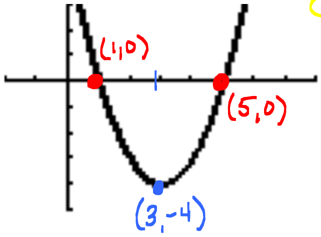
Integrated Math 3 Name:__________________________

Semester 1 Exam Review Period:_____

Chapter 4: Quadratics

1. Write the function in intercept form for the graph below. [-2, 8] by [-5, 3]



Vertex form

$y = a(x-h)^2 + k$

$y = a(x-3)^2 - 4$

$0 = a(1-3)^2 - 4$

$0 = a(-2)^2 - 4$

$0 = 4a - 4$

$4 = 4a$

$1 = a \longrightarrow y = 1(x-3)^2 - 4$

* Intercept form: $y = a(x-r_1)(x-r_2)$

$y = a(x-1)(x-5)$

$y = a(x-1)(x-5)$

$-4 = a(3-1)(3-5)$

$-4 = a(2)(-2)$

$-4 = a(-4)$

$1 = a$

$y = 1(x-1)(x-5)$

2. What is the vertex of the function $f(x) = 3x^2 - 6x + 5$?

vertex $= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \frac{6}{2(3)}$

$= 1$

$f(1) = 3(1)^2 - 6(1) + 5$

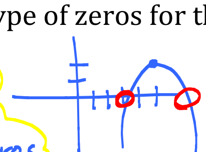
$= 3 - 6 + 5$

$= 2$

vertex @ (1, 2)

3. What are the number and type of zeros for the function $v(x) = -3(x-5)^2 + 2$?

reflect over x; right 5; up two



Since the vertex is in Q1 and opens down there will be 2 Real Zeros.

4. Simplify the expression. $(-5-6i)(-3+2i)$

Double Distribute — $15 - 10i + 18i - 12i^2$

Collect like Terms — $15 + 8i - 12i^2$

Evaluate $i^2 = -1$ — $15 + 8i - 12(-1)$

Multiply & Collect like Terms. — $15 + 8i + 12$

$27 + 8i$

5. Using a system of equations or a quadratic regression, write an equation in standard form with the points: (0, 4), (1, -2), (2, -4)

Using Grapher:

- Go to STAT edit, enter in 3 ordered pairs.
- Go to STAT calc, #5 Quad Reg, enter to find a, b, c's.
- $y = ax^2 + bx + c$, $a = 2$, $b = -8$, $c = 4$

$y = 2x^2 - 8x + 4$

$\boxed{OR}$

Using a 3x3 System of Equations:

$y = ax^2 + bx + c$

1. Plug in for (x, y) to get each of the 3 equations

① $-4 = a(2)^2 + b(2) + c$

$-4 = 4a + 2b + c$

② $-2 = a(1)^2 + b(1) + c$

$-2 = 1a + 1b + c$

③ $4 = a(0)^2 + b(0) + c$

$-4 = 4a + 2b + c$

$-2 = a + b + c$

$\boxed{4 = c}$

② Replace the variable c.

$-4 = 4a + 2b + 4$

$-2(-2 = a + b + 4)$

③ Eliminate a variable (a/b)

$4 = -2a - 2b - 8$

$0 = 2a - 4$

$4 = 2a$

$\boxed{2 = a}$

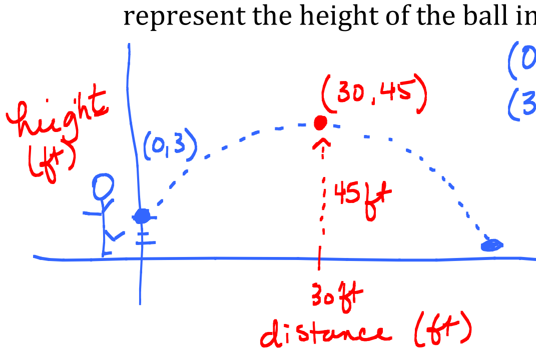
④ Solve for b.

$-2 = 2 + b + 4$

$-2 = 6 + b$

$\boxed{-8 = b}$

6. A football player kicks a football across the football field. The ball leaves the player's foot at a height of 3 feet. It follows a path in the shape of a parabola. At its highest point, the ball is a horizontal distance of 30 feet from the player and 45 feet above the ground. Write a function to represent the height of the ball in terms of its distance from the player.



