(3)}{2}$

$= 18$

15. −4, −8, −12, −16, . . .

$a_1 = -4$ and $d = -4$

$a_n = a_1 + (n - 1)d$

$= -4 + (n - 1)(-4)$

$= -4n$

$S_3 = -4 + (-8) + (-12)$

$= -24$

$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$

$= \frac{n[-4 + (-4n)]}{2}$

$= -2n^2 - 2n$

$S_3 = f(3) = -2(3)^2 - 2(3)$

$= -24$

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16. −4, 0, 4, 8, . . .

$a_1 = -4$ and $d = 4$

$a_n = a_1 + (n - 1)d$

$= -4 + (n - 1)(4)$

$= 4n - 8$

$S_3 = -4 + 0 + 4$

$= 0$

$$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$$

$$= \frac{n[-4 + (4n - 8)]}{2}$$

$$= 2n^2 - 6n$$

$S_3 = f(3) = 2(3)^2 - 6(3)$

$= 0$

17. −2, −4, −6, −8, . . .

$a_1 = -2$ and $d = -2$

$a_n = a_1 + (n - 1)d$

$= -2 + (n - 1)(-2)$

$= -2n$

$S_3 = -2 + (-4) + (-6)$

$= -12$

$$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$$

$$= \frac{n[-2 + (-2n)]}{2}$$

$$= -n^2 - n$$

$S_3 = f(3) = -(3)^2 - 3$

$= -12$

18. $-2, -3, -4, -5, \ldots$

$a_1 = -2$ and $d = -1$

$$a_n = a_1 + (n - 1)d$$
$$= -2 + (n - 1)(-1)$$
$$= -n - 1$$

$$S_3 = -2 + (-3) + (-4)$$
$$= -9$$

$$S_n = f(n) = \frac{n(a_1 + a_n)}{2}$$
$$= \frac{n[-2 + (-n - 1)]}{2}$$
$$= \frac{-n^2 - 3n}{2}$$

$$S_3 = f(3) = \frac{-(3)^2 - 3(3)}{2}$$
$$= -9$$

A military band marches in a formation consisting of 8 rows. The first row has 2 band members, and each successive row has 3 more band members than the previous row. Use the given information to answer each question.

19. Write an arithmetic series to represent the number of band members in the formation. Then, rewrite the series using sigma notation.

$2 + 5 + 8 + 11 + 14 + 17 + 20 + 23$

$$S_8 = \sum_{i=1}^{8} a_i$$
$$= 2 + 5 + 8 + 11 + 14 + 17 + 20 + 23$$
$$= 100$$

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20. Write an explicit formula to calculate the number of band members in any given row. Then, use the explicit formula to verify that the last row has 23 band members, or $a_8 = 23$.

$a_1 = 2$ and $d = 3$

$a_n = a_1 + (n - 1)d$

$a_n = 2 + (n - 1)(3)$

$a_n = 3n - 1$

I will let $n = 8$ to verify the number of band members in the last row.

$a_8 = 3(8) - 1$

$a_8 = 23$

Using the explicit formula, I verified that the number of band members in the last row is 23.

21. Use Gauss's formula to determine the number of band members in the first 5 rows.

$n = 5$, $a_1 = 2$, and $a_5 = 14$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_5 = \frac{5(2 + 14)}{2}$$

$$= 40$$

There are 40 band members in the first 5 rows.

22. Use Gauss's formula to determine the total number of band members in the formation.

$n = 8$, $a_1 = 2$, and $a_8 = 23$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_8 = \frac{8(2 + 23)}{2}$$

$$= 100$$

There are 100 band members in the formation.

23. Determine the additional number of band members needed to create 4 more rows.

I know that adding 4 more rows would create 12 rows in the formation.

First, I can use an explicit formula to determine the number of band members in the 12th row.

$n = 12$, $a_1 = 2$, and $d = 3$

$a_n = a_1 + (n - 1)d$

$a_{12} = 2 + (12 - 1)(3)$

$a_{12} = 2 + 33$

$a_{12} = 35$

There would be 35 band members in the 12th row.

Next, I can use Gauss's formula to determine the number of band members needed to create the original 8 row formation and the new 12 row formation.

$n = 8$, $a_1 = 2$, and $a_8 = 23$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_8 = \frac{8(2 + 23)}{2}$$

$$= 100$$

To create the 8 row formation, the band needs 100 members.

$n = 12$, $a_1 = 2$, and $a_{12} = 35$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_{12} = \frac{12(2 + 35)}{2}$$

$$= 222$$

To create the 12 row formation, the band needs 222 members.

Finally, I can subtract the number of band members needed to create the 8 row formation from the number of band members needed to create the 12 row formation to determine how many new members are needed to create 4 more rows.

$$S_{12} - S_8 = 222 - 100$$

$$= 122$$

The band would need to add 122 new members to create 4 more rows.

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24. Determine the number of band members in the formation if the last row in the formation contains 44 members.

First, I can use an explicit formula to determine how many rows are needed so that the last row contains 44 members.

$a_n = 44$, $a_1 = 2$, and $d = 3$

$a_n = a_1 + d(n - 1)$

$44 = 2 + 3(n - 1)$

$42 = 3(n - 1)$

$14 = n - 1$

$15 = n$

There would be 15 rows in the formation if the last row contained 44 members.

Next, I can use Gauss's formula to determine the number of band members in a formation containing 15 rows.

