Integrated Math 3 Name KEY

Chapters 6-7 Test Review Packet

Section 6.1 Analyzing Polynomial Functions

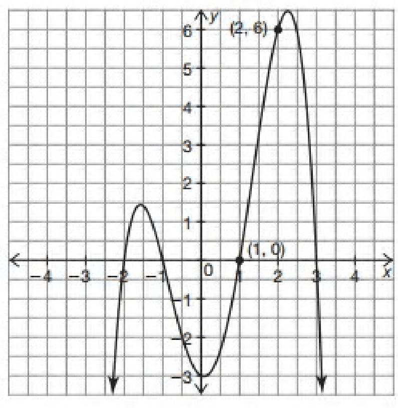
→ I can answer questions about a real-life scenario that is modeled by a polynomial function.

→ I can compute the average rate of change of a function over an interval of the domain.

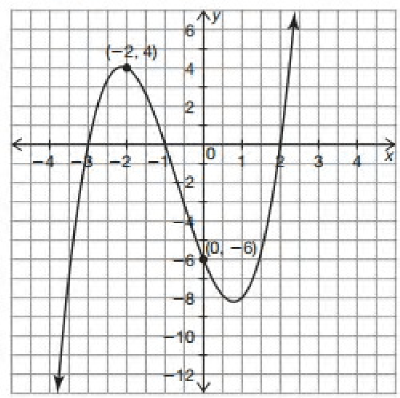
For #'s 1-2, determine the average rate of change for the given interval for each polynomial function.

1. (1, 2)

$\frac{6-0}{1}$



2. (−2, 0)



$\frac{-6-4}{0-2}$ $\frac{-16}{2}$

1. 6

2. −5

Section 6.2 Polynomial Division

→ I can use long division for dividing polynomials.

→ I can use the remainder of polynomial division to determine if a polynomial is a factor of another polynomial.

→ I can write a polynomial as a product of its factors, as determined by polynomial division.

→ I can use synthetic division to divide by a linear polynomial of the form x – c .

→ I can understand that synthetic division cannot be used to divide by non-linear polynomials.

For #'s 3-5, divide the following polynomials using either long or synthetic division.

3. $x^4 - 5x^3 - 8x^2 + 13x - 12 \div (x - 6)$

3. $x^3 + x^2 - 2x + x - \frac{6}{x-6}$

6	1	−5	−8	13	−12
	↓	6	6	−12	6
	1	1	−2	1	−6

3a. Is (x – 6) a factor of $x^4 - 5x^3 - 8x^2 + 13x - 12$? Yes/No Why or why not? Remainder is Not 0

4. $(5x^4 + 2x^3 - 9x + 12) \div (x^2 - 3x + 4)$

4. $5x^2+17x+31+\dfrac{16x-112}{x^2-3x+4}$

$$
\begin{array}{r}
5x^2+17x+31 \\
x^2-3x+4\;\overline{)\;5x^4+2x^3+0x^2-9x+12} \\
-5x^4+15x^3\mp20x^2 \qquad\qquad \\
\hline
17x^3-20x^2-9x \qquad \\
-17x^3+51x^2\mp68x \qquad \\
\hline
31x^2-77x+12 \\
-31x^2+93x\mp124
\end{array}
$$

$16x-112$

4a. Is $(x^2 - 3x + 4)$ a factor of $5x^4 + 2x^3 - 9x + 12$? Yes/No Why or why not? Remainder is Not 0

5. $(x^4 - x + 2) \div (x + 1)$

5. $x^3-x^2+x-2+\dfrac{4}{x+1}$

$$
\begin{array}{r|rrrrr}
-1 & 1 & 0 & 0 & -1 & 2 \\
 & \downarrow & -1 & 1 & -1 & 2 \\
\hline
 & 1 & -1 & 1 & -2 & 4
\end{array}
$$

5a. Is $(x + 1)$ a factor of $x^4 - x + 2$? Yes/No Why or why not? Remainder is Not 0

6. Describe in full sentences why you choose synthetic or long division for #'s 3-5. Be sure to discuss why one of the problems had to be done using long division.