(0,3)

(30,45)

$y = a(x-h)^2 + k$

$h(d) = a(d-30)^2 + 45$

$3 = a(0-30)^2 + 45$

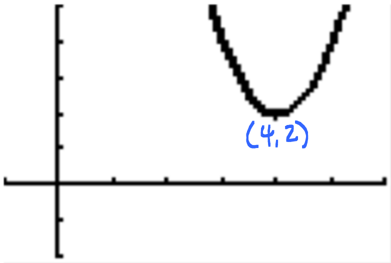
$3 = 900a + 45$

$-42 = 900a$

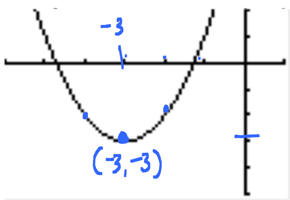
$\frac{-42}{900} \approx -.047 = a$

$h(d) = \frac{-7}{150}(d-30)^2 + 45$

7. The function graphed below has an absolute minimum or maximum of <u>2</u> at x = <u>4</u>.



8. Write a possible function to model the given graph in vertex form.



$y = 1(x+3)^2 - 3$

Chapter 5: Polynomial Functions

1. Determine the product, $h(x)$, of the given linear and quadratic factors.

$f(x) = 5x + 3$ and $g(x) = 2x^2 - 12x + 1$

$h(x) = (5x+3)(2x^2 - 12x + 1)$

$h(x) = 10x^3 - 60x^2 + 5x + 6x^2 - 36x + 3$

$h(x) = 10x^3 - 54x^2 - 31x + 3$

2. List the number of possible extrema for an 11th degree polynomial.

10, 8, 6, 4, 2, or 0

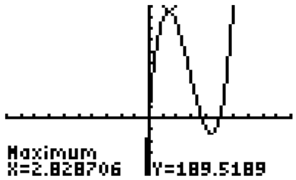
3. Reflect the function $f(x) = x^3$ about the x-axis and translate it 2 units to the right and 4 units up to produce $g(x)$. Write an equation that represents the function $g(x)$.

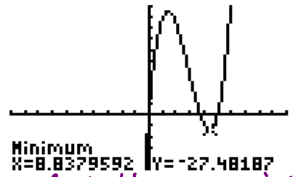
$g(x) = -(x-2)^3 + 4$

4. Determine if the function is even, odd, or neither. $f(x) = 3x^4 - 2x^2$

Even

5. The volume $V(x)$ of a box is defined by the function $V(x) = x(15 - 2x)(10 - x)$, where each factor represents a dimension of the box. Using the window [-20, 20] by [-100, 200],find the extrema for the volume situation .





Relative maximum at 189.5 when $X=2.8$

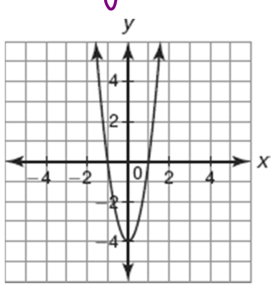
Relative minimum at -27.5 when $X=8.8$

6. The equation for $f(x)$ is given. The equation for the transformed function $g(x)$ in terms of $f(x)$ is also given. Describe the transformation(s) performed on $f(x)$ that produced $g(x)$.

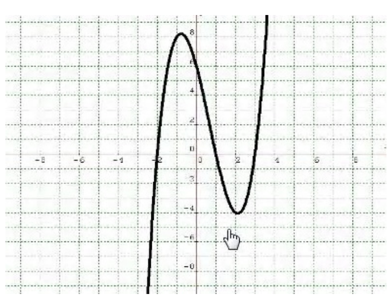
$f(x) = x^4; g(x) = \frac{1}{2}f(x+3) - 5$

The graph is vertically compressed/shrunk by a factor of 1/2. Shifted 3 units left and 5 units down.

7. Fill in the blanks. This function is <u>even</u> (even, odd, or neither) because it is symmetric to <u>y-axis</u> (x-axis, y-axis, origin, or NO symmetry).



8. Fill in the blanks. This function is <u>neither</u> (even, odd, or neither) because it is symmetric to <u>NO symmetry</u>(x-axis, y-axis, origin, or NO symmetry).