$n = 15$, $a_1 = 2$, and $a_{15} = 44$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_{15} = \frac{15(2 + 44)}{2}$$

$= 345$ band members

There are 345 members in the formation if the last row contains 44 members.

8

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I Am Having a Series Craving (For Some Math)!
Geometric Series

Vocabulary

Write the term that best completes the sentence.

1. A ___geometric series___ is the sum of the terms of a geometric sequence.

Problem Set

Use Euclid's Method to compute each series.

1. 2 + 6 + 18 + 54 + 162

$r = 3, \quad g_1 = 2, \quad g_5 = 162$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_5 = \frac{162(3) - 2}{3 - 1}$$

$$= 242$$

2. 1 + (−4) + 16 + (−64) + 256 + (−1024)

$r = -4, \quad g_1 = 1, \quad g_6 = -1024$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_6 = \frac{-1024(-4) - 1}{-4 - 1}$$

$$= -819$$

3. $\frac{1}{2} + \frac{1}{10} + \frac{1}{50} + \frac{1}{250} + \frac{1}{1250}$

$r = \frac{1}{5}$, $g_1 = \frac{1}{2}$, $g_5 = \frac{1}{1250}$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_5 = \frac{\frac{1}{1250}\left(\frac{1}{5}\right) - \frac{1}{2}}{\frac{1}{5} - 1}$$

$$= \frac{781}{1250}$$

4. $-0.2 + (-0.02) + (-0.002) + (-0.0002)$

$r = 0.1$, $g_1 = -0.2$, $g_4 = -0.0002$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_4 = \frac{-0.0002(0.1) - (-0.2)}{0.1 - 1}$$

$$= -0.2222$$

5. $\sum_{i=0}^{7} 2^i$

$r = 2$, $g_1 = 2^0 = 1$, $g_8 = 2^7 = 128$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_8 = \frac{128(2) - 1}{2 - 1}$$

$$= 255$$

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6. $3\sum_{i=0}^{6}(-3)^i$

$r = -3, \quad g_1 = (-3)^0 = 1, \quad g_7 = (-3)^6 = 729$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$3 \cdot S_7 = 3\left[\frac{729(-3) - 1}{-3 - 1}\right]$$

$$= 3[547]$$

$$= 1641$$

7. $5\sum_{i=0}^{8}\left(\frac{1}{2}\right)^i$

$r = \frac{1}{2}, \quad g_1 = \left(\frac{1}{2}\right)^0 = 1, \quad g_9 = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$5 \cdot S_9 = 5\left[\frac{\left(\frac{1}{256}\right)\left(\frac{1}{2}\right) - 1}{\frac{1}{2} - 1}\right]$$

$$= 5\left[\frac{511}{256}\right]$$

$$= \frac{2555}{256}$$

8. $\sum_{i=0}^{6}4^i$

$r = 4, \quad g_1 = 4^0 = 1, \quad g_7 = 4^6 = 4096$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_7 = \frac{4096(4) - 1}{4 - 1}$$

$$= 5461$$

9. A geometric sequence with 7 terms, a common ratio of 3, and a first term of −2.

$r = 3$, $g_1 = -2$, $g_7 = -2(3)^6 = -1458$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_7 = \frac{(-1458)(3) - (-2)}{3 - 1}$$

$$= -2186$$

10. A geometric sequence with 6 terms, a common ratio of 0.1, and a first term of 6.

$r = 0.1$, $g_1 = 6$, $g_6 = 6(0.1)^5 = 0.00006$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_6 = \frac{(0.00006)(0.1) - 6}{0.1 - 1}$$

$$= 6.66666$$

Write each geometric series in the form $g(1 + r^1 + r^2 + r^3 + \cdots + r^{n-1})$ where g is a constant, r is the common ratio, and n is the number of terms. Then, compute each series using the formula $S_n = \frac{g_1(r^n - 1)}{r - 1}$.

11. $7 + 14 + 28 + 56 + 112$

$= 7(1 + 2 + 4 + 8 + 16)$

$= 7(2^0 + 2^1 + 2^2 + 2^3 + 2^4)$

$$S_n = \frac{g_1(r^n - 1)}{r - 1}$$

$$S_5 = \frac{7(2^5 - 1)}{2 - 1}$$

$$= 217$$

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12. $-3 + (-9) + (-27) + (-81)$

$= -3(1 + 3 + 9 + 27)$

$= -3(3^0 + 3^1 + 3^2 + 3^3)$

$S_n = \dfrac{g_1(r^n - 1)}{r - 1}$

$S_4 = \dfrac{-3(3^4 - 1)}{3 - 1}$

$= -120$

13. $\dfrac{-1}{4} + \dfrac{1}{16} + \left(\dfrac{-1}{64}\right) + \dfrac{1}{256} + \left(\dfrac{-1}{1024}\right)$

$= -\dfrac{1}{4}\left[1 + \left(-\dfrac{1}{4}\right) + \dfrac{1}{16} + \left(-\dfrac{1}{64}\right) + \dfrac{1}{256}\right]$

$= -\dfrac{1}{4}\left[\left(-\dfrac{1}{4}\right)^0 + \left(-\dfrac{1}{4}\right)^1 + \left(-\dfrac{1}{4}\right)^2 + \left(-\dfrac{1}{4}\right)^3 + \left(-\dfrac{1}{4}\right)^4\right]$