I used synthetic division for #'s 3 & 5 because I divided by a linear factor. It was necessary to use long division for #4 because I had to divide by a quadratic.

Section 6.3 Factor Theorem and Remainder Theorem

→ I can verify that x – c is a factor of a polynomial f(x) by showing that f(c) = 0.

→ I can find f(c) using direct substitution or by finding the remainder with synthetic division.

7. Verify $(x + 1)$ is a factor of $x^3 + 2x^2 - x - 2$ by using the Remainder Theorem with Synthetic Division.

$$
\begin{array}{r|rrrr}
-1 & 1 & 2 & -1 & -2 \\
 & \downarrow & -1 & -1 & 2 \\
\hline
 & 1 & 1 & -2 & \boxed{0}
\end{array}
$$

← Remainder is 0!

Section 6.4 Factoring Higher Order Polynomials

→ I can factor any polynomial completely using the following strategies: Difference of two squares, trinomial into two binomials, grouping, sum/difference of cubes and multiple steps.

For #'s 8-13, factor the polynomial COMPLETELY!!

8. $x^4 + x^2 - 20$

$(x^2-4)(x^2+5)$

$(x+2)(x-2)(x^2+5)$

9. $y^3 - 8$

$(y-2)(y^2+2y+4)$

10. $(c^3 + 4c^2)(-9c - 36)$

$c^2(c+4)-9(c+4)$

$(c^2-9)(c+4)$

$(c+3)(c-3)(c+4)$

11. $x^4 - 36$

$(x^2-6)(x^2+6)$

12. $(x^3 + 6x^2)(+ 7x + 42)$

$x^2(x+6)+7(x+6)$

$(x^2+7)(x+6)$

13. $2w^3 + 54$

$2(w^3+27)$

$2(w+3)(w^2-3w+9)$

Section 6.5 Rational Root Theorem

→ I can make a list of possible rational roots for a polynomial function.

→ I can use synthetic division to verify that a number is a root of a polynomial.

→ I can find all complex roots of a polynomial function using synthetic division and/or the quadratic formula.

For #'s 14-15, list the possible rational zeros for the function.

14. $x^3 - 2x^2 + 27x - 12$

$$\frac{\pm1 \quad \pm2 \quad \pm3 \quad \pm4 \quad \pm6 \quad \pm12}{\pm1}$$

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

15. $2x^3 - 14x^2 + 5x + 8$

$$\frac{\pm1 \quad \pm2 \quad \pm4 \quad \pm8}{\pm1 \quad \pm2}$$

$\pm\frac{1}{2}, \pm1, \pm2, \pm4, \pm8$

Zeros: $2, -1+\sqrt{3}i, -1-\sqrt{3}i$

For #'s 16-17, find ALL the complex roots of a function.

16. $x^3 + 6x^2 + 7x + 42$ Deg: 3

One Zero From Calc: -6

$$\begin{array}{r|rrrr} -6 & 1 & 6 & 7 & 42 \\ & \downarrow & -6 & 0 & -42 \\ \hline & 1 & 0 & 7 & \boxed{0} \end{array}$$

$x^2 + 7 = 0$

$x^2 = -7$

$x = \pm\sqrt{-7}$

$x = \pm\sqrt{7}\,i$

Zeros: -6, $\sqrt{7}i$ & $-\sqrt{7}i$

17. $y^3 - 8$ Deg: 3

One Zero From Calc: 2

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & \downarrow & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & \boxed{0} \end{array}$$

