Review 12.1-12.3 REVIEW Name Key

Exponential Functions

(12.1)--Geometric Sequence to Exponential function

Growth vs Decay

1. Construct an exponential function from each geometric sequence. Class Example: $a_n = 12 \cdot 3^{n-1}$

$f(n) = 12 \cdot 3^n \cdot 3^{-1} = \boxed{4 \cdot 3^n}$

a. $a_n = 1.2 \cdot 3^{n-1}$

$= 1.2(3)^n(3)^{-1}$

$= .4(3)^n$

b. $a_n = 64 \cdot \left(\frac{1}{4}\right)^{n-1}$

$= 64\left(\frac{1}{4}\right)^n\left(\frac{1}{4}\right)^{-1}$

a) $f(n) = .4(3)^n$

b) $f(n) = 256\left(\frac{1}{4}\right)^n$

2. Identify the base of the exponential function, determine whether each function represents exponential growth or exponential decay and explain why.

a. $h(x) = 2\left(\frac{4}{3}\right)^x$ **base:** $\frac{4}{3}$ **growth or decay** **why?** $b > 1$

b. $m(x) = 6(0.45)^x$ **base:** $.45$ **growth or decay** **why?** $0 < b < 1$

e. $g(x) = (6)5^{-x}$ **base:** $\frac{1}{5}$ **growth or decay** **why?** $0 < b < 1$

(12.1)---Half-Life

3. Class example: The **half-life** of a certain radioactive substance is 2 seconds. The initial amount of the substance is 500 grams. Write a function that expresses the amount of the substance remaining in the bloodstream, $A(t)$, as a function of time, *t*, in seconds.

$A(t) = 500\left(\frac{1}{2}\right)^{t/2}$

Use this function to predict how much of the substance remains after 6 seconds. 62.5g

$A(6) = 500\left(\frac{1}{2}\right)^{6/2}$

Use this function to predict how much of the substance remains after 10 seconds. 15.625g

$A(10) = 500\left(\frac{1}{2}\right)^{10/2}$

4. The **half-life** of a certain medication is 3 hours and there are 120 milligrams present initially. Write an exponential function that expresses the amount of the medication left, $A(t)$, as a function of time, *t,* in hours.

$A(t) = 120\left(\frac{1}{2}\right)^{t/3}$

Use this function to predict how much of the medication remains after 9 hours. 15mg

$A(9) = 120\left(\frac{1}{2}\right)^{9/3}$

Use this function to predict how much of the medication remains after 12 hours. 7.5mg

$A(12) = 120\left(\frac{1}{2}\right)^{12/3}$

5. The table shows the approximate amounts of two types of substances in a person's body each hour after the substance was taken. The starting dose for substance A was 300 grams, and the starting dose for substance B was 450 grams.

$b = .8$ $b = .7$

Time Taken (Hours)	Substance A (grams)	Substance B (grams)
0	300	450
1	240	315
2	192	220.5
3	154	154

$= 300(.8)^6$ $= 450(.7)^6$

$= 78.6$ $= 52.9$

a. Will a greater amount of medicine A or medicine B remain in the body after 6 hours? Explain your answer.

Medicine a will have a greater amount remaining in the body after 6 hours, since medicine A has 78.6 g while medicine B only has 52.9 grams.

b. Write equations that describe the amount of medicine $A(t)$ and the amount of medicine $B(t)$ that remain in the body after t hours.

$A(t) = 300(.8)^t$ $B(t) = 450(.7)^t$

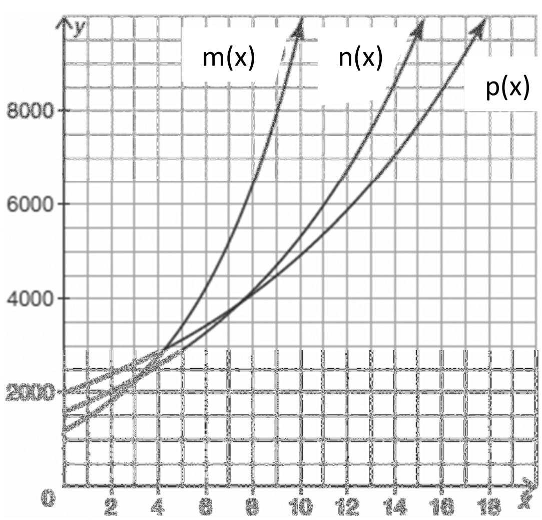
c. Calculate the amount of each medicine that remains in the body after 15 hours.