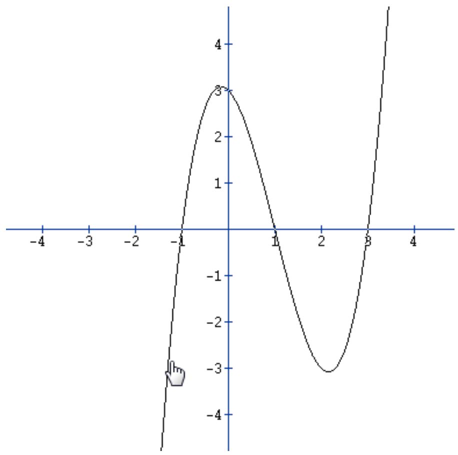


9. Describe the end behavior using limits. $f(x) = 5x^3 - 2x^3 + 6x - 4$

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

10. Write a function that represents the following graph.



$f(x) = (x+1)(x-1)(x-3)$

Chapters 6-7: Polynomial Equations

1. Analyze the functions. Determine which function has the higher degree

$f(x) = x^2 + 2x + 1$

Deg: 2

x	f(x)
-2	-9
-1	-2
0	-1
1	0
2	7

+7, +1, +1, +7

-6, +6, +6

+6, +6

Deg: 3 B/c 3rd Differences are the same

2. Solve the following inequality and write your answer in set-builder notation: $x^2 + 2x > 8$

$x^2 + 2x - 8$

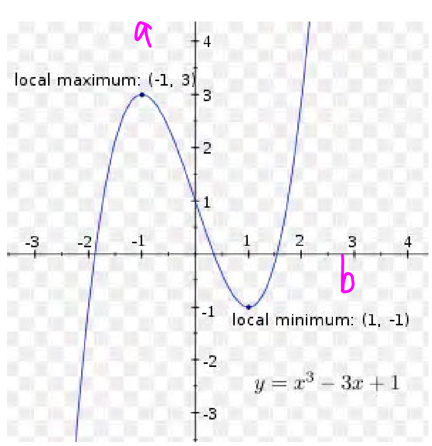
$(x+4)(x-2)$

-10, 0, 10

-4, 2

$x < -4$ or $x > 2$

3. Find the average rate of change between the local maximum and the local minimum.



$$\frac{f(b) - f(a)}{b - a} \quad \frac{-1-3}{1-(-1)} = \frac{-4}{2} = -2$$

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4. List all the POSSIBLE rational roots for the following function: $g(x) = 3x^3 - 2x^2 + 7x - 6$.

$$\frac{\text{Factors of Constant}}{\text{Factors of LC}} \quad \frac{\pm 1 \ \pm 2 \ \pm 3 \ \pm 6}{\pm 1 \ \pm 3}$$

$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6$

5. Use the Binomial Theorem or Pascal's Triangle to expand the following binomial: $(2x - 3)^4$.

$1a^4 \quad 4a^3b \quad 6a^2b^2 \quad 4ab^3 \quad 1b^4$

$1(2x)^4 \quad 4(2x)^3(-3) \quad 6(2x)^2(-3)^2 \quad 4(2x)(-3)^3 \quad 1(-3)^4$

$16x^4 \quad 4\cdot 8x^3\cdot -3 \quad 6\cdot 4x^2\cdot 9 \quad 8x\cdot -27$

$-96x^3 \quad 216x^2 \quad -216x \quad 81$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$16x^4 - 96x^3 + 216x^2 - 216x + 81$