$S_n = \dfrac{g_1(r^n - 1)}{r - 1}$

$S_5 = \dfrac{-\dfrac{1}{4}\left[\left(-\dfrac{1}{4}\right)^5 - 1\right]}{-\dfrac{1}{4} - 1}$

$= -\dfrac{205}{1024}$

14. $\displaystyle\sum_{i=0}^{5} 2(3)^i$

$= 2(3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5)$

$S_n = \dfrac{g_1(r^n - 1)}{r - 1}$

$S_6 = \dfrac{2(3^6 - 1)}{3 - 1}$

$= 728$

15. $\sum_{i=0}^{4} 4(0.2)^i$

$= 4(0.2^0 + 0.2^1 + 0.2^2 + 0.2^3 + 0.2^4)$

$S_n = \frac{g_1(r^n - 1)}{r - 1}$

$S_5 = \frac{4(0.2^5 - 1)}{0.2 - 1}$

$= 4.9984$

16. $\sum_{i=0}^{5} -\frac{1}{2}\left(\frac{1}{3}\right)^i$

$= -\frac{1}{2}\left[\left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5\right]$

$S_n = \frac{g_1(r^n - 1)}{r - 1}$

$S_6 = \frac{-\frac{1}{2}\left[\left(\frac{1}{3}\right)^6 - 1\right]}{\frac{1}{3} - 1}$

$= -\frac{182}{243}$

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Use Euclid's Method to solve each problem.

17. A rubber ball is dropped from a height of 600 centimeters and bounces on the ground. The table shows the height reached by the ball after each successive bounce. What is the total distance traveled by the ball on the first five return bounces?

Bounce	Height (centimeters)
1	300
2	150
3	75
4	37.5
5	18.75

$n = 5, g_5 = 18.75, g_1 = 300, r = \frac{1}{2}$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_5 = \frac{18.75\left(\frac{1}{2}\right) - 300}{\frac{1}{2} - 1}$$

$$= 581.25 \text{ cm}$$

The ball traveled 581.25 centimeters on the first five return bounces.

18. A small local grocery store reviewed their net profits for the past 7 years and observed that profits increased by 2% per year. The table shows the net profits. What is the grocery store's total net profit for the past 7 years?

Year	Net Profit (dollars)
1	20,000
2	20,400
3	20,808
4	21,224.16
5	21,648.64
6	22,081.62
7	22,523.25

$n = 7$, $g_7 = 22{,}523.25$, $g_1 = 20{,}000$, $r = 1.02$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_7 = \frac{22{,}523.25(1.02) - 20{,}000}{1.02 - 1}$$

$$= 148{,}685.75$$

The grocery store's net profit for the past 7 years is $148,685.75.

19. A soccer tournament has 64 participating teams. In the first round of the tournament, 32 games are played, with the winning team from each game moving on to the next round. In the second round, 16 games are played, with the winning team from each game moving on to the next round. This pattern continues until one team emerges as the winner of the tournament. How many games are played in the tournament to determine the winner?

$n = 6$, $g_6 = 1$, $g_1 = 32$, $r = \frac{1}{2}$

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_6 = \frac{1\left(\frac{1}{2}\right) - 32}{\frac{1}{2} - 1}$$

$$= 63$$

To determine the winner, 63 games are played in the tournament.

Name ______________________________ Date ____________

20. Serena paid \$8700 to attend college her freshman year, then found out that her cost would increase by 8% each year she stayed in college. If it takes Serena 6 years to graduate, how much will it cost her to complete college?

To use Euclid's Method, I need to determine how much Serena spent on her 6th year in college.

$n = 6$, $g_1 = 8700$, $r = 1.08$

$g_n = g_1 \cdot r^{n-1}$

$g_6 = 8700 \cdot 1.08^5$

$\approx 12{,}783.15$

Serena will spend approximately \$12,783.15 on her 6th year of college.

I can use Euclid's Method to determine Serena's total cost to complete college.

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_6 = \frac{12{,}783.15(1.08) - 8700}{1.08 - 1}$$

$\approx 63{,}822.53$

It will cost Serena \$63,822.53 to complete college.

21. A manufacturing company has an available position in its accounting department. The company advertises that the job will pay $40,000 the first year with an annual raise of 3%. How much money will the new accounting hire earn over 10 years?

To use Euclid's Method, I need to determine how much money the new hire will earn in the 10th year.

$n = 10$, $g_1 = 40{,}000$, $r = 1.03$,

$$g_n = g_1 \cdot r^{n-1}$$

$$g_{10} = 40{,}000 \cdot 1.03^9$$

$$\approx 52{,}190.93$$

The new hire will earn $52,190.93 in the 10th year.

I can use Euclid's Method to determine how much the new hire will earn over 10 years.

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_{10} = \frac{52{,}190.93(1.03) - 40{,}000}{1.03 - 1}$$

$$\approx 458{,}555.26$$

The new hire will earn $458,555.26 over the course of 10 years.

22. A pile driver is a mechanical device used to drive poles into the ground to provide support for a structure. Each time the device is used, the pole is driven further and further into the ground. Suppose on the first drive the pole is driven 6 feet into the ground and on each successive drive the pole is driven 80% of the distance achieved on the previous drive. How far has the pole been driven into the ground after 6 drives?

To use Euclid's Method, I need to determine how far the pile driver drove the pole on the 6th drive.