Round each answer to the nearest gram. Medicine A: 11 g

$A(15) = 300(.8)^{15}$ $B(15) = 450(.7)^{15}$

Medicine B: 2g

6. The figure shows the graphs of three exponential functions.



a. Which of the functions has the greatest rate of increase? m(x)

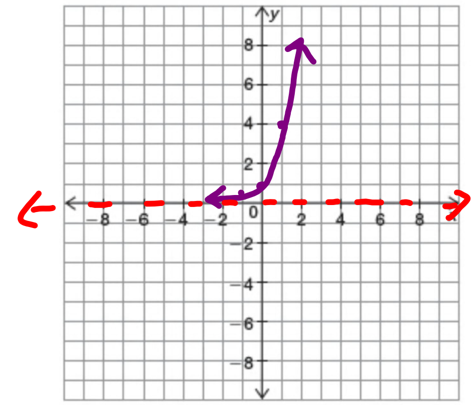
b. At which value of x do $n(x)$ and $p(x)$ have the same value? $x = 7.5$

c. Which function has the greatest initial value and what is that value?

Function p(x) has the greatest initial value of 2000

d. Which function has the greatest value at $x = 3$? p(x)

6. Graph and identify the characteristics of the function $(x) = 4^x$.



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptotes: $y = 0$

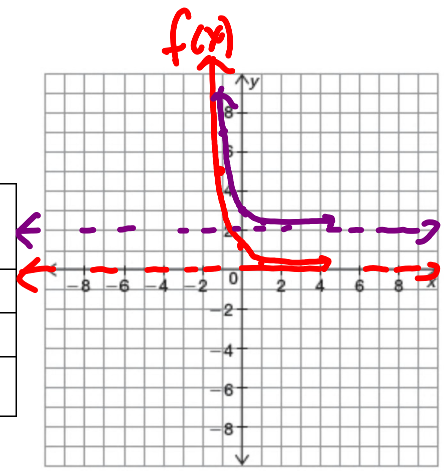
Intercepts: $(0, 1)$

x	y
-1	$\frac{1}{4}$
0	1
1	4

End behavior: $\lim_{x \to -\infty} m(x) = $ 0 $\lim_{x \to \infty} m(x) = \infty$

Intervals of increase or decrease: Increasing on the interval of $(-\infty, \infty)$

7. Given $f(x) = \left(\frac{1}{5}\right)^x$, graph $f(x)$ and $g(x) = f(x) + 2$.
Describe the following key characteristics of g(x).



Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

Asymptotes: $y = 2$

$(x, y+2)$

Reference points of $f(x)$	Corresponding points on $g(x)$
(-1, 5)	(-1, 7)
(0, 1)	(0, 3)
(1, $\frac{1}{5}$)	(1, $2\frac{1}{5}$)

End behavior: $\lim_{x \to -\infty} g(x) = \infty$ $\lim_{x \to \infty} g(x) = 2$

Intervals of increase or decrease: Decreasing on the interval of $(-\infty, \infty)$

Describe the transformations performed on f(x) to create g(x).
Then write an equation for g(x) in terms of f(x).

The graph of the f(x) functions is vertically translated up 2 units to create the g(x) function. $g(x) = \left(\frac{1}{5}\right)^x + 2$

Helpful formulas: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ or $N(t) = N_0 e^{rt}$

8. The population of Autin, Texas was approximately 790,000 in the year 2010 and has been continuously growing at a rate of 3.9% each year.

a. Write a function that describes the population as a function of the number of years, *t*, since 2010.

$N(t) = 790{,}000e^{.039t}$

b. Use this function to predict the population of Autin, Texas in the year 2015. b. 960,096 people
Assume the population continues to grow at the same rate.

$N(5) = 790{,}000e^{.039(5)}$

c. Use this function to estimate the population of Martin in 1998. c. 494,740 people
Assume the population growth had been the same since that time.

$2010 - 1998 = 12$ $N(12) = 790{,}000e^{.039(-12)}$

Helpful formulas: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ or $N(t) = N_0 e^{rt}$

9. An investor deposits $1,000 in an account that promises 5% interest compounded quarterly.

a. Write a function that describes the amount of money as a function of the number of years, *t*.

$A(t) = 1000\left(1 + \frac{.05}{4}\right)^{4t}$

b. Use this function to predict the amount of money will be in the account after seven years.

$A(7) = 1000\left(1 + \frac{.05}{4}\right)^{4 \cdot 7}$ b. $1415.99

c. Use this function to predict the amount of money will be in the account after fifteen years.

$A(15) = 1000\left(1 + \frac{.05}{4}\right)^{4 \cdot 15}$ c. $2,107.18

10. Write an equation that transforms the graph of the function $f(x) = 5^x$ in the following ways:

a. reflection across the *x*-axis a. $g(x) = -1 \cdot 5^x$

b. reflection across the *y*-axis b. $g(x) = 5^{-x}$

c. translated horizontally right six units c. $g(x) = 5^{(x-6)}$

11. For each, describe the graphical transformations on $f(x) = 3^x$ to produce $g(x)$.

a. $g(x) = 3^{-x} + 5$ The graph of g(x) is reflected over the y-axis and vertically translated

b. $g(x) = 3^{x+2} - 7$ The graph of the f(x) is translated horizontally left 2 units and vertically down 7 units to create the g(x) function.

c. $g(x) = -6 \cdot 3^{x-8}$ The graph of the f(x) function is reflected over the x-axis, vertically stretched by a factor of 6, and horizontally translated 8 units to the right.