6. Factor completely: $4x^4 - 16x^2$.

$4x^2(x^2 - 4)$

$4x^2(x+2)(x-2)$

7. Factor completely: $x^3 + 9x^2 - 9x - 81$.

$(x^3 + 9x^2)(-9x - 81)$

$x^2(x+9) - 9(x+9)$

$(x^2 - 9)(x+9)$

$(x+3)(x-3)(x+9)$

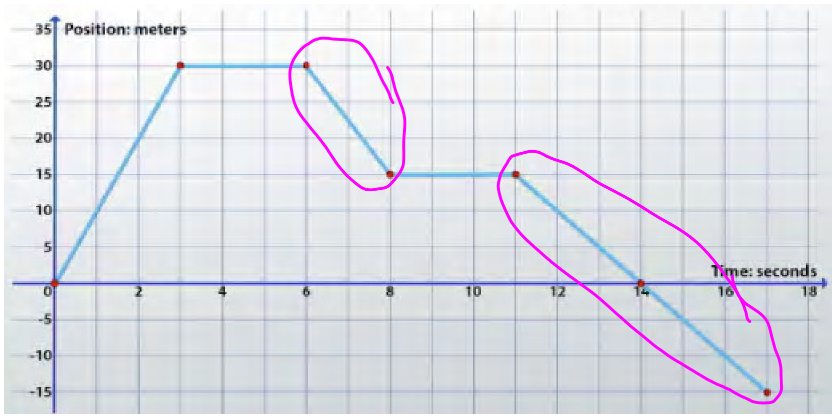
8. Given $(x - 2)$ is one of the factors of $f(x) = x^3 - 5x^2 + 2x + 8$, factor completely.

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & \downarrow & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

$x^2 - 3x - 4$

$(x-4)(x+1)(x-2)$

9.The following function models a situation of average velocity in regards to position over time. Use interval notation to describe all the intervals where the function is decreasing.



$(6, 8)$

$(11, 17)$

Chapter 9: Graphing Rational Functions

1. Given the function f(x)=$f(x) = \frac{x+1}{x^2-16}$, identify the vertical and horizontal asymptotes.

Vertical Asymptotes: x = 4 and x = - 4

Horizontal Asymptotes: y = 0

2. A) What is a rational function?

A rational function is any function that can be written as a ratio of two polynomials.

B) Determine whether each function is a rational function or not a rational function. If the function is not rational, explain why.

c) $f(x) = \frac{x^2+2x}{x+5}$ d) $q(x) = \frac{x^3 - 2x + 1}{\sqrt{x}}$

Yes, C is a rational function.

D is NOT a rational function because $\sqrt{x}$ is not polynomial.

3. Given the function $g(x) = \frac{x^2-2x-24}{x-6}$, where does the removable discontinuity occur?

The removable discontinuity is a hole at (6, 10).

$g(x) = \frac{\cancel{(x-6)}(x+4)}{\cancel{(x-6)}} = x+4 = 10$

$x = 6$

4. State the **RANGE** of the function $f(x) = \boxed{\frac{2x}{x-5}}$.

H.A.: $y = 2$

$\boxed{\text{Range: } (-\infty, 2) \cup (2, \infty)}$

5. Solve the equation $\frac{5x-6}{3x+4} = \frac{2}{1}$ and list any restrictions.

$x \neq -4/3$

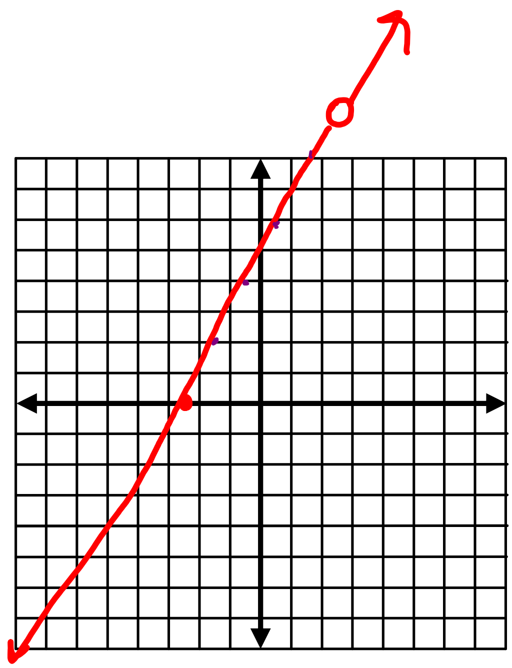
$2(3x+4) = 5x-6$

$6x + 8 = 5x - 6$

$x + 8 = -6$

$\boxed{x = -14}$

6. Analyze the graph of the function $f(x) = \frac{4x^2 - 25}{2x - 5}$. State the domain, range, and discontinuities. Graph the function.



$f(x) = \frac{\cancel{(2x-5)}(2x+5)}{\cancel{2x-5}} = 2x + 5 = 10$

$x = \frac{5}{2}$ or 2.5

D: $(-\infty, 2.5) \cup (2.5, \infty)$

R: $(-\infty, 10) \cup (10, \infty)$

Hole at $(2.5, 10)$

7. What is the domain of the function $h(x) = \frac{5}{3x^2 - 5x + 2}$?

$(3x+1)(x-2)$

D: $(-\infty, -1/3) \cup (-1/3, 2) \cup (2, \infty)$ or All real numbers except $x = \frac{-1}{3}$ and $x = 2$.