$n = 6$, $g_1 = 6$, $r = 0.8$

$$g_n = g_1 \cdot r^{n-1}$$

$$g_6 = 6 \cdot 0.8^5$$

$$= 1.96608$$

The pole was driven 1.96608 feet into the ground on the 6th drive.

I can use Euclid's Method to determine how far the pole was driven into the ground after 6 drives.

$$S_n = \frac{g_n(r) - g_1}{r - 1}$$

$$S_6 = \frac{1.96608(0.8) - 6}{0.8 - 1}$$

$$= 22.13568$$

The pole was driven a total of 22.13568 feet into the ground after 6 drives.

Name ____________________ Date __________

These Series Just Go On . . . And On . . . And On . . .

Infinite Geometric Series

Vocabulary

Describe how the key terms are similar and how they are dissimilar.

1. convergent series

divergent series

They are similar in that they are both a series. They are dissimilar in that a convergent series has a finite sum while a divergent series does not have a finite sum. The sum of a divergent series is infinite.

Problem Set

Determine whether each geometric series is convergent or divergent. Explain your reasoning.

1. $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots$

$r = \frac{1}{5}$

The series is convergent because the common ratio is between 0 and 1.

2. $\frac{5}{7} + \frac{10}{21} + \frac{20}{63} + \cdots$

$r = \frac{2}{3}$

The series is convergent because the common ratio is between 0 and 1.

3. $\frac{2}{5} + \frac{8}{15} + \frac{32}{45} \cdots$

$r = \frac{4}{3}$

The series is divergent because the common ratio is greater than 1.

4. $0.3 + 0.9 + 2.7 + \cdots$

$r = 3$

The series is divergent because the common ratio is greater than 1.

5. $\sum_{i=1}^{\infty} \left(\frac{2}{7}\right)^i$

$r = \frac{2}{7}$

The series is convergent because the common ratio is between 0 and 1.

6. $\sum_{i=0}^{\infty} 100(0.1)^i$

$r = 0.1$

The series is convergent because the common ratio is between 0 and 1.

7. $\frac{1}{50}\sum_{i=1}^{\infty}(4)^i$

$r = 4$

The series is divergent because the common ratio is greater than 1.

8. $\frac{3}{2}\left[1 + \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots\right]$

$r = \frac{1}{3}$

The series is convergent because the common ratio is between 0 and 1.

9. $1 + \left(\frac{8}{3}\right)^1 + \left(\frac{8}{3}\right)^2 + \left(\frac{8}{3}\right)^3 + \cdots$

$r = \frac{8}{3}$

The series is divergent because the common ratio is greater than 1.

10. $\sum_{i=0}^{\infty}(6^{-1})^i$

$r = \frac{1}{6}$

The series is convergent because the common ratio is between 0 and 1.

Determine whether each geometric series is convergent or divergent. If the series is convergent, compute the series.

11. $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

$r = \frac{1}{3}$; The series is convergent.

$$S = \frac{g_1}{1 - r} = \frac{\frac{2}{3}}{1 - \frac{1}{3}} = 1$$

12. $\frac{1}{10} + \frac{3}{20} + \frac{9}{40} + \cdots$

$r = \frac{3}{2}$; The series is divergent.

13. $1.05 + 2.1 + 4.2 + \cdots$

$r = 2$; The series is divergent.

14. $17 + 1.7 + 0.17 + \cdots$

$r = 0.1$; The series is convergent.

$$S = \frac{g_1}{1 - r} = \frac{17}{1 - 0.1} = \frac{170}{9}$$

Name ______________________________ Date ____________

15. $\sum_{i=1}^{\infty} \frac{7}{5}\left(\frac{1}{5}\right)^{i-1}$

$r = \frac{1}{5}$; This series is convergent.

$$S = \frac{g_1}{1-r} = \frac{\frac{7}{5}}{1-\frac{1}{5}} = \frac{7}{4}$$

16. $32\sum_{i=1}^{\infty}(1.01)^i$

$r = 1.01$; This series is divergent.

17. $\sum_{i=1}^{\infty} \frac{1}{4^i}$

$r = \frac{1}{4}$; This series is convergent.

$$S = \frac{g_1}{1-r} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$$

18. $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{2}\right)^{-3} + \cdots$

$r = 2$; This series is divergent.

19. $12\left[1 + \left(\frac{5}{3}\right)^1 + \left(\frac{5}{3}\right)^2 + \cdots\right]$

$r = \frac{5}{3}$; This series is divergent.

20. $12\left[1 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \cdots\right]$

$r = \frac{2}{3}$; This series is convergent.

$$S = \frac{g_1}{1-r} = \frac{12}{1-\frac{2}{3}} = 36$$

Identify which formula should be used for each type of series, and then compute each series.

Formula for the first n terms of an arithmetic series: $S_n = \frac{n(a_1 + a_n)}{2}$

Formula for the first n terms of a geometric series: $S_n = \frac{g_1(r^n - 1)}{r - 1}$ or $\frac{g_n \cdot r - g_1}{r - 1}$

Formula for an infinite convergent geometric series: $S = \frac{g_1}{1 - r}$

Formula for an infinite divergent geometric series: S = infinity

21. $\frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \frac{2}{243}$

$r = \frac{1}{3}$

The geometric series is finite.

