

Integrated Math 3. Chapter 9. Rational Functions. Day 6 Review

Name: \_\_\_\_\_

Complete the following steps for each rational function.

- Identify the discontinuities and tell what type each is.
- Give the end behavior using limits.
- Write the equations of any horizontal asymptotes.
- Find the x-intercept(s) and y-intercept.
- Make a table of values.
- Graph.
- Use limits to give the behavior of the function at the vertical asymptotes.
- Give the DOMAIN and RANGE of each function.

1.)  $g(x) = \frac{x^2 + 3x}{x + 2} = \frac{x(x+3)}{(x+2)}$

a.  $x = -2$  vertical asymptote

b.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$      $\lim_{x \rightarrow \infty} f(x) = \infty$

c. no horizontal asymptote

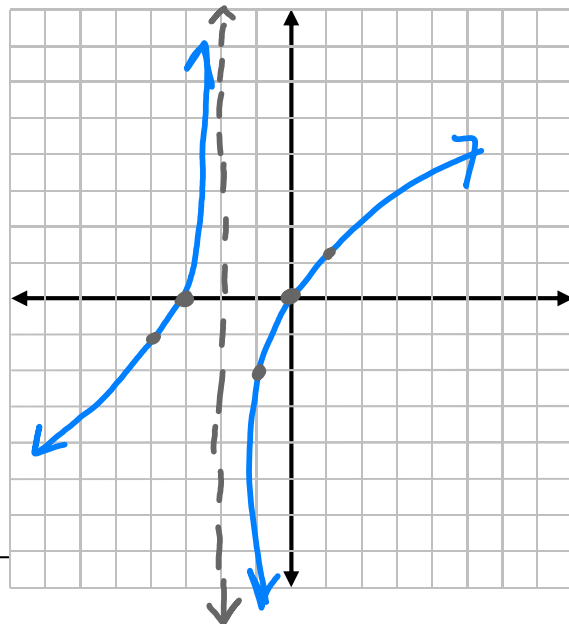
d.  $y = \frac{0}{2} = 0$  (0,0) /  $0 = \frac{x(x+3)}{x+2}$      $x(x+3) = 0$   
 $x = 0$  or  $-3$     (-3,0)

e.

x	-4	-3	-2	-1	0	1
y	-2	0	und	-2	0	1.3

g.  $\lim_{x \rightarrow -2^-} f(x) = \infty$      $\lim_{x \rightarrow -2^+} f(x) = -\infty$

h. D:  $(-\infty, -2) \cup (-2, \infty)$     R:  $(-\infty, \infty)$



2.)  $f(x) = \frac{4 - 2x}{x + 2} = \frac{-2(x-2)}{(x+2)}$

a.  $x = -2$  vertical asymptote

b.  $\lim_{x \rightarrow -\infty} f(x) = -2$      $\lim_{x \rightarrow \infty} f(x) = -2$

c.  $y = -2$

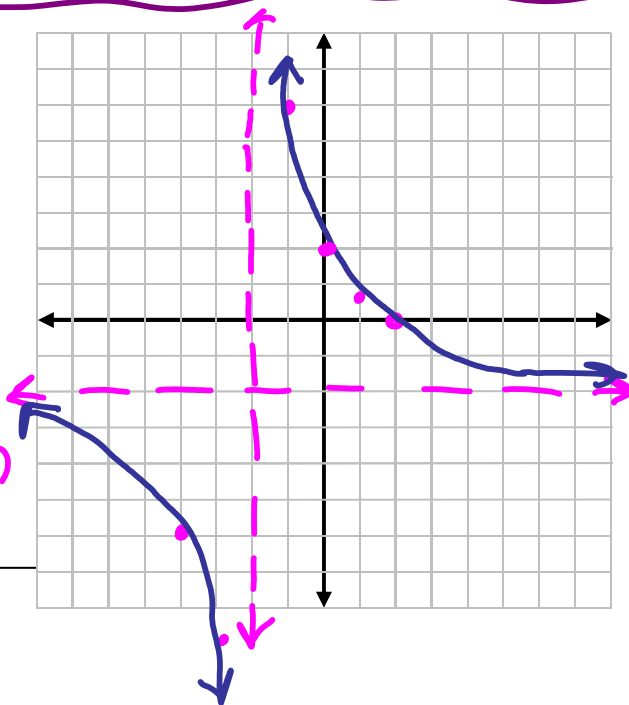
d.  $y = \frac{4}{2} = 2$  (0,2) /  $0 = \frac{-2(x-2)}{x+2}$      $-2(x-2) = 0$   
 $x - 2 = 0$   
 $x = 2$  (2,0)

e.