$x^2 + 2x + 4$

$\frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$

$\frac{-2 \pm \sqrt{4-16}}{2}$

$\frac{-2 \pm \sqrt{-12}}{2}$

$\frac{-2 \pm \sqrt{4}\sqrt{3}i}{2}$

$\frac{-\cancel{2} \pm \cancel{2}\sqrt{3}i}{\cancel{2}}$

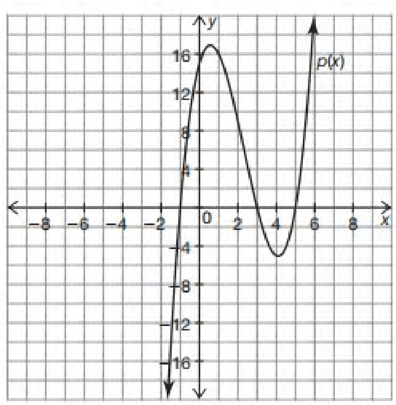
$-1 \pm \sqrt{3}i$

<u>Section 7.1 Solving Polynomial Inequalities</u>

→ I can solve a polynomial inequality graphically.

→ I can solve a polynomial inequality algebraically using the zeros of the function and a number line.

18. Analyze the following graph and identify when the x-values show f(x) < 0 and f(x) > 0.



f(x) < 0: $x < -1$ & $3 < x < 5$ $(-\infty, -1) \cup (3, 5)$

f(x) > 0: $-1 < x < 3$ & $x > 5$ $(-1, 3) \cup (5, \infty)$

19. Solve algebraically. Write in interval notation.

$2x^2 + 3x \geq 5$

$2x^2 + 3x - 5 \geq 0$

$(2x + 5)(x - 1) \geq 0$

$x = -\frac{5}{2}$ $x = 1$

-10 ✓   0 X   10 ✓

$-\frac{5}{2}$ 1

$(-\infty, -\frac{5}{2}] \cup [1, \infty)$

Section 7.3 Piecewise Functions

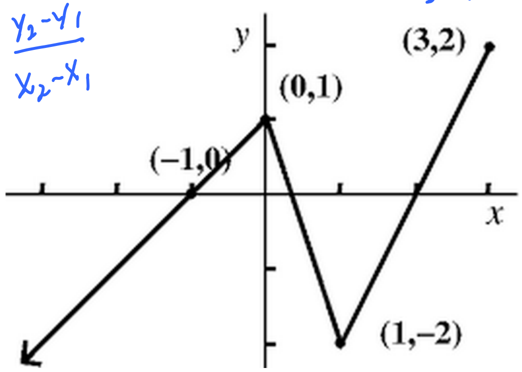
→ I can graph a piecewise function.

→ I can write a piecewise function given its graph.

20. Write a piecewise function to represent the following graph. Be sure to set your domain for each piece.

$\frac{2+2}{3-1} = \frac{4}{2} = 2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$



$f(x) = \begin{cases} x+1; & x \le 0 \\ -3x+1; & 0 < x \le 1 \\ 2x-4; & 1 < x \le 3 \end{cases}$

$\frac{-2-1}{1-0} = \frac{-3}{1} = -3$

Section 6.7 Pascal's Triangle and the Binomial Theorem

→ I can generate Pascal's Triangle.

→ I can use the values in Pascal's Triangle to expand binomials.

→ I can use the Binomial Theorem to expand binomials.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

21. Evaluate ${}_{19}C_4$ ____3876____

For #'s 22-23, use Pascal's Triangle or the Binomial Theorem to expand the binomial.

22. $(x + 3)^4$

$1a^4 \quad 4a^3b^1 \quad 6a^2b^2 \quad 4a^1b^3 \quad 1b^4$

$x^4 \quad 4 \cdot x^3 \cdot 3 \quad 6 \cdot x^2 \cdot 9 \quad 4 \cdot x \cdot 27 + 81$

$x^4 + 12x^3 + 54x^2 + 108x + 81$

23. $(2x - 5)^3$

$1a^3 \quad 3a^2b \quad 3ab^2 \quad 1b^3$

$(2x)^3 \quad 3(2x)^2(-5) \quad 3(2x)(-5)^2 \quad (-5)^3$

$8x^3 - 60x^2 + 150x - 125$

24. Find the coefficient for x^6 in the expansion of $(2x + 3)^{11}$.

${}_{11}C_6 \; a^6 b^5$

$462 \, (2x)^6 (3)^5$

$462 \cdot 64x^6 \cdot 243$

$7{,}185{,}024 \, x^6$