$S_n = \frac{g_n \cdot r - g_1}{r - 1}$

$S_4 = \frac{\frac{2}{243}\left(\frac{1}{3}\right) - \frac{2}{9}}{\frac{1}{3} - 1}$

$= \frac{80}{243}$

22. $\sum_{i=1}^{\infty} 5\left(\frac{2}{11}\right)^i$

$r = \frac{2}{11}$

The infinite geometric series is convergent.

$S = \frac{g_1}{1 - r}$

$= \frac{5\left(\frac{2}{11}\right)}{1 - \frac{2}{11}}$

$= \frac{10}{9}$

23. $\frac{5}{4} + \frac{25}{16} + \frac{125}{64} + \cdots$

$r = \frac{5}{4}$

The infinite geometric series is divergent.

S = infinity

24. $5 + (-1) + (-7) + (-13) + (-19)$

$n = 5$

The arithmetic series is finite.

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_5 = \frac{5[5 + (-19)]}{2}$

$= -35$

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25. $0.3[1 + (0.3)^1 + (0.3)^2 + (0.3)^3 + \cdots]$

$r = 0.3$

The infinite geometric series is convergent.

$$S = \frac{g_1}{1 - r} = \frac{0.3}{1 - 0.3} = \frac{3}{7}$$

26. $1 + 1.1 + 1.21 + 1.331 + 1.4641 + \cdots$

$r = 1.1$

The infinite geometric series is divergent.

$S = \text{infinity}$

27. $\sum_{n=1}^{7} 2n$

$n = 7$

The arithmetic series is finite.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_7 = \frac{7(2 + 14)}{2} = 56$$

28. $\sum_{i=1}^{5} \left(\frac{1}{8}\right)^{i-1}$

$r = \frac{1}{8}$

The geometric series is finite.

$$S_n = \frac{g_n \cdot r - g_1}{r - 1}$$

$$S_5 = \frac{\left(\frac{1}{8}\right)^4 \left(\frac{1}{8}\right) - 1}{\frac{1}{8} - 1} = \frac{4681}{4096}$$

Name ____ Date ____________

The Power of Interest (It's a Curious Thing)
Geometric Series Applications

Problem Set

Write an explicit formula for the *n*th term of the geometric sequence that models each problem situation. Identify and interpret the meaning of the first term, g_1, and the common ratio, r.

1. Henry invested $500 in a bank account which earns 3% interest annually.

   The first term, g_1, is $500 and represents the amount of money Henry initially invests in the bank account. The common ratio, r, is 1.03 which represents the factor by which Henry's investment will increase each year.

   $g_n = 500 \cdot 1.03^{n-1}$

2. Gamble's Manufacturing makes widgets. The table shows the number of widgets produced by the company over a period of 5 years.

Year	Widgets Produced (thousands)
1	10
2	11
3	12.1
4	13.31
5	14.641

   The first term, g_1, is 10 and represents the 10,000 widgets produced by Gamble's Manufacturing in the first year. The common ratio, r, is 1.1 and represents the factor by which the production of widgets has grown each year.

   $g_n = 10 \cdot 1.1^{n-1}$

3. Panna initially invested \$1500 in her 401K plan. Due to the slow economy, her investment began to decrease by 1.5% per year.

The first term, g_1, is \$1500 and represents Panna's initial investment in her 401K plan. The common ratio, r, is 0.985 and represents the factor by which her previous investment will decrease each year.

$g_n = 1500 \cdot 0.985^{n-1}$

4. The table shows the population of a city at the end of four consecutive decades.

Decade	Population (thousands)
1	11
2	13.2
3	15.84
4	19.008

The first term, g_1, is 11 or 11,000 people and represents the city's population at the end of the first of four decades. The common ratio, r, is 1.2 and represents the factor by which the city's population has increased each successive decade.

$g_n = 11 \cdot 1.2^{n-1}$

5. Monty Maris is a professional baseball player whose end of the year batting averages have recently been declining. In the past 4 seasons his consecutive batting averages have been: 0.340, 0.306, 0.275, and 0.248.

The first term, g_1, is 0.340 and represents Monty's end of the year batting average in the first of four seasons. The common ratio, r, is 0.9 and represents the factor by which his batting average decreased each season.

$g_n = 0.340 \cdot 0.9^{n-1}$

6. Juanita is offered a job that pays \$50,000 the first year with an expected annual increase of 5%.

The first term, g_1, is \$50,000 and represents Juanita's initial salary. The common ratio, r, is 1.05 and represents the factor by which Juanita's salary is expected to increase each year.

$g_n = 50{,}000 \cdot 1.05^{n-1}$

Name ____ Date ____________

Use your knowledge of geometric sequences and series to solve each problem.

7. Aiden takes a job paying $34,000 dollars a year with a guaranteed increase of 5% per year. If he knows that he will earn $45,916.31 in his 20th year of employment, what is the total amount of money he earns over 20 years?

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 20$, $g_{20} = 45{,}916.31$, $r = 1.05$, and $g_1 = 34{,}000$, to determine how much money Aiden earns in 20 years.

$$S_{20} = \frac{g_{20}(r) - g_1}{r - 1}$$

$$= \frac{45{,}916.31(1.05) - 34{,}000}{1.05 - 1}$$

$$= 284{,}242.51$$

Aiden earns $284,242.51 over 20 years.

8. When Madie got her first job, she opened a savings account and deposited $14. With each paycheck she increased the amount of money she deposited in her savings account by 2%. If she deposited $18.11 in her account with her 14th paycheck, how much money does she have in her account after making her 14th deposit?