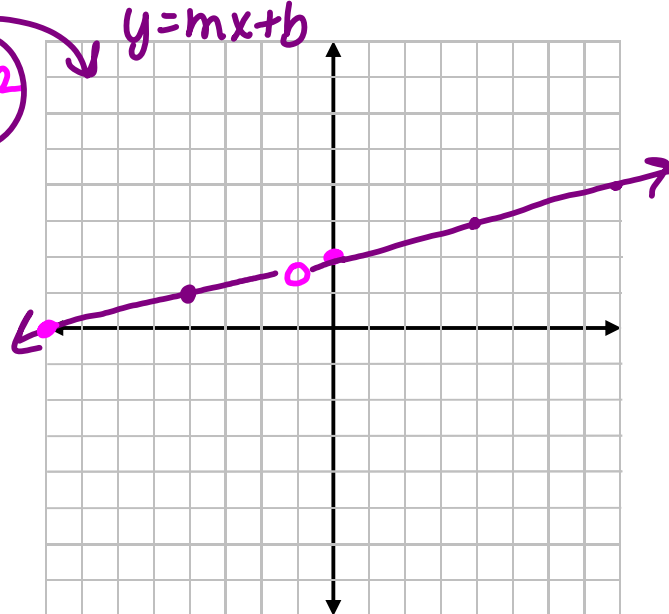
x	-4	-3	-2	-1	0	1	2
y	-6	-10	und	6	2	-6	0

g.  $\lim_{x \rightarrow -2^-} f(x) = -\infty$      $\lim_{x \rightarrow -2^+} f(x) = \infty$

h. D:  $(-\infty, -2) \cup (-2, \infty)$     R:  $(-\infty, -2) \cup (-2, \infty)$



3.)  $f(x) = \frac{x^2 + 9x + 8}{4x + 4} = \frac{(x+8)(x+1)}{4(x+1)} = \frac{x+8}{4}$  or  $\frac{1}{4}x + 2$



a.  $x = -1$  hole at  $(-1, 7/4) = (-1, 1.75)$

b.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$      $\lim_{x \rightarrow \infty} f(x) = \infty$

c. no h.A.

d.  $y = \frac{0+8}{4} = 2$  (0, 2) /  $\frac{0}{1} = \frac{x+8}{4}$      $0 = x+8$   
 $x = -8$   
 (-8, 0)

e. use  $y = mx + b$

x	
y	

g. no v.A.

h. D:  $(-\infty, -1) \cup (-1, \infty)$     R:  $(-\infty, 1.75) \cup (1.75, \infty)$

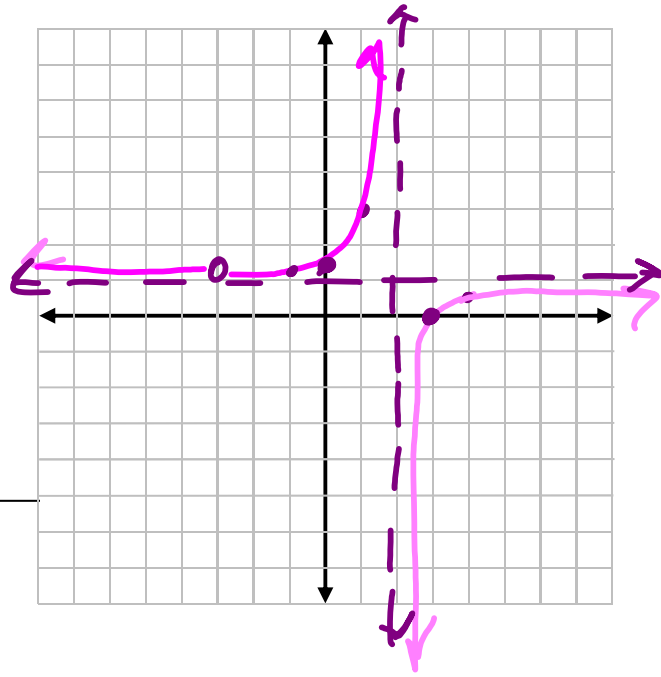
4.)  $h(x) = \frac{x^2 - 9}{x^2 + x - 6} = \frac{(x-3)(x+3)}{(x-2)(x+3)} = \frac{x-3}{x-2}$

a.  $x = 2$  vert. asympt.     $x = -3$  hole  $\frac{-3-3}{-3-2} = (-3, 1.2)$

b.  $\lim_{x \rightarrow -\infty} f(x) = 1$      $\lim_{x \rightarrow \infty} f(x) = 1$

c.  $y = 1$

d.  $y = \frac{0-3}{0-2} = 3/2$  (0, 1.5) /  $0 = \frac{x-3}{x-2}$      $0 = x-3$   
 $x = 3$   
 (3, 0)



e.