8. The Community Wellness Center charges a monthly membership fee of \$25, plus a one-time initiation fee of \$45 to join. Write an equation that gives the average cost y in dollars for x months of membership.

x = months of membership

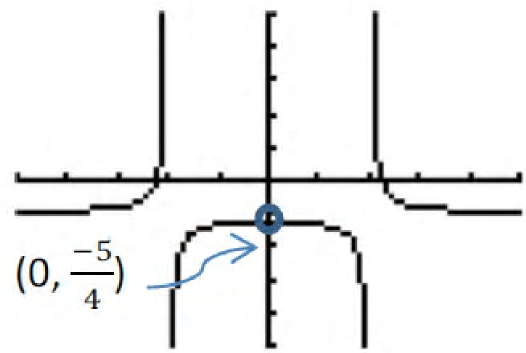
$y = \frac{45 + 25x}{x}$

9. Identify the vertical and horizontal asymptotes for the function $f(x) = \frac{3}{x^2 + 16}$.

Vertical Asymptote: None

Horizontal Asymptote: y = 0

10. Which function is represented by the graph shown?
It has vertical asymptotes at x = 2 and x = -2, horizontal asymptote at y= -1 and a hole in the graph at $(0, \frac{-5}{4})$.



a. $f(x) = x^2 - 4x$

b. $g(x) = x^2 - 4$

c. $h(x) = \frac{-x^3 + 5x}{x^3 - 4x}$ $\frac{-x(x^2-5)}{x(x+2)(x-2)}$

d. $j(x) = \frac{x^2 - 4}{x}$

Chapter 10: Rational Equations

1. What is (are) the solution(s) of the equation $\frac{8}{3x-2} = \frac{2}{x-1}$?

$8(x-1) = 2(3x-2)$ $\quad x \neq \frac{2}{3}, 1$

$8x - 8 = 6x - 4$

$2x = 4$

$\boxed{x = 2}$

2. Determine the restriction(s) for the value of x in the expression $\frac{1}{3x^2 + 12x}$.

$3x(x+4)$

$3x = 0 \qquad x + 4 = 0$

$\boxed{x \neq 0} \qquad \boxed{x \neq -4}$

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LCD: 20

3. Simplify: $\frac{4x}{5}+\frac{3x}{10}-\frac{7y}{4}$

$$\frac{4x(4)}{5(4)}+\frac{3x(2)}{10(2)}-\frac{7y(5)}{4(5)}=\frac{16x+6x-35y}{20}=\boxed{\frac{22x-35y}{20}}$$

4. Simplify: $\frac{3x}{6x+42}\div\frac{12x^3}{2x^2-98}$

$$\frac{\cancel{3x}^{1}}{\cancel{6}_{2}(\cancel{x+7})}\cdot\frac{\cancel{2}^{1}(x-7)(\cancel{x+7})}{\cancel{12}_{6}\cancel{x^3}_{x^2}}=\boxed{\frac{x-7}{12x^2}}$$

LCD: 3x

5. Simplify: $\frac{\left(\frac{x}{3}-6\right)3x}{\left(\frac{4}{x}+10\right)3x}=\boxed{\frac{x^2-18x}{12+30x}=\frac{x(x-18)}{6(5x+2)}}$

6. Solve the equation $\frac{x}{2}=\frac{x^2-3x}{4}$.

$2(x^2-3x)=4x$
$2x^2-6x=4x$
$2x^2-10x=0$

$2x(x-5)=0$
$\boxed{x=0 \text{ or } x=5}$

7. For which of the following would you need to determine and use the least common denominator (LCD) in order to calculate?

a. $\frac{2}{xy}+\frac{16}{xy}$ → already same denominator

b. $\frac{14}{x}\cdot\frac{5}{6x}$

c. $\frac{5}{2x}\div\frac{8}{x}$

d. $\frac{8}{9x}+\frac{3}{x}$

8. For her birthday party Kathryn mixed together 3 gal. of Brand A fruit punch and 6 gal. of Brand B fruit punch. Brand A contains 17% fruit juice and Brand B contains 26% fruit juice. What percent of the mixture is fruit juice?