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 14$, $g_{14} = 18.11$, $r = 1.02$, and $g_1 = 14$, to determine how much money Madie has in her account after making her 14th deposit.

$$S_{14} = \frac{g_{14}(r) - g_1}{r - 1}$$

$$= \frac{18.11(1.02) - 14}{1.02 - 1}$$

$$= 223.61$$

Madie has $223.61 in her savings account after making her 14th deposit.

9. Contagious diseases like the flu can spread rather rapidly. Suppose in a large community initially 3 people have the flu. By the end of the 1st week, 12 people have the flu. If this pattern continues 786,432 will have the flu by the end of the 10th week. What is the total number of people who will have the flu during the 10-week period?

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 10$, $g_{10} = 786{,}432$, $r = 4$, and $g_1 = 3$, to determine how many people have the flu during the 10-week period.

$$S_{10} = \frac{g_{10}(r) - g_1}{r - 1}$$

$$= \frac{786{,}432(4) - 3}{4 - 1}$$

$$= 1{,}048{,}575$$

The total number of people who will have the flu during the 10-week period is 1,048,575 people.

10. Each month Gil withdraws 10% of the total amount of money he has invested in his savings account. He gives 60% of what he withdraws to charity and spends the rest. Complete the table.

Month	Savings Account	Amount Withdrawn (10%)	Amount Given to Charity (60%)	Amount Spent (40%)
1	$1000	$100	$60	$40
2	$900	$90	$54	$36
3	$810	$81	$48.60	$32.40
4	$729	$72.90	$43.74	$29.16
n	$1000 \cdot 0.9^{n-1}$	$100 \cdot 0.9^{n-1}$	$60 \cdot 0.9^{n-1}$	$40 \cdot 0.9^{n-1}$

Name ____ Date ____________

11. Del Corn, a canning company, processes canned vegetables. In the fall they harvest corn from the 10,000 acres of corn planted in the spring. The table shows the number of acres remaining to be harvested at the beginning of any given week. If this pattern continues, write a formula to predict the number of acres remaining to be harvested at the beginning of any given week. Use your formula to predict the number of acres remaining to be harvested at the beginning of the 9th week.

Week	Acres Remaining to be Harvested
1	10,000
2	8000
3	6400
4	5120

The first term, g_1, is 10,000 acres and represents the total number acres to be harvested. The common ratio, r, is 0.8 and represents the factor by which the acreage remaining to be harvested is decreasing each week.

$g_n = 10{,}000 \cdot 0.8^{n-1}$

To predict the number of acres remaining to be harvested at the beginning of the 9th week, evaluate the equation for $n = 9$.

$g_9 = 10{,}000 \cdot 0.8^{9-1}$

≈ 1678

At the beginning of the 9th week, approximately 1678 acres remain to be harvested.

12. At the end of each year, Wendy withdraws 20% of the total amount of money she has invested in her savings account. She gives 50% of what she withdraws to charity and uses the other 50% to buy gifts for her friends and relatives. Complete the table. Determine how much money she has given to charity over the period of 5 years. How much money did she spend on gifts over the same time period?

Year	Savings Account	Amount Withdrawn (20%)	Amount Given to Charity (50%)	Amount Spent on Gifts (50%)
1	\$8000	\$1600	\$800	\$800
2	\$6400	\$1280	\$640	\$640
3	\$5120	\$1024	\$512	\$512
4	\$4096	\$819.20	\$409.60	\$409.60
5	\$3276.80	\$655.36	\$327.68	\$327.68
n	$8000 \cdot 0.8^{n-1}$	$1600 \cdot 0.8^{n-1}$	$800 \cdot 0.8^{n-1}$	$800 \cdot 0.8^{n-1}$

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 5$, $g_5 = 327.68$, $r = 0.8$, and $g_1 = 800$, to determine how much money Wendy has given to charity over the period of 5 years.

$$S_5 = \frac{g_5(r) - g_1}{r - 1}$$

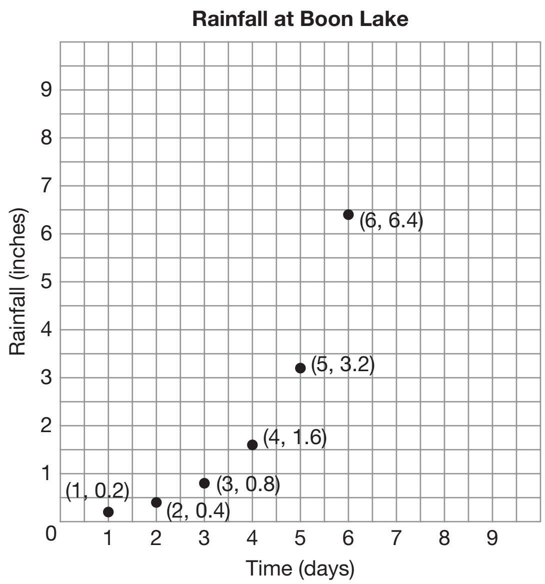
$$= \frac{327.68(0.8) - 800}{0.8 - 1}$$

$$= 2689.28$$

Wendy has given \$2689.28 to charity over the period of 5 years. Since the amount spent on gifts is the same as the amount given to charity, Wendy spent \$2689.28 on gifts for friends and relatives over the same time period.