x	-1	0	1	2	3	4
y	1.3	1.5	2	und	0	0.5

g.  $\lim_{x \rightarrow 2^-} f(x) = \infty$      $\lim_{x \rightarrow 2^+} f(x) = -\infty$

h. D:  $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

R:  $(-\infty, 1) \cup (1, 1.2) \cup (1.2, \infty)$

5.)  $f(x) = \frac{3x^2}{x^2-9} = \frac{3x^2}{(x-3)(x+3)}$

a.  $x=3$   
 $x=-3$  } v.A.

b.  $\lim_{x \rightarrow -\infty} f(x) = 3$     $\lim_{x \rightarrow \infty} f(x) = 3$

c.  $y = 3$

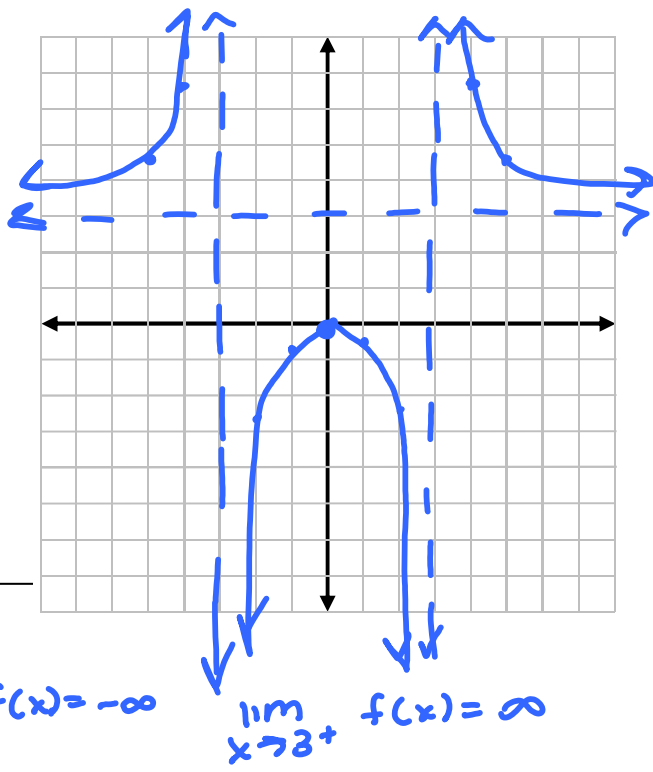
d.  $y = \frac{0}{-9} = 0$  (0,0)

e.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	4.7	6.9	und	-2.4	-.4	0	-.4	-2.9	und	6.9	4.7

g.  $\lim_{x \rightarrow -3^-} f(x) = \infty$     $\lim_{x \rightarrow -3^+} f(x) = -\infty$  /  $\lim_{x \rightarrow 3^-} f(x) = -\infty$     $\lim_{x \rightarrow 3^+} f(x) = \infty$

h. D:  $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$   
R:  $(-\infty, 0] \cup (3, \infty)$



6.)  $g(x) = \frac{3x-6}{x^2+2x-8} = \frac{3(x-2)}{(x-2)(x+4)} = \frac{3}{x+4}$

a.  $x = -4$  v.A.  
 $x = 2$  hole (2, 0.5)

b.  $\lim_{x \rightarrow -\infty} f(x) = 0$     $\lim_{x \rightarrow \infty} f(x) = 0$