$3(.17) + 6(.26) = 9x$

$2.07 = 9x$

$x = 0.23$

$\boxed{23\%}$

9. Simplify $\frac{48x^5y^3}{y^4} \cdot \frac{x^2y}{6x^3y^2} \cdot \frac{y}{12x^2}$

$\frac{48x^7y^5}{72x^5y^6} = \boxed{\frac{2x^2}{3y}}$

10. Solve the equation $(x-2)(x+2)\left(\frac{5}{x-2} - \frac{2}{x+2}\right) = \left(\frac{26}{x^2-4}\right)(x-2)(x+2)$.

LCD: $(x-2)(x+2)$

$x \neq 2, -2$

$5(x+2) - 2(x-2) = 26$

$5x + 10 - 2x + 4 = 26$

$3x + 14 = 26$

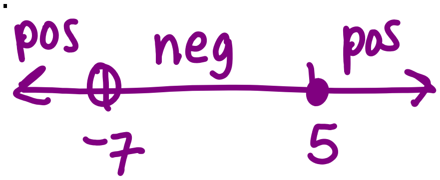
$3x = 12$

$\boxed{x = 4}$

11. Solve the inequality $\frac{x-5}{x+7} > 0$.

zero : 5

discontinuity: -7



$\boxed{(-\infty, 7) \cup (5, \infty)}$

Chapter 11: Inverse and Radical Functions

1. Write an equation that shifts the graph of the function $f(x) = \sqrt{x}$ to the right 2 units.

$K(x) = \sqrt{x-2}$

2. Simplify the expression $\sqrt{25x^6y^2z^4}$ for all real numbers x, y, and z.

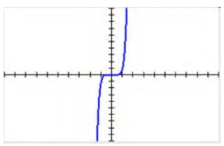
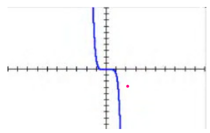
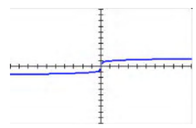
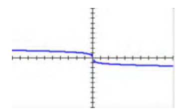
$5z^2|x^3y|$

3. The relationship between the length of a pendulum L (in feet) and its period T (in seconds) is modeled by the equation $T = 2\pi\sqrt{\frac{L}{32}}$. To the nearest foot, find the length of a pendulum with period 40 seconds.

$\frac{40}{2\pi} = \frac{\cancel{2\pi}\sqrt{\frac{L}{32}}}{\cancel{2\pi}}$ $\left(6.37 = \sqrt{\frac{L}{32}}\right)^2 = 32 \cdot 40.58 = \frac{L}{\cancel{32}} \cdot 32$

$= \boxed{1299 \text{ feet}}$

4. Which is the graph of the inverse of the function $C(x) = x^7$? $\sqrt[7]{x} = \sqrt[7]{y^7} \rightarrow \boxed{y = \sqrt[7]{x}}$

a. 
b. 
c. 
d. 

5. Find the extraneous solution of the equation $-5 - \sqrt{x+7} = -x$.

$\cancel{-5} - \sqrt{x+7} = -x$
$\cancel{+5} \qquad +5$
$-1(-\sqrt{x+7} = -x+5)$
$(\sqrt{x+7} = x-5)^2$

$x+7 = (x-5)(x-5)$
$\cancel{x+7} = x^2 - 10x + 25$
$\cancel{-x-7} \qquad - x - 7$
$0 = x^2 - 11x + 18$

$0 = (x-9)(x-2)$
$x = 9 \quad x = 2$
Plug values in original.
$\boxed{x = 2}$

6. Find the solution of the equation $2\sqrt[3]{x+1} + 8 = 0$.

$\cancel{-8} \; -8$
$\frac{2\sqrt[3]{x+1}}{2} = \frac{-8}{2}$
$(\sqrt[3]{x+1} = -4)^3$
$x + 1 = -64$
$-1 \qquad -1$
$\boxed{x = -65}$

7. Describe how the graphs of a function and its inverse are related.

The graphs are symmetric about the line $y=x$

8. Write $\sqrt[7]{x^5}$ in exponential form.

$x^{\frac{5}{7}}$

9. Which equation has no solution?

a. $\sqrt{x-3} = 2$ b. $\sqrt[3]{x} + 4 = 0$ c. $\sqrt{2x-5} + 11 = 4$ d. $\sqrt{2x+8} - 4 = 1$

$-11 \quad -11$
$\sqrt{2x-5} = -7$
* cannot get a negative when $\sqrt{\ }$