Name ______________________________ Date ____________

13. Tanya lives on the shore of Boon Lake. This summer was particularly rainy, and the water level of Boon Lake began to rise so quickly that Tanya thought her cottage was going to flood. The graph shows the amount of rainfall over 6 consecutive days. Determine the total rainfall at Boon Lake over the 6 days.



I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 6$, $g_6 = 6.4$, $r = 2$, and $g_1 = 0.2$, to determine the total rainfall at Boon Lake over the 6 days.

$$S_6 = \frac{g_6(r) - g_1}{r - 1}$$

$$= \frac{6.4(2) - 0.2}{2 - 1}$$

$$= 12.6$$

The total rainfall at Boon Lake was 12.6 inches over the 6 days.

14. Santiago began collecting coins. At the end of the first year, his collection had 12 coins. At the end of the second year he added 24 more coins, and at the end of the third year he added 48 more coins. This pattern continued and at the end of the seventh year he added 768 more coins to his collection. How many total coins does Santiago have in his collection at the end of his seventh year?

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 7$, $g_7 = 768$, $r = 2$, and $g_1 = 12$, to determine how many coins Santiago has in his collection at the end of his seventh year.

$$S_7 = \frac{g_7(r) - g_1}{r - 1}$$

$$= \frac{768(2) - 12}{2 - 1}$$

$$= 1524$$

Santiago has 1524 coins in his collection at the end of his seventh year.

Name ______________________________ Date ____________

A Series of Fortunate Events

Applications of Arithmetic and Geometric Series

Problem Set

Determine whether each situation is best modeled by an arithmetic or geometric series. Explain your reasoning.

1. Shania loves to run. Currently she is running about 20 miles per week and decides to increase her weekly mileage by 3 miles per week.

   The situation is arithmetic because Shania's mileage is increasing by a constant amount of 3 miles per week.

2. Production of muffins at Puffery's Bakery continues to increase from month to month. The bakery increases production by a factor of 1.5 each month.

   The situation is geometric because the ratio between consecutive terms is 1.5.

3. Claudette bought stock in a new high tech company. The table shows the dividends she received at the end of each year.

Year	Dividends (dollars)
1	84.00
2	86.52
3	89.12
4	91.79
5	94.54

   The situation is geometric because the ratio between consecutive terms is 1.03.

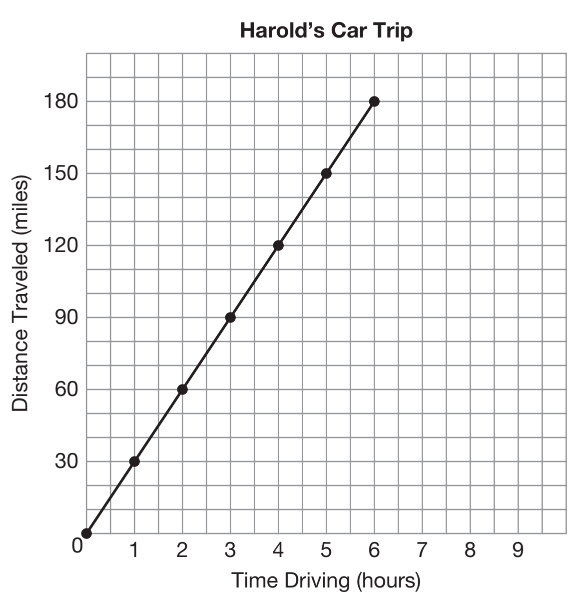
4. A local call center has an opening for a new associate that pays $14.50 per hour with a quarterly increase of $0.10 per hour.

The situation is arithmetic because the associate's salary is increasing quarterly by a constant amount of $0.10 per hour.

5. Neddy ran a regression analysis on some data, whose domain is positive integers, using his graphing calculator. The equation of best fit, according to the calculator, is $y = 3 \cdot 1.01^x$.

The situation is geometric because the ratio between consecutive terms is 1.01.

6. The graph shows the relationship between the number of hours Harold drove his car and the distance he traveled during a 6-hour trip.



The situation is arithmetic because the number of miles traveled is increasing hourly by a constant amount of 30 miles per hour.

Name ____________________ Date ____________

7. Shawana is considering the following two data plans for her mobile phone.

Data Plan 1	
Year	**Cost (dollars)**
1	148.00
2	153.00
3	158.00
4	163.00
5	168.00

Data Plan 2	
Year	**Cost (dollars)**
1	120.00
2	124.80
3	129.79
4	134.98
5	140.38

The situation is both arithmetic and geometric.

Data Plan 1 is arithmetic because the cost of the plan is increasing yearly by a constant amount of $5 per year.

Data Plan 2 is geometric because the ratio between consecutive terms is 1.04.

8. Chloe opened a clothing store. During the first 4 years the store's net revenue has been "flat," meaning that the net revenue remained the same during each of the 4 years.

The situation is both arithmetic and geometric.

It is arithmetic if you consider the common difference to be 0.

It is geometric if you consider the common ratio to be 1.

Use your knowledge of arithmetic and geometric series to solve each problem.

9. Billy Kidd owns a horse ranch. Initially the number of horses in his herd was small but over time the number increased. In the spring of the first year he had 42 horses, in the spring of the second year he had 60 horses, and in the spring of the third year he had 78 horses. If the pattern continues, write an explicit formula to predict the number of horses Billy has in his herd any given spring.