c.  $y = 0$

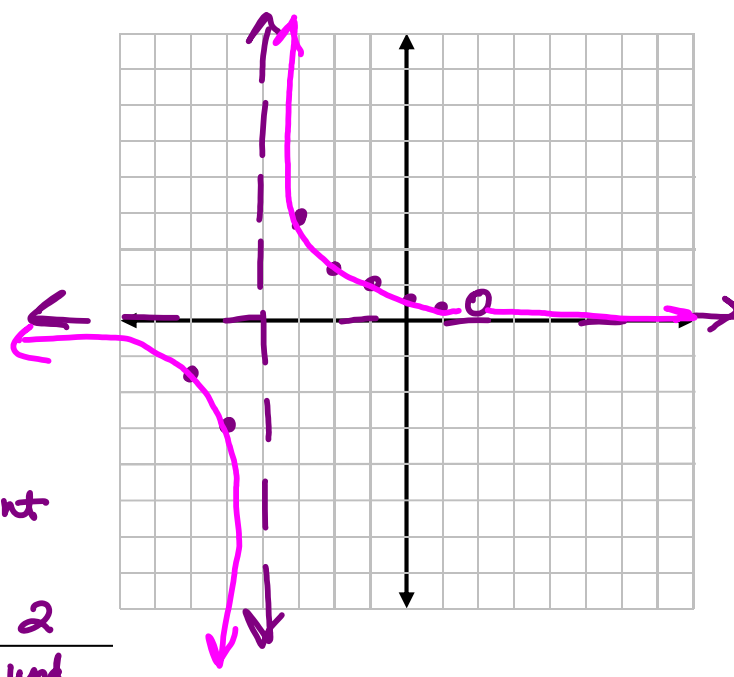
d.  $y = \frac{3}{0+4} = 3/4$  (0, .75) /  $\frac{0}{1} = \frac{3}{x+4}$  no x-int

e.

x	-6	-5	-4	-3	-2	-1	0	1	2
y	-1.5	-3	und	3	1.5	1	.75	.6	und

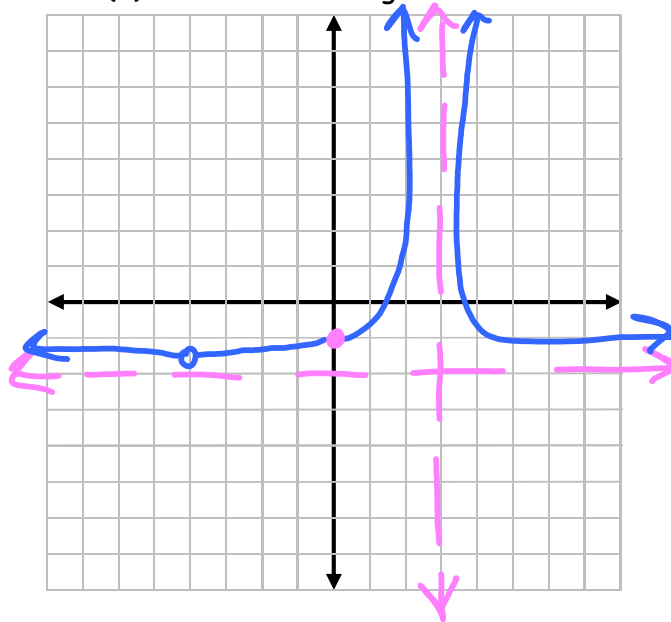
g.  $\lim_{x \rightarrow -4^-} f(x) = -\infty$     $\lim_{x \rightarrow -4^+} f(x) = \infty$

h. D:  $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$   
R:  $(-\infty, 0) \cup (0, 0.5) \cup (0.5, \infty)$

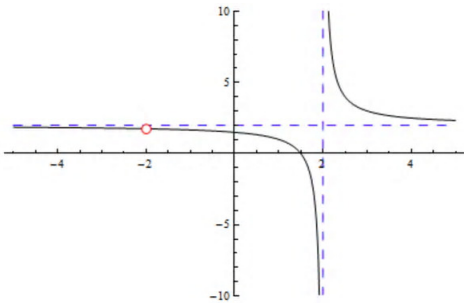


7.) Sketch the graph of a function  $f(x)$  with the following characteristics:

- $\lim_{x \rightarrow 3^-} f(x) = \infty$
- $\lim_{x \rightarrow 3^+} f(x) = \infty$
- $\lim_{x \rightarrow \infty} f(x) = -2$
- $\lim_{x \rightarrow -\infty} f(x) = -2$
- $f(-4) = \text{Undefined}$
- $f(0) = -1$



8.) For the rational function  $f(x)$  shown, USE LIMITS to give the **END BEHAVIOR** and the **BEHAVIOR AT THE VERTICAL ASYMPTOTE**. Also give the **DOMAIN** and **RANGE**. \*Hole is located at  $(-2, 1.75)$ \*



$$\lim_{x \rightarrow -\infty} f(x) = 2$$

$$\lim_{x \rightarrow \infty} f(x) = 2$$

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 2^+} f(x) = \infty$$

$$D: (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

$$R: (-\infty, 1.75) \cup (1.75, 2) \cup (2, \infty)$$