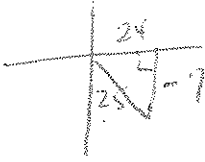


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Show work for all problems. Write out formula used.

1. If  $\cos \theta = \frac{24}{25}$  and  $\theta$  is in quadrant IV, find the five remaining trigonometric function values of  $\theta$ .



1)  $\sin \theta = \frac{-7}{25}$      $\csc \theta = \frac{25}{-7}$   
 $\cos \theta = \frac{24}{25}$      $\sec \theta = \frac{25}{24}$   
 $\tan \theta = \frac{-7}{24}$      $\cot \theta = \frac{24}{-7}$

2. Express  $\sec \theta - \sin \theta \tan \theta$  as a single function of  $\theta$ .

$$\frac{1}{\cos \theta} - \sin \theta \left( \frac{\sin \theta}{\cos \theta} \right) = \frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta}$$

2)  $\cos \theta$

3. Express  $\tan^2 x - \sec^2 x$  in terms of  $\sin x$  and  $\cos x$ , and simplify.

$$\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = -\frac{\cos^2 x}{\cos^2 x} = -1$$

3)  $-1$

4. Find the exact value of  $\cos \frac{5\pi}{12}$ .

$$\cos(45 + 30) = \cos 45 \cos 30 - \sin 45 \sin 30 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

4)  $\frac{\sqrt{6} - \sqrt{2}}{4}$

5. Express as a function of  $x$  alone.

(a)  $\cos(270^\circ - x)$     (b)  $\tan(\pi + x)$   
 $\cos 270^\circ \cos x + \sin 270^\circ \sin x = 0$   
 $\tan(\pi + x) = \frac{\tan \pi + \tan x}{1 - \tan \pi \tan x} = \tan x$

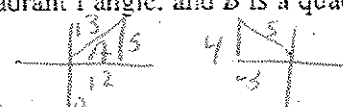
5) a)  $-\sin x$   
 b)  $\tan x$

6. Use a half-angle identity to find the exact value of  $\sin(-22.5^\circ)$ .

$$\sin(-22.5^\circ) = -\sqrt{\frac{1 - \cos 45^\circ}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = -\frac{\sqrt{2 - \sqrt{2}}}{2}$$

6)  $-\frac{\sqrt{2 - \sqrt{2}}}{2}$

8. Given that  $\sin A = \frac{5}{13}$ ,  $\cos B = -\frac{3}{5}$ ,  $A$  is a quadrant I angle, and  $B$  is a quadrant II angle, find each of the following.



(a)  $\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{5}{13} \left(-\frac{3}{5}\right) + \frac{12}{13} \left(\frac{4}{5}\right) = \frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$   
 (b)  $\cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{12}{13} \left(-\frac{3}{5}\right) - \frac{5}{13} \left(\frac{4}{5}\right) = \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65}$   
 (c)  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{5}{12} - \frac{-4}{3}}{1 + \frac{-20}{36}} = \frac{\frac{5}{12} + \frac{16}{12}}{\frac{16}{36}} = \frac{21}{16}$   
 (d) the quadrant of  $A + B$  is II

8) a)  $\frac{33}{65}$   
 b)  $-\frac{56}{65}$   
 c)  $\frac{63}{16}$   
 d) II

~~$\cos(A+B) = \cos A \cos B$~~   
 $\frac{15 + 48}{36 - 20} = \frac{63}{16}$



9. Given that  $\cos \theta = -\frac{3}{5}$  and  $\frac{\pi}{2} < \theta < \pi$ , find each of the following.

(a)  $\cos 2\theta = 2\left(-\frac{3}{5}\right)^2 - 1 = 2\left(\frac{9}{25}\right) - 1 = \frac{18}{25} - \frac{25}{25} = -\frac{7}{25}$

(b)  $\sin 2\theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$

(c)  $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2\left(\frac{4}{3}\right)}{1 - \frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}} = \frac{-\frac{8}{3} \cdot \frac{3}{-1}}{-\frac{7}{3}} = \frac{8}{-7} = -\frac{8}{7}$

(d)  $\cos \frac{1}{2}\theta = \pm \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

(e)  $\tan \frac{1}{2}\theta = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{1 - \frac{3}{5}}{1 + \frac{3}{5}}} = \sqrt{\frac{\frac{2}{5}}{\frac{8}{5}}} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

- 9) a)  $-\frac{7}{25}$   
 b)  $-\frac{24}{25}$   
 c)  $\frac{24}{7}$   
 d)  $\frac{\sqrt{5}}{5}$   
 e)  $2$

Answers:

[5.1] 1.  $\sin \theta = -\frac{7}{25}$

$\tan \theta = -\frac{7}{24}$ ,  $\cot \theta = -\frac{24}{7}$

$\sec \theta = \frac{25}{24}$ ,  $\csc \theta = -\frac{25}{7}$

2.  $\cos \theta = 3$ ,  $-1$

[5.3] 4.  $\frac{\sqrt{6} - \sqrt{2}}{4}$

[5.3, 5.4] 5. (a)  $\sin \alpha$   
 (b)  $\tan \alpha$

[5.6] 6.  $\frac{\sqrt{2} - \sqrt{2}}{2}$

[5.3, 5.4] 8. (a)  $\frac{33}{65}$  (b)  $-\frac{56}{65}$

(c)  $\frac{63}{16}$  (d)  $11$

[5.5, 5.6] 9. (a)  $-\frac{7}{25}$  (b)  $-\frac{24}{25}$

(c)  $\frac{24}{7}$  (d)  $\frac{\sqrt{5}}{5}$  (e)  $2$