The sequence is arithmetic.

I can use the formula, $a_n = a_1 + d(n - 1)$, where $a_1 = 42$ and $d = 18$.

$a_n = a_1 + d(n - 1)$

$a_n = 42 + 18(n - 1)$

$a_n = 18n + 24$

An explicit formula to predict the number of horses in Billy's herd any given spring is $a_n = 18n + 24$.

10. Jimmy does not like to write short stories. His teacher, Ms. Mundy, told him that he can become a better writer and enjoy writing more if he starts out limiting the number of words in his stories. As he writes more stories, he can increase the number of words per story. The table shows an overview of Ms. Mundy's suggestion. Determine how many words Jimmy uses in his first 5 stories.

Story	Number of Words per Story
1	75
2	150
3	300
4	600
5	1200

The series is geometric.

I can use Euclid's Method, $S_n = \frac{g_n(r) - g_1}{r - 1}$, where $n = 5$, $g_5 = 1200$, $r = 2$, and $g_1 = 75$, to determine how many words Jimmy uses in his first 5 stories.

$$S_5 = \frac{g_5(r) - g_1}{r - 1}$$

$$= \frac{1200(2) - 75}{2 - 1}$$

$$= 2325$$

Jimmy uses 2325 words in his first 5 stories.

Name ____ Date ______________

11. When Kodda received her first paycheck she opened a savings account and deposited \$10 in the account. Upon receiving her second paycheck she deposited \$12 in the account, and after receiving her third paycheck she deposited \$14 in the account. If this pattern continues, determine how much money Kodda deposits in her account when she receives her twelfth paycheck.

The sequence is arithmetic.

I can use the formula $a_n = a_1 + d(n - 1)$ where $n = 12$, $a_1 = 10$, and $d = 2$ to determine how much money Kodda deposits in her account when she receives her twelfth paycheck.

$a_n = a_1 + d(n - 1)$

$a_{12} = 10 + 2(12 - 1)$

$= 32$

Kodda will place \$32 in her account upon receiving her twelfth pay check.

12. Grace buys a powerboat with a 90 horsepower motor. She learns that the wear on the motor is measured by the number of hours the motor has been running. The table shows the number of hours the powerboat was used during the first six months. Determine how many hours the motor has been used in the first six months.

Month	Number of Hours Motor Used
1	21
2	32
3	43
4	54
5	65
6	76

The series is arithmetic.

I can use the formula $S_n = \frac{n(a_1 + a_n)}{2}$ where $n = 6$, $a_1 = 21$, and $a_6 = 76$, to determine how many hours the motor was used in the first six months.

$S_n = \frac{n(a_1 + a_n)}{2}$

$S_6 = \frac{6(21 + 76)}{2}$

$= 291$

The motor was used for 291 hours in the first six months.

13. Malinda and Otto are a song writing team. During their first year of collaboration they wrote only 3 songs but in each succeeding year they were able to triple the number of songs written each year. Determine how many songs they were able to write in the fourth year.

The sequence is geometric.

I can use the formula $g_n = g_1 \cdot r^{n-1}$ where $n = 4$, $g_1 = 3$, and $r = 3$, to determine how many songs they wrote in the fourth year.

$g_n = g_1 \cdot r^{n-1}$

$g_4 = 3 \cdot 3^{4-1}$

$= 81$

They were able to write 81 songs in the fourth year.

14. Mr. Greenwell, Maxwell's math teacher, challenged him to determine the sum of the following infinite sequence:

$$2, \frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \ldots$$

Maxwell thought about the challenge and then smiled. What did Maxwell know?

The infinite sequence is geometric, with a common ratio of $\frac{1}{5}$.

Maxwell can use the formula $S = \frac{g_1}{1 - r}$ where $g_1 = 2$, and $r = \frac{1}{5}$, to determine the sum of the sequence.

$S = \frac{g_1}{1 - r}$

$S = \frac{2}{1 - \frac{1}{5}}$

$= \frac{5}{2}$

Maxwell knew that the infinite sequence is convergent where $0 < r < 1$, so its sum is finite.

The sum of this sequence is $\frac{5}{2}$.

Name ____ Date ____________

15. At six years old, Tanya is a prolific block builder. Her mother walked into the playroom to find that Tanya had built a wall with blocks. On further inspection her mother realized that there was a pattern to how the wall was built. The top row contained 1 block, the next row contained 7 blocks, the next row contained 13 blocks, and the bottom row contained 55 blocks. In total, there were 10 rows. Determine how many blocks Tanya used to build her wall.

The series is arithmetic.

I can use the formula $S_n = \frac{n(a_1 + a_n)}{2}$ where $n = 10$, $a_1 = 1$, and $a_{10} = 55$, to determine how many blocks Tanya used.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_{10} = \frac{10(1 + 55)}{2}$$

$$= 280$$

Tanya used 280 blocks to build her wall.

16. Turner just learned to count by fives. Determine the fifteenth number in Turner's recitation of the multiples of 5.

The sequence is arithmetic.

I can use the formula $a_n = a_1 + d(n - 1)$ where $n = 15$, $a_1 = 5$, and $d = 5$, to determine the fifteenth number in the sequence.

$$a_n = a_1 + d(n - 1)$$

$$a_{15} = 5 + 5(15 - 1)$$

$$= 75$$

The fifteenth number in Turner's recitation is 